fluctuations of the current in an open chain

Vincent PASQUIER

IPhT Saclay

Collaboration with Alexandre Lazarescu

Open XXZ with nonconserving boundaries

Many people made important contribution to the subject:

Open XXZ with nonconserving boundaries

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From integrability point of view:

- Fabian Essler
- Ohlger Frahm
- Nicolai Kitanine
- Raphael Nepomechie
- Yupeng Wang
- Christian Korff

Open XXZ with nonconserving boundaries

From nonequilibrium physics

- Bernard Derrida
- Joel Lebowitz
- Martin Evans
- Kirone Mallick
- Sylvain Prolhac

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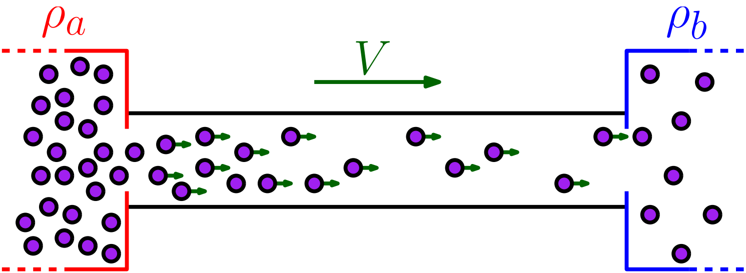
This Talk based on:
Alexandre thesis: arXiv 1311.7370
A.Lazarescu and V.P:arXiv 1403.6963

Introduction

Particles propagating and interacting between reservoirs.



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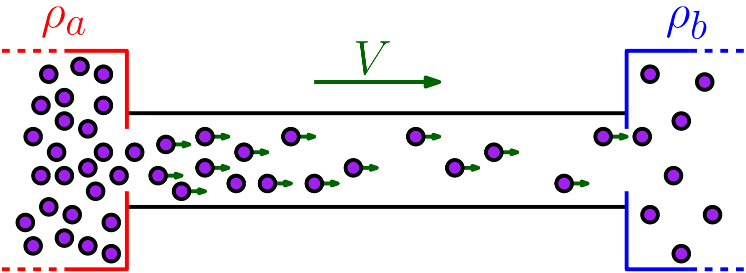
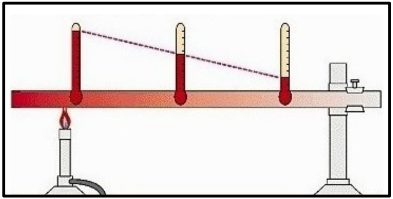
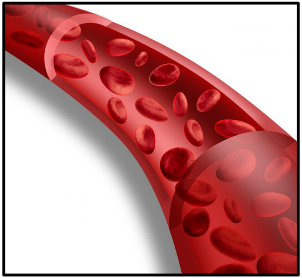


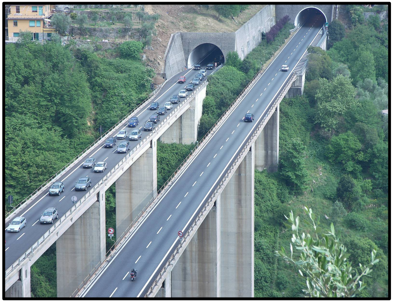
The field or the unbalance between reservoirs $\Rightarrow$ macroscopic current particles (entropy production).

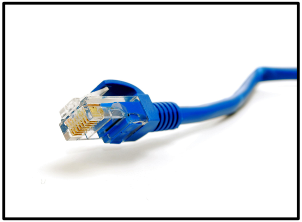
Introduction

examples:







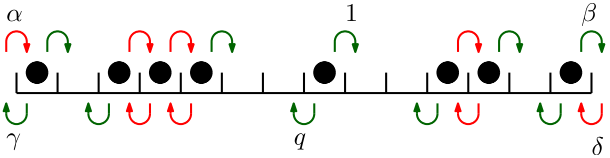


Plan

- Introduction

- I – Open ASEP : Definition of stationary state
- II – From Matrix to Bethe Ansatz

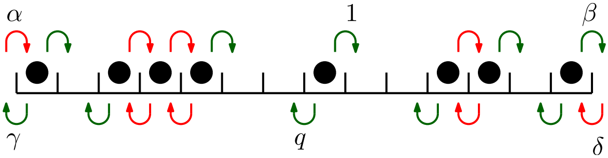
- Conclusion



Asymmetric Simple Exclusion Process, ASEP :

- One dimensional lattice L
- enter left with rate α and right with rate δ
- leave right with rate β and left with rate γ
- jump with rate $p = 1$ to the right et $q < 1$ to the left (if target is unoccupied)

I - Asymetric exclusion model



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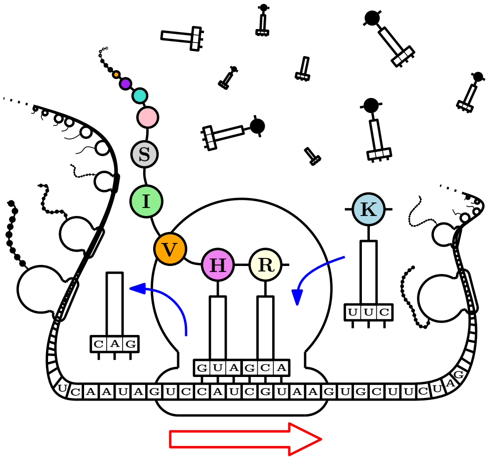
totally asymmetric (TASEP): $q = \gamma = \delta = 0$

I - Asymetric exclusion model

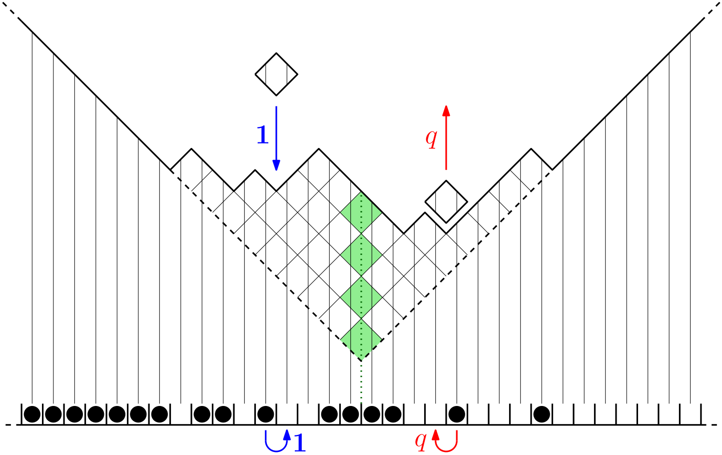
Invented for biology.



[C. T. MacDonald, J. H. Gibbs, A. C. Pipkin, **Biopolymers**, 1968]

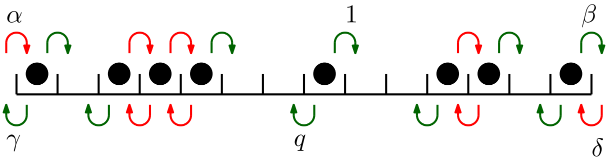
I - Asymetric exclusion model

Related to other models like interface growth.



[M. Kardar, G. Parisi, Y.-C. Zhang, **P. R. L.**, 1986]

I - Motivation



why ?

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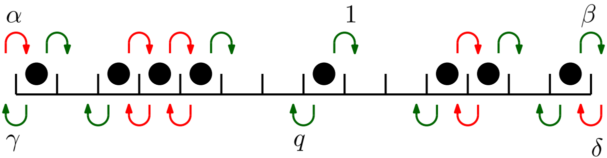

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- Simplicity

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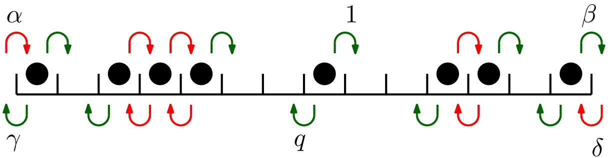

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- Simplicity
- Related to other topics (quantum XXZ, directed polymers, traffic, random matrices, orthogonal polynomials ...)

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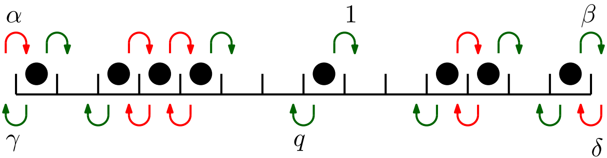

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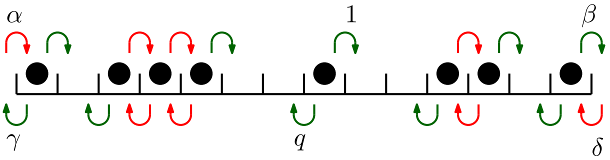

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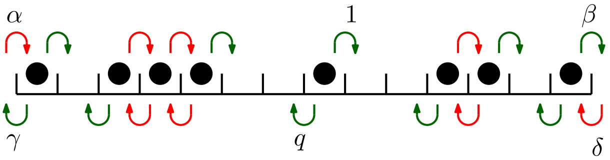
Interesting quantity :



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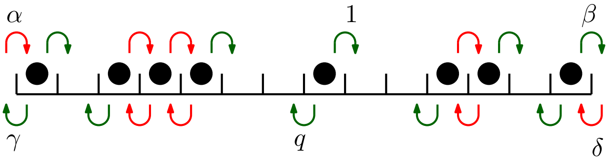
Interesting quantity : The macroscopic current.

I - Master equation



The vector $|P_t\rangle$ components are the probabilities to observe a configuration $\mathcal{C}$ at time t. It obeys the master equation:

$$\frac{d}{dt}|P_t\rangle = M|P_t\rangle$$

with M is a sum of local matrices M_i (one for each link i) (in the basis $\{0,1\}$ and $\{00,01,10,11\}$)

$$M_0 = \begin{bmatrix} -\alpha & \gamma \\ \alpha & -\gamma \end{bmatrix}, \quad M_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -q & 1 & 0 \\ 0 & q & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_L = \begin{bmatrix} -\delta & \beta \\ \delta & -\beta \end{bmatrix}$$

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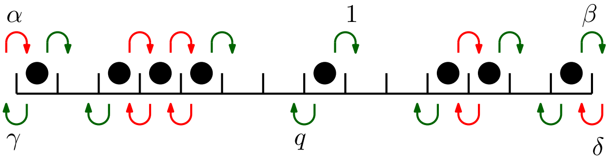

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I - Matrix Ansatz

[B. Derrida, M. R. Evans, V. Hakim, V.P., **J. Phys. A**, 1993]

Define matrices D et E and states $\langle\langle W\|$ et $\|V\rangle\rangle$ obeying

$$
\begin{aligned}
DE - qED &= (1-q)(D+E) \\
\langle\langle W\|(\alpha E - \gamma D) &= (1-q)\langle\langle W\| \\
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$$P^\star(\mathcal{C}) = \frac{1}{Z_L}\langle\langle W \| \prod_{i=1}^{L} \left(n_i D + (1-n_i)E\right) \| V \rangle\rangle$$

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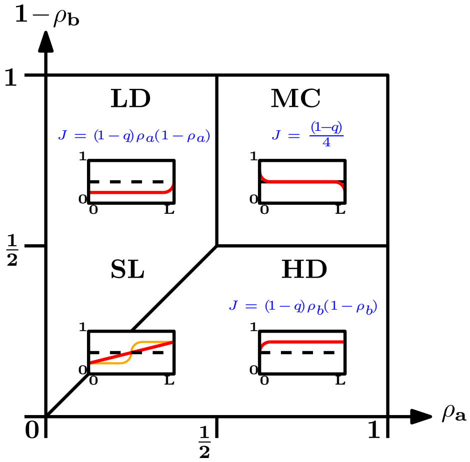
One deduces the mean current $J = (1-q)\frac{Z_{L-1}}{Z_L}$

As a function of $\rho_a(\alpha, \gamma, q)$ and $\rho_b(\beta, \delta, q)$:



In each case : $J = (1-q)\rho_c(1-\rho_c)$.

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History $\mathcal{C}(t)$ with a current $Q_t[\mathcal{C}]$.

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Large deviations for J_t

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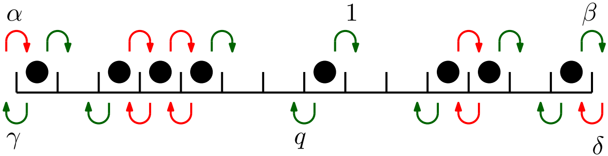
Gärtner-Ellis

Theorem:

$$E(\mu) = \mu j^{\star} - g(j^{\star}) \quad , \quad \frac{\mathrm{d}}{\mathrm{d}j} g(j^{\star}) = \mu$$

II - Measure the current



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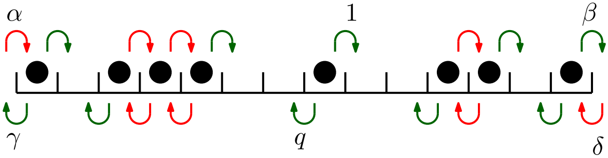

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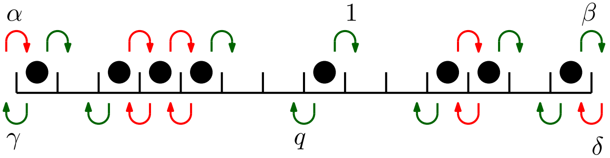

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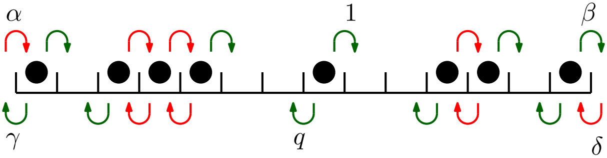

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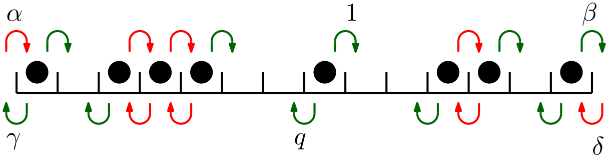

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$E(\mu)$ largest eigenvalue M_μ

Corresponding eigenvectors : $|P_\mu\rangle$ and $\langle\tilde{P}_\mu|$

IV - Strategy

- Obtain a Two Parameters $T(u, v)$ family of transfer matrices commuting with the Hamiltonian

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- Use the above equation with $k = 1, 2$ to obtain the spectrum.

Factorized T(u,v)

- For any u_- and u_+, the transfer matrix is a product of L matrices:

$$T(u,v) = tr(L(u_-,u_+).L(u_-,u_+)\cdots .L(u_-,u_+))$$

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- After projection:

$$L = \begin{pmatrix} 1 - u_+ X & -Y^{-1}(1-X) \\ Y(1 - u_+ u_- X) & u_- X \end{pmatrix}$$

Factorized T(u,v)

- Consequence of Yang-Baxter

$$T(u_-, u_+)T(v_-, v_+)$$

Invariant under permutations u_-, u_+, v_-, v_+, u_-, v_-, u_+, v_+.

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So, you can factorize:

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- If $u_- u_+ = q^{1-k}$ T becomes triangular:

$$T(q^{1-k}/v, v) = \begin{pmatrix} t^{(k)} & * \\ . & T(q/v, q^k v) \end{pmatrix}$$

Bethe Ansatz

- For any integer $k \geq 1$, we find that

$$\boxed{P(v)Q(1/q^{k-1}v) = t^{(k)}(v) + \mathrm{e}^{-k\mu}P(q^k v)Q(q/v)}$$

where $t^{(k)}(x)$ is the Bethe transfer matrix with a k-dimensional auxiliary space.

- First two relations:

$$P(v)Q(1/v) = h(v) + \mathrm{e}^{-\mu}P(qv)Q(q/v)$$
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- which combine into

$$t^{(2)}(v)Q(1/v) = h(v)Q(1/qv) + \mathrm{e}^{-2\mu}h(qv)Q(q/v)$$

First relation is the the Wronsky identity.

Solving Bethe Anzats

- Wronsky

$$\boxed{P(v)Q(1/v) = h(v) + \mathrm{e}^{-\mu} P(qv)Q(q/v)}$$

$$\boxed{h(v) = \frac{(1+v)^L}{v^N}}$$

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- P(v) degree $[0, L-N]$, $Q(v)$ degree $[-N, 0]$, h(v) degree $[-N, L]$, Laurent polynomials.

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- $L+1$ equations and $L+1$ unknown ($P(0) = 1$ monique).
- Current generator $P = Q = 1$ for $\mu = 1$ Translates into nonlinear equation with $P(v)$ entire without zeros in and $Q(1/v)$ entire without zeros out of the unit disc.

$$e^{-W} + 1 = Bhe^{-X*W}$$

with X Szego Kernel. $B = e^{\mu}/Q(0)$ is an expansion parameter.

- Open chain - Commuting Matrices

- For any u and v, obtain a transfer matrix which commutes with the Hamiltonian:

$$[\ M_\mu\ ,\ U_\mu(u)T_\mu(v)\ ] = 0$$

- Open chain - Commuting Matrices

- For any u and v, obtain a transfer matrix which commutes with the Hamiltonian:

$$[\ M_\mu\ ,\ U_\mu(u) T_\mu(v)\ ] = 0$$

- The transfer matrix is a product of L matrices sandwiched between states:

$$U(u) = \langle V | L(u,u).L(u,u) \cdots .L(u,u) | W \rangle$$

$$T(v) = \langle V' | L(v,v).L(v,v) \cdots .L(v,v) | W' \rangle$$

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• Boundary states are coherent states determined by the boundary Hamiltonians.

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From U_μ and T_μ, we construct 'Q-operators' P and Q, which commute:

$$P(u) = U_\mu(u)\Big[U_\mu(0)\Big]^{-1} \quad , \quad Q(v) = U_\mu(0)\,T_\mu(v)$$

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- For any integer $k \geq 1$, we find that

$$\boxed{P(v)Q(1/q^{k-1}v) = t^{(k)}(v) + \mathrm{e}^{-2k\mu} P(q^k v)Q(q/v)}$$

where $t^{(k)}(v)$ is the Bethe transfer matrix with a k-dimensional auxiliary space.

Solving Bethe Anzats for open chain

- Wronsky

$$\boxed{P(v)Q(1/v) = F(v) + e^{-2\mu} P(qv)Q(q/v)}$$

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- P(v) entire in Q(v) out the unit disc.

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- P(v) entire in Q(v) out the unit disc.
- Current generator $P = Q = 1$ for $\mu = 1$ Translates into nonlinear equation:

$$e^{-W} = 1 - BFe^{-X*W}$$

Exactly the same equation as in the closed case.

- Bethe Ansatz

- First two relations:

$$P(v)Q(1/v) = F(v) + \mathrm{e}^{-2\mu}P(qv)Q(q/v)$$
$$P(v)Q(1/qv) = t^{(2)}(v) + \mathrm{e}^{-4\mu}P(q^2v)Q(q/v)$$

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II - Cumulants : ASEP [Gorissen, Lazarescu, Mallick, Vanderzande; P.R.L., 2012]

$$E(\mu) = -(1-q) \oint_S \frac{dz}{\imath 2\pi (1+z)^2} W_B(z) \qquad = -(1-q) \sum_{k=1}^{\infty} D_k \frac{B^k}{k}$$

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- $j \to \infty$

$$g(j) \sim L\ j(\log(j\pi) - 1)$$

Current fluctuations: large size limit

For the TASEP:

$$C_k = \frac{1}{2} \oint_S \frac{dz}{\imath 2\pi z} F^k(z)$$

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For L large, there's a saddle point at $z = 1$.

- if $a < 1$ and $b < 1$, i.e. $\rho_a > 1/2$ and $\rho_b < 1/2$ (MC), then we can integrate on the unit circle.
- otherwise, extra pole at $max[a, b]$, which dominates.

Conclusion

We have:

- Constructed a Q matrix commuting wth the ASEP Hamiltonian
- Used it to obtain functional equations equivalent to the Bethe Ansatz
- Used analiticity properties to obtain a particular eigenvalue yielding the large deviation function.

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- Used it to obtain functional equations equivalent to the Bethe Ansatz
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We would like to:

- Explore more ahead our results especially in the maximal current phase.
- Obtain the correct analiticity properties to solve XXZ chain and other models with open boundary.
- Simplify our derivations.

Thank you!

Bethe Ansatz

- K Matrix:

$$\boxed{(K_\psi)_{v_1,v_2} = \int_0^\infty dt t^{v_1+v_2-1} G_{v_1,v_2}(t,\mu,x,r_1,r_2) G_{v_1,v_2}(t,-\mu,y,r_1,r_2)}$$

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On which you can observe the symmetry:

$$K(x,y,\mu) = K(y,x,-\mu)$$

- Commutation:

$$\boxed{\Lambda(v_1, v_2)\Lambda(v_3, v_4)}$$

Invariant under S_4 permutations.

IV - Bethe Ansatz

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Invariant under S_4 permutations.

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Then, use factorization to write:

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Then, use factorization to write:

$$\bar{\Lambda}(v) = P(1 - v - p)Q(v) - \tilde{N}_p(v)P(1 - v)Q(v + p)$$