

UNIVERSITY OF GLASGOW  
DEPARTMENT OF MATHEMATICS

**E1: first mathematics course for aeronautical engineers  
2010–2011**

**Information for students**

This course covers fundamental topics from Algebra, Calculus, Geometry and Vectors. Its aim is to provide the essential grounding in mathematics appropriate to Engineering Students at the beginning of their Degree Course.

## **WORK AND ORGANISATION**

The Class normally meets daily from 9–10 am throughout the teaching year. There are four lectures and one tutorial in a normal week. For tutorials the class is divided into two equally sized groups. Tutorial work will be set each week. Students will be expected to hand in solutions to one or two specified problems prior to the tutorial, and to provide solutions to additional assigned problems for inspection and correction at the tutorial. Lecturers will also be available for consultation at certain set times ('Office Hours') which will be posted on their office doors.

The lecturers will be: Prof X.Y. Luo /Dr. M. England (Calculus), Dr D.M. Haughton/Dr. D. Stevenson (Algebra).

**RECOMMENDED BOOKS** (For reference only; you are not expected to buy a copy)

*Modern Engineering Mathematics* by Glyn James (Addison Wesley).

*Engineering mathematics, a programmed approach* by C.W. Evans (Van Nostrand Reinhold).

*Calculus: A First Course* (3rd edition) by D.J. Moore (Moodiesburn Press)

## **EXAMINATIONS**

There will be a 2 hour examination (Class Test) in December and an examination in May/June consisting of two papers each of 2 hours duration. The final mark will be a weighted average of the percentage marks obtained in January and May/June, the weighting being 20:80. Thus the final mark for the session will be obtained by adding 20% of the January percentage mark to 80% of the overall percentage total for the May/June papers. The minimum requirement for a Pass is an overall mark of 40%.

Students who miss the January examination will be required to provide a written explanation for their absence. **Those who provide a valid medical certificate** or, exceptionally, some other acceptable reason for absence will be assessed solely on their performance in the

May/June examination. Their final mark will be the percentage total of the two May/June papers. **In any other case of absence** from the January examination, a mark of 0 will be recorded as the mark in that examination and the final mark will be calculated as in the previous paragraph, that is, it will be 80% of the May/June percentage total.

Students who fail to attain a pass following the May/June examination may take a resit examination in August/September. This consists of two two-hour papers as in May/June. The overall mark will be calculated by combining the percentage total of the resit examination with the total from the January examination as above. It is not possible to resit the January examination.

## **PAST EXAMINATION PAPERS**

Past Degree Examination papers may be consulted in the University Library. In addition, copies of examination papers for previous years are available from the Department web-site.

## **CALCULATORS**

Calculators are of limited importance in the work of the class. Calculators taken into Class and Degree Examinations may be programmable but they must not feature full alphabetic character sets on or associated with their keys nor may they have graphical displays.

## SYLLABUS, AIMS AND OBJECTIVES

The general aim of this course is to provide the essential grounding in mathematics appropriate to Engineering Students at the beginning of their Degree Course.

### ALGEBRA

## 1 Revision of Elementary Algebra [9 lectures]

- *Index laws*
- *Completing the square*
- *Factorization*
- *Remainder theorem*
- *Least common denominator*
- *Partial fractions*

**Objectives** The student should be able to

- *apply the index laws in simplifying algebraic expressions*
- *express  $ax^2 + bx + c$  in the form  $a(x - h)^2 + k$  and 2 or 3 variable variations to be used later in geometry*
- *apply factorisation techniques (taking out common factor, quadratics, difference of squares, sum and difference of cubes) in simplifying algebraic expressions*
- *use remainder theorem in factorisation of cubics*
- *put sums of rational functions over a common denominator and investigate the sign of such a function*
- *express a rational function in partial fractions*

## 2 Vectors [8 lectures]

- *Representation of 3-dimensional vectors by directed line segments*
- *Elementary operations on vectors*
- *Vectors as triples*
- *Scalar and vector products with applications*
- *Work done, moment of a force*
- *Scalar and vector triple products*
- *Volume of a parallelepiped*

**Objectives** The student should be able to

- *carry out elementary operations of addition, subtraction, multiplication by a scalar of vectors represented either as directed line segments or as triples of real numbers*
- *state the section formula and use it to solve simple geometrical problems*
- *state the midpoint of  $AB$ , centroid of triangle  $ABC$*
- *state the definitions of scalar and vector products and calculate these for two vectors given as triples*
- *apply products to find the angle between two vectors and, in particular, determine whether two vectors are parallel or perpendicular*
- *resolve a vector along and perpendicular to a given vector*
- *state the definition of work done by a force and the moment of a force, and to calculate these*
- *state definition of scalar and vector triple products and the formula for the vector triple product*
- *use the scalar triple product for calculating volumes of parallelepiped and tetrahedron*

### 3 Coordinate Geometry [6 lectures]

- *Equations of a line*
- *Equation of a plane*
- *Angle between planes*
- *Equations of the line of intersection of two planes*
- *Distance from a point to a plane*
- *Problems on lines and planes*
- *Equation of a sphere*
- *Intersection of a plane and sphere*
- *Tangent plane to a sphere*

**Objectives** The student should be able to

- *give the equations of a line given a direction vector and a point on the line*
- *give the equation of a plane given a normal vector and a point on the plane*
- *state the formula for the distance of a point from a plane*
- *apply the above to various problems involving intersections of lines and planes in three-dimensional space*
- *find the angle between two planes*
- *find the equation of a sphere given its centre and radius and, conversely, given the equation of a sphere, determine its centre and radius*
- *use this in various problems involving the intersection of a sphere with lines and planes in three-dimensional space*

### 4 Complex Numbers [7 lectures]

- *Elementary operations on complex numbers*
- *Conjugate*
- *Polar form*
- *de Moivre's theorem*

- *Geometry of the complex plane*
- *$e^{i\theta}$  as a rotation*
- *Root extraction*

**Objectives** The student should be able to

- *carry out elementary operations of addition, subtraction, multiplication and division of complex numbers*
- *solve simple equations in a complex variable  $z$*
- *express complex number  $a + ib$  in polar form and calculate  $(a + ib)^n$*
- *recognise simple regions of complex plane expressed algebraically, and recognise the effect of multiplying by a complex number*
- *use knowledge of complex roots to factorise real polynomials*
- *find  $n^{\text{th}}$  roots of a complex number*

## 5 Matrices [5 lectures]

- *Matrix algebra*
- *Non-singular matrices*
- *Solution of linear equations using inverses*

**Objectives** The student should be able to

- *carry out operations of addition, subtraction, multiplication of matrices and multiplication by a scalar*
- *state definition of a nonsingular matrix and its inverse*
- *state formula for the inverse of a  $2 \times 2$  nonsingular matrix*
- *solve equation  $AX = B$  given a matrix  $P$  such that  $PA$  is diagonal*

## 6 Series [5 lectures]

- *Finite series*
- *Arithmetic, Geometric,  $\sum r^s (s = 1, 2, 3)$*
- *Binomial coefficients and theorem*
- *Infinite geometric series*

**Objectives** The student should be able to

- *find the sums of arithmetic and geometric series*
- *find sums that can be reduced to combinations of sums  $\sum r^s (s = 1, 2, 3)$*
- *know the Binomial theorem, Pascal's Triangle, and the properties of the Binomial Coefficients*

## CALCULUS

### 1 Functions [4 lectures]

- *Polynomials, rational functions*
- *Powers, general*
- *Exponential  $a^x$  (accepted as intuitive)*
- *Trigonometry (including formulae for  $\cos(A \pm B)$ , etc).*

**Objectives** The student should be able to

- *recognise and use the standard notations for intervals of the real line*
- *identify the maximal domains for some simple functions*
- *draw the graphs of linear and quadratic functions*
- *recognise symmetry in a function and use this in sketching the graph*
- *deal with simple rational functions and rational powers*
- *draw the graphs of the general exponential function and the modulus function*
- *convert between radian and degree measures*
- *apply the addition formulae and formulae for sums into products and products into sums*

## 2 Limits, Continuity and Differentiation [5 lectures]

- *Limits and Continuity*
- *Derivatives*
- *Product, quotient and chain rules*
- *Derivative of  $x^n$  and of the trigonometric functions*
- *Higher derivatives*
- *Implicit functions and differentiation*

**Objectives** The student should

- *know the basic properties of limits and the definition of continuity*
- *be able to differentiate simple functions from first principles*
- *be able to apply the basic rules of differentiation*
- *know the derivatives of powers and of the trigonometric functions*
- *be able to find higher derivatives*
- *be able to find the tangent to a curve given implicitly*

## 3 The exponential and hyperbolic functions [3 lectures]

- *Index laws*
- *Asymptotic behaviour*
- *The hyperbolic functions*

**Objectives** The student should be able to

- *graph and differentiate the exponential and hyperbolic functions*
- *establish simple identities involving the hyperbolic functions*

## 4 Inverse functions and their derivatives [4 lectures]

- *Inverse trigonometric functions*
- *Logarithmic function*
- *Logarithmic differentiation*
- *Inverse hyperbolic functions*

**Objectives** The student should

- *be able to graph and differentiate the inverse trigonometric functions*
- *know the properties of the logarithm function*
- *be able to differentiate logarithmically*
- *be able to graph and differentiate the inverse hyperbolic functions*

## 5 Applications of differentiation [8 lectures]

- *Rates of change and simple problems on related rates*
- *Stationary points*
- *Turning points*
- *Problems on extrema*
- *Curve sketching of rational and other functions, using sign, asymptotic behaviour, vertical asymptotes and stationary points and concavity*
- *Parametric curves — expressions for  $y'$ ,  $y''$*

**Objectives** The student should be able to

- *deal with rates of change and related rates*
- *find stationary and turning points, solve problems on extrema*
- *sketch the graphs of rational and other simple functions, with an appreciation of asymptotic behaviour*

## 6 The integral [10 lectures]

- *Brief introduction to the indefinite integral as an antiderivative*
- *Standard integrals*
- *Definite integral as an area*
- *The fundamental theorem of calculus*
- *Techniques of integration (simple examples only)*
  - (a) *change of variable*
  - (b) *trigonometric functions*
  - (c) *rational functions (by partial fractions)*
  - (d) *integration by parts*
- *Infinite integrals (simple convergent integrals)*

**Objectives** The student should

- *know the standard indefinite integrals*
- *know the connection between definite integrals and area, and the Fundamental Theorem of Calculus*
- *be able to apply the standard techniques of integration*
- *be able to integrate simple rational functions (by partial fractions)*
- *be able to integrate simple trigonometric functions*
- *be able to find simple infinite integrals*

## 7 Applications of integration [6 lectures]

- *Arc length, volume and surface area of revolution*
- *Laws of growth and decay and other practical problems involving first order separable differential equations*

**Objectives** The student should be able to

- *find the arc length and the volume and surface area of revolution determined by simple functions*
- *solve simple problems of growth and decay and of cooling*