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Steady and unsteady flows in collapsible channels

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Abstract: This paper presents a review and discussion on the numerical studies on steady and unsteady flow in collapsible channels using both the fluid-membrane model and the fluid-beam model. The aim of the research is to explore the possible mechanisms of self-excited oscillations observed in experiments on flow in collapsible tubes. The existence and stability of the steady solutions for different control parameters are examined using numerical methods. Results from the different models are compared and discussed. The limitations of these models are also stated.

Introduction

The collapse of compressed elastic tubes conveying a flow occurs naturally in several physiological applications (Pedley, 1980), e.g. (a) blood flow in veins, either above the level and the heart where the internal pressure may be subatmospheric because of the effect of gravity (the jugular vein of the giraffe is particularly interesting in this context (Pedley, et al., 1996), or being squeezed by contracting skeletal muscle as in the 'muscle pump' used to return blood to the heart from the feet of an upright mammal; (b) blood flow in arteries, such as intramyocardial coronary arteries during contraction of the left ventricle, or actively squeezed by an external agency such as blood-pressure cuff; (c) flow of air in the lungs during forced expiration, because in an increase in alveolar air pressure, intended to increase the expiratory flow rate, is also exerted on the outside of the airways. In this case, increasing alveolar pressure above a certain level does not increase the expiratory flow rate, a process known as flow limitation (Shapiro, 1977; Kamm & Pedley, 1989); and (d) urine flow in the urethra during micturition, where flow limitation is again commonplace. These and other examples are discussed in greater details by Shapiro (1977). Note that in all these cases mentioned, the Reynolds number of the flow (Re) is in the order of hundreds.

In laboratory experiments on a finite length of collapsible tube, mounted on rigid tubes and contained in a chamber whose pressure can be controlled, with flow driven through at realistic values of Re, self-excited oscillations invariably arise in particular regions of parameter space (Conrad, 1969; Brower & Sholten, 1975). The thorough experiments of Bertram, in particular, have revealed a rich variety of periodic and chaotic oscillations types, demonstrating that the system is a nonlinear dynamical system of great complexity (Bertram 1982; Bertram et al. 1990, 1991). This has stimulated great interests of many researchers who tried to explain the mechanisms of dynamical behaviour of the system. Numbers of theories, most of them one-dimensional, have been put forward to explain the physical mechanisms responsible for the generation of the self excited oscillations (e.g. Reyn, 1974; Shapiro, 1977; Cancelli & Pedley, 1985; Jensen, 1992; Matsuzaki & Kujimura, 1995; Pedley, 1992). However, due to the great complexity of the system, involving three-dimensional dynamic behaviour and fluid-structure interactions, there is as yet no complete theoretical description of the oscillations in any realizable experimental conditions.

From the mathematical point of view, a self-excited oscillation can arise only when a steady solution fails to exist or becomes unstable in a system with constant control parameters. Hence it is essential to investigate the existence and the stability of the steady flow in a rationally described model, including important effects such as the non-linearity of the flow and wall dynamics.

As the three-dimensional simulations of the system requires computing resources in excess of any available to us except in the cases where flow can be greatly simplified (Heil & Pedley, 1996; Heil,1997), two dimensional models on elastic walled channel flow are considered instead (Lowe & Pedley, 1995; Luo & Pedley, 1995, 1996, 1998, 2000; Cai & Luo, 2001; Cai, *et al.* 2001, Luo *et al.* 2001). In this paper, both steady and unsteady simulations of these models will be discussed. First, the membrane model is used and the study is mainly concentrated on the existence of steady solutions (Lowe & Pedley, 1994; Rast, 1994; Luo & Pedley, 1995). It was found that in the given range of Reynolds number and transmural pressure, although a steady flow solution should exist for all values of longitudinal tension according to one-dimensional analytic models, the numerical simulations could only achieve these solutions for a sufficiently large tension. We have also found that such problems are extremely ill-conditioned when the boundaries are highly complaint (e.g., tension very small). Secondly, the stability of the steady solutions are investigated, and self-excited oscillations are found to occur if the longitudinal tension of the membrane is small enough (Luo & Pedley, 1996). Finally, a new fluid-beam model is introduced where the wall mechanics is properly presented. Results from the different models are compared and discussed.

The fluid-membrane model:

The system configuration is shown in figure 1. The rigid channel has width of D. One part of the upper wall is replaced by an elastic membrane subjected to an external pressure p_e. Steady Poiseuille flow with average velocity U_o is assumed either at the entrance or exit, depending on which type of boundary condition is used (see below); the fluid pressure at either the upstream or the downstream end is taken to be zero. The flow is incompressible and laminar, the fluid having density ρ and viscosity μ. The longitudinal tension T is taken to be constant, i.e., variations due to the wall shear stress or the overall change of the membrane length are considered to be small relative to the initial stretching tension.

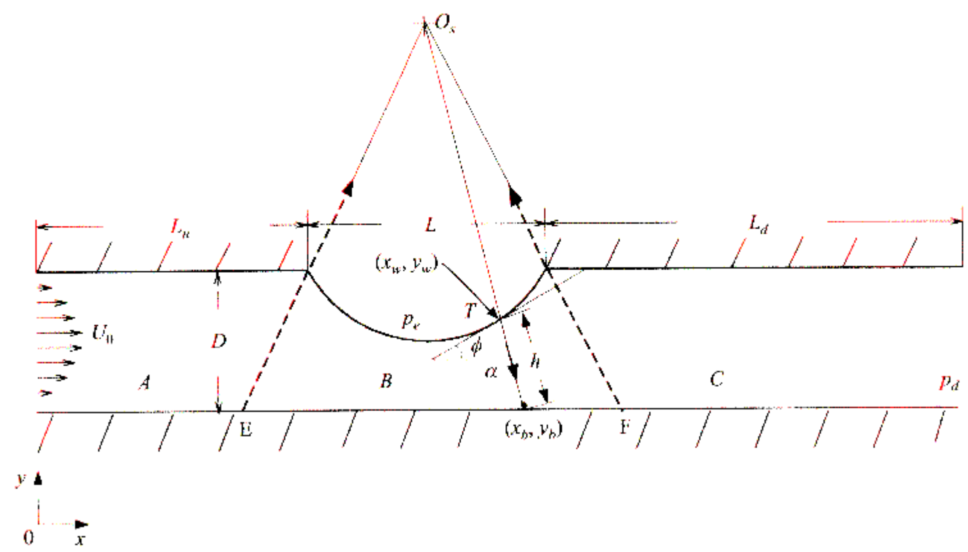

FIGURE 1. Two-dimensional flow configuration. Steady Poiseuille flow with average velocity U_0 entering upstream; p_d is pressure at the downstream end of the channel, p_e is the external pressure and T is the tension in the membrane. For explanation of other symbols, see text.

Steady flow solutions for a fixed p_e-p_d

The typical non-dimensional parameters in the model are chosen to be L=5, L_u=5, L_d=5 (scaled to *D*), T_o=179 (scaled to $D \bullet U_o^2$), p_e-p_d=1.03 (scaled to • U_o^2), and *Re*=300 (Luo & Pedley, 1995). The main results from the steady flow simulations for *Re*=300, and a fixed transmural pressure p_e-p_d are shown in figure 2. It is seen that the membrane started to collapse as tension is reduced. At small • (large *T*), the membrane is stretched tight and is not deformed. As • is increased, the deformation increases, the minimum channel width y_{min} occurring close to the midpoint of the membrane. As the construction becomes more severe, it tends to move downstream and a point of deflection appears in the upstream half. When • increases above 30 two, possibly independent, phenomena are seen: the upstream part of the membrane begins to bulge out, y_{min} increases somewhat as • increases. The membrane slope becomes very large.

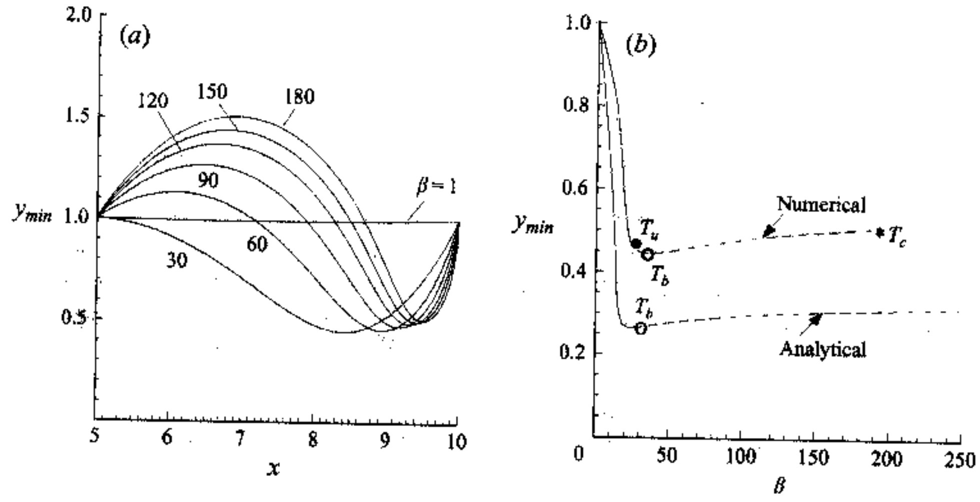

Figure 2. Left: The wall shapes of the elastic section for different values of the membrane tension $T=T_o$/•, where • is a scaling parameter. Right: The minimum wall position y_{min} plotted against •. Re=300.

Both the above phenomena are also seen in the corresponding high *Re* one-dimensional model, which is the same of that of Jensen & Pedley (1989) but applied to channel flow. Indeed, the shape of the graph of y_{min} against β predicted by the 1-D model is very similar to that given by the full computation, as shown in figure 2 (right).

Unsteauy flows of the fluid-membrane model:

Stability and self-excited oscillations

If the steady solutions shown in figure 2 are perturbed in the time-dependent domain, then the stability of these solutions can be checked numerically. It is discovered that for small enough tension (• > 25), these solutions do become unstable. Further more, self-excited oscillations are developed for • > 30. These oscillations are almost regular sinusoidal ones for higher tension. However, as the tension is reduced further, the oscillations become irregular, and the system seems to have gone though a period-doubling bifurcation. The instantaneous streamlines as self-excited oscillations occurring (•=35) are shown in figure 3.

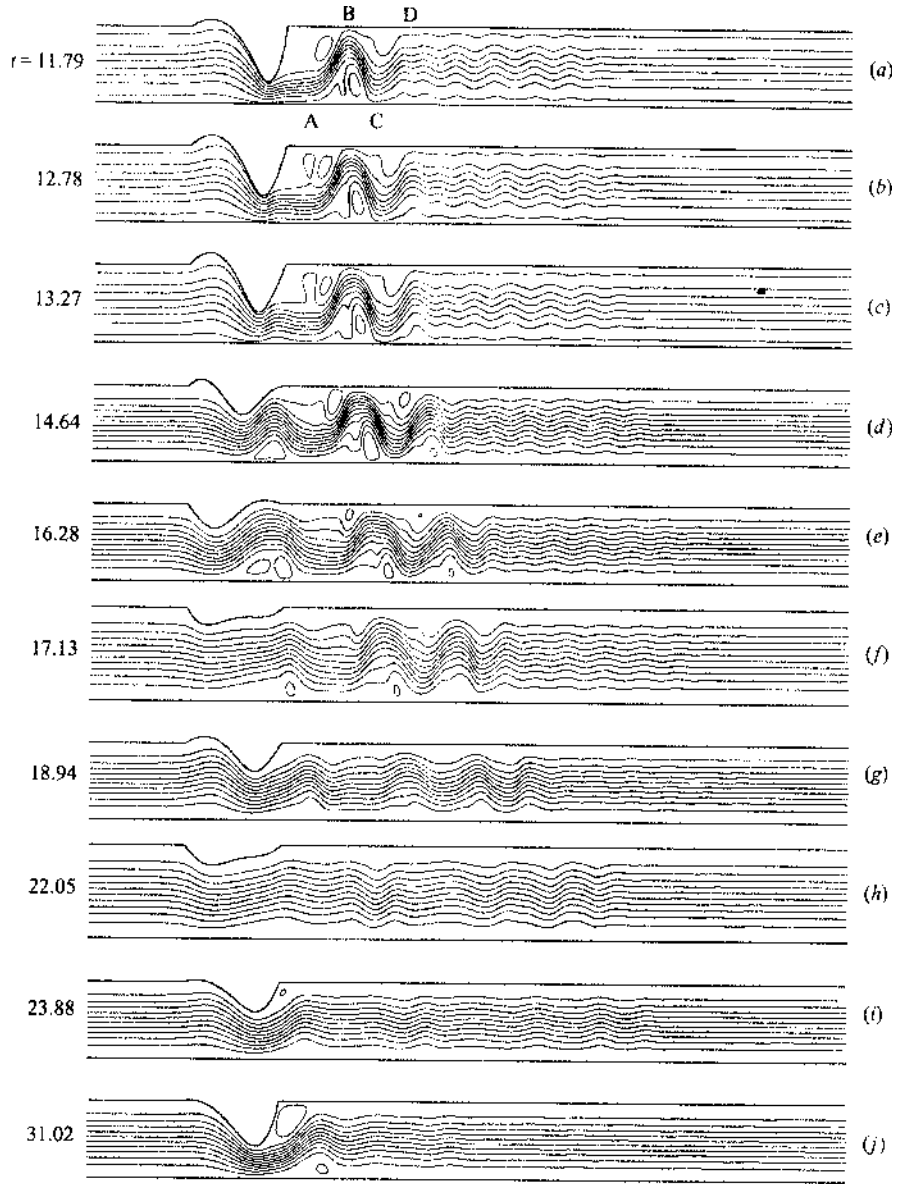

Figure 3: Instantaneous streamlines during the time period as marked on the left.

The fluid-beam model:

The fluid-membrane model discussed above, however, involves several ad hoc assumptions. First, for the unsteady flow simulations, the direction of the wall movement has to be assumed because the membrane equation alone cannot determine the movement of the material points of the elastic wall. Secondly, the membrane model ignores the axial stiffness and hence the longitudinal stretch of the elastic wall is only balanced by a uniform tension and the transmural pressure. Thirdly, the bending stiffness of the wall is ignored in the membrane model. While these assumptions are reasonable for a certain range of parameters, they may lead to unrealistic results when the wall is not so thin. In addition, ignoring the longitudinal stretch makes it extremely difficult to make comparison with the experimental results, where the initial tension is often zero or very low (Ikeda, et. al. 1998). To overcome these shortcomings, Cai & Luo (2001) have proposed a new model in which a plane strained elastic beam with large deflection and incrementally linear extension is used to replace the membrane.

The fluid-beam model takes the same configuration as in figure 1, except now that the membrane section is replaced by an elastic beam. The dimensionless governing equations for the system are:

$$x' = \lambda\cos\theta\ ,\quad y' = \lambda\cos\theta\ ,\ \theta' = \lambda\kappa$$

$$\frac{\rho_m}{\lambda}(x'\frac{d^2x}{dt^2} + y'\frac{d^2y}{dt^2}) = c_\kappa\kappa\kappa' + c_\lambda\lambda' + \lambda\tau_n$$

$$\frac{\rho_m}{\lambda}(y'\frac{d^2x}{dt^2} - x'\frac{d^2y}{dt^2}) = c_\kappa\left(\frac{1}{\lambda}\kappa'\right)' - \lambda\kappa T - c_\lambda\lambda\kappa(\lambda-1) - \lambda\sigma_n + \lambda p_e$$

$$u_j u_{i,j} = -p_{,i} + \frac{1}{\mathrm{Re}}u_{i,jj}\ ,\ u_{i,i} = 0\ ,\ \ i, j = 1,2$$

where conventional symbols are used for flow variables, c_κ, c_λ denote the bending and extensional stiffness, respectively, T is now the pre-tension, and λ, θ, κ are the slope, the principal stretch and the curvature of the beam, respectively. Notice that as both c_κ and $c_\lambda \to 0$, we recover the fluid-membrane model (Luo & Pedley, 1995).

An adaptive finite element (FEM) approach utilizing rotating spines is adopted to solve this fluid-structure interaction problem The mesh is divided into three subdomains, one of which is placed with many spines originating from the bottom rigid wall to the movable beam. These spines are straight lines which can rotate around the fixed nodes at the bottom, hence all the other nodes on the spines can be stretched or compressed depending on the beam deformation. A numerical code is developed to solve the fluid and the beam equations simultaneously using the weighted residual methods. The boundary conditions for the flow are: no-slip on the walls, parabolic entry flow, and stress free at the far downstream boundary. Clamped boundary conditions are imposed at the beam ends.

If $c_\kappa \to 0$, a singularity may exist at the corners between the elastic and rigid walls. Following Lowe & Pedley (1996), an asymptotic approach to the beam model for very small c_κ in the small region near the corner (inner region) is employed to analyse the local behaviour of the flow field (Cai, *et al*. 2001). The boundary conditions for this inner problem are determined by matching the numerical results for the outer region.

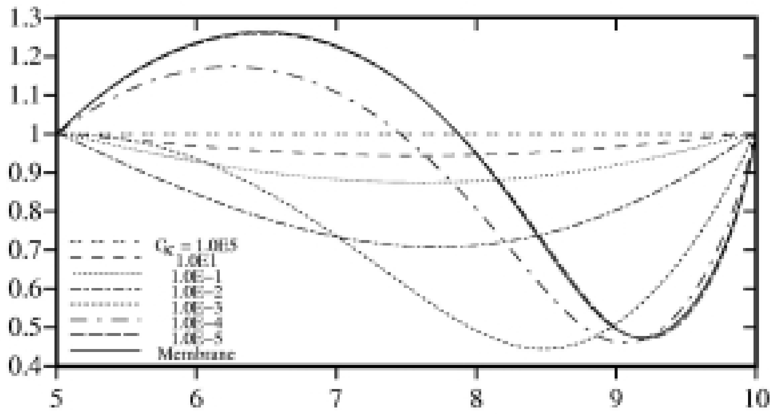

Figure 4. The wall shapes for different values of c_κ, and β=90

Following Luo & Pedley (1996), the pre-tension T is also chosen to be T=178.8/β. The bending stiffness is in the range of $c_\kappa = 10^5 - 10^{-5}$, and a fixed ratio of $c_\kappa / c_\lambda = 10^{-5}$ is used, which is equivalent to choosing the thickness of the wall to be 1% of the channel width; all other parameters are the same as in Luo & Pedley (1996), with a Reynolds number of 300.

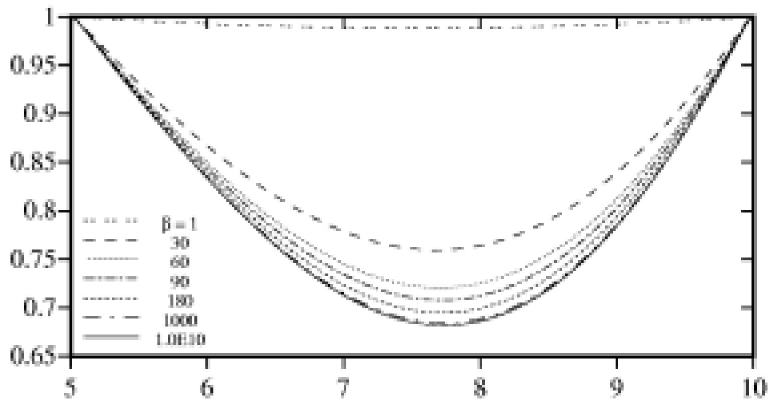

Figure 5 The wall shape for different values of pre-tension for $c_{\kappa} = 10^{-2}$

When c_{κ} is very large, the beam behaves like a rigid wall. The deformation of the elastic wall increases as c_{κ} decreases. The upstream bulging phenomenon observed in the fluid-membrane model when the tension is below a certain value (Luo & Pedley, 1995) also occurs here when c_{κ} falls below 10^{-3}. For an even smaller value of c_{κ}, say 10^{-5}, this model behaves almost identically to the fluid-membrane model. In other words, the fluid-membrane model seems to be a good approximation if c_{κ} is $O(10^{-5})$.

One important difference between the beam and the membrane models is that in the latter, when the tension falls below a critical value (λ=181), the numerical scheme breaks down and a steady solution is not attainable (Luo & Pedley, 1995). We found this is only true in the beam model for very small values of c_{κ}. For $c_{\kappa} > 10^{-5}$, however, a solution is always attainable, and the elastic wall approaches to a limiting shape as $T \rightarrow 0$. This limiting shape is found to depend the value of c_{κ}.

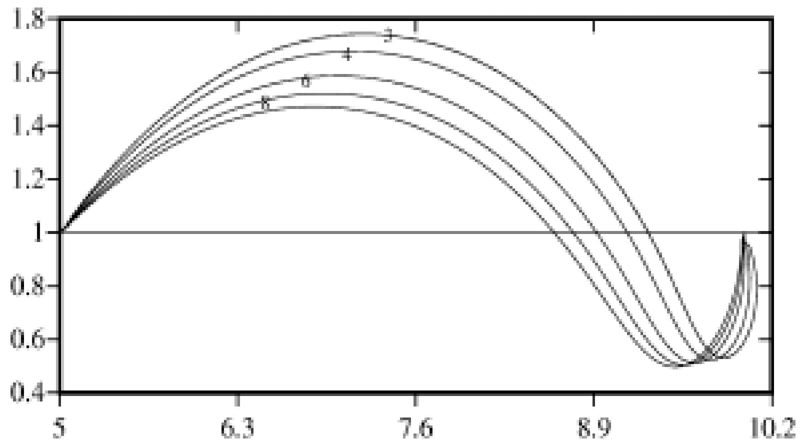

Figure 6 The limiting wall shapes when $T \rightarrow 0$ for different values of c_{ε}

In the limit when $c_{\kappa} \rightarrow 0$, asymptotic approach shows that $u_i \sim R$, and $p \sim p_O$ near the beam end, i.e., there is no singularity at the corners for any nonzero bending stiffness (Cai, Luo & Pedley, 2001). In addition, the beam solution is independent of the flow in the inner region when $Re \sim o(\varepsilon^{-1}L^{-2})$, where $\varepsilon = c_{\kappa}/(c_{\kappa} L^2)$. This means that the beam shape near the corner is solely determined by the boundary conditions at the ends of the inner region, and the influence of the flow only comes in through the interface to the outer solution. For $\varepsilon \rightarrow 0$, the asymptotic approach also predicted that the flow field would approach the sharp corner singularity in the manner of $O(\epsilon^{-1/2})$. Excellent agreement is found between the numerical and asymptotic solutions near the corner when $c_\kappa \rightarrow 0$.

Concluding Remarks

In order to explore the mechanisms of self-excited oscillations in flow in collapsible tubes, both steady and unsteady flow are simulated numerically on two-dimensional models with the elastic section being replaced by a membrane or by a beam. A finite element code is developed for the strongly coupled fluid-structure interaction problem. Comparisons of the beam and membrane models are made. It is found that for $c_\kappa < 10^{-5}$, the fluid-membrane model is a very good approximation to fluid-beam model, although when applied to unsteady flows, it imposes almost too strict constrains for the system.

In the membrane model, it was found that there is a limit value of tension when a steady solution is not attainable, but for the beam model, this again depends on the wall stiffness. If these values are not too small, then as the pre-tension $T \rightarrow 0$, the elastic wall approaches a finite limiting shape, and there a steady solution is always attainable. This is due to the fact that tension used in the membrane really means the final tension which approximates to pre-tension only for very small values of the wall stiffness.

For greater values of the wall stiffness, however, there are significant differences between the two models. This is highly important, since for most of the bioengineering applications, the wall properties of tissue materials can be much greater than the ones of the membrane (Fung, 1993). Although only steady flow is considered in this paper, the model is valid for unsteady flow. Using this new model will allow us to explore the self-excited mechanisms in a more realistic parameter region and make it possible to compare with the corresponding two dimensional experiments.

The limitations of the fluid-beam model is that the physical linearity assumed for the beam, which may not be valid if the wall deformation is too large. In addition, it is still a two dimensional approach to the original complicated three dimensional system of flow in collapsible tubes. Therefore the direct application of this model to the collapsible tube flow is likely to be limited. Nevertheless, a study on the simpler version of the original three dimensional fluid-structure interaction problem will serve to improving our understanding of the fundamental behaviour of the system. Most importantly, this work makes it possible to have direct comparisons with two-dimensional channel flow experiments, which in principle can be conducted. Finally, it is worth pointing out that although it is possible to apply the current numerical code to two-dimensional axisymmetric tube flow, which appears to be a step closer to the three dimensional tube flow, it is not pursued here since we are mainly interested in the mechanisms of self-excited oscillations, which when they occur, are not axisymmetric.

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Xiaoyu Luo was born in Xi'an, China. She became a student at the Xi'an Jiaotong University in 1978, and obtained her MEng in Solid Mechanics four years later. She continued to study as a post-graduate and obtained MSc. in Applied Mechanics in 1985. She then became a lecturer at the department of Engineering Mechanics, Xi'an Jiaotong University. In 1986, she got interested in Biomechanics, and started her Ph.D study on the blood flow in arteries.

Sponsored by the World Bank in 1987, Xiaoyu spent a year at the Royal Hallamshire Hospital of Sheffield, UK, where she had a chance to do some blood flow experiments and was exposed to surgeon's view of vascular problems. Returned to China in 1988 to work with her Ph.D supervisor, Professor Z. B. Kuang, she simulated stenotic arterial blood flows, and modelled the non-Newtonian behaviour of the blood. In 1990, her research was awarded the Tang ZhaoQian Scholarship at Xi'an Jiaotong University. In 1992, she became an editor of the Chinese Journal of Applied Mechanics.

Supported by an UK EPSRC Grant, Xiaoyu returned to the UK in 1992 to work with Professor T.J. Pedley, FRS, at the Department of Applied Mathematics, University of Leeds (who later became the G.I. Taylor Professor of DAMTP, University of Cambridge). Her research was mainly on fluid flow in collapsible tubes, where the non-rigid nature of the vessel has a dominant effect on the flow. Flow in collapsible tubes causes many interesting phenomena not found in rigid tubes. By employing a finite element code with adaptive mesh techniques, important features of the complex dynamic system have been revealed, which led to the development of new analytical models.

In 1997, Xiaoyu became a lecturer at the Queen Mary & Westfield College, University of London, UK, and started to build up a research group. From May, 2000, she moved to the Department of Mechanical Engineering, the University of Sheffield, and since has carried out research in close collaborations with colleagues ranging from surgeons, experimentalists to mathematicians. She has also attracted funding from EPSRC, the Royal Society, and the University. In addition to studying flow in collapsible tubes, she is also interested in the bile flow in the human gallbladder and cystic duct, influences of turbulent flows on particle deposition in the human airway; anisotropic behaviour of tissue heart valves; blood flow in a cardiopulmonary bypass and mechanisms of haemodynamic induced atherosclerosis. .