1D and 3D Blood Flow

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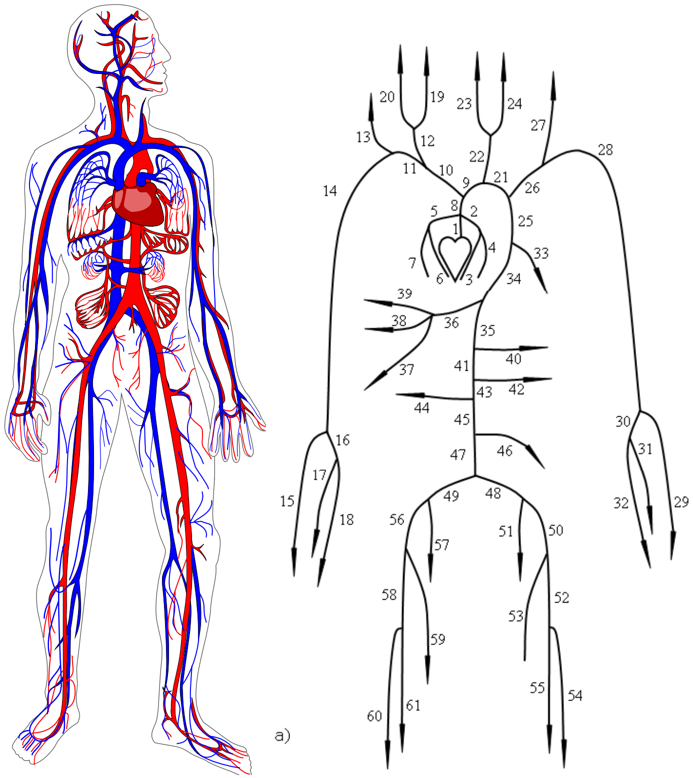
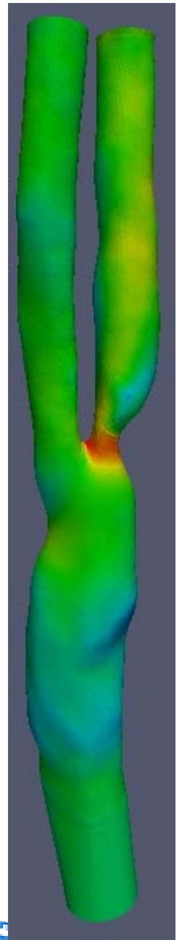
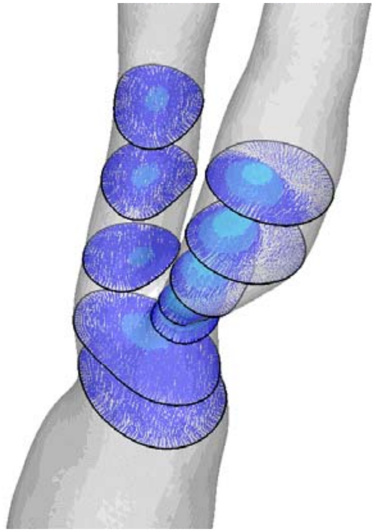
Part I – 1D

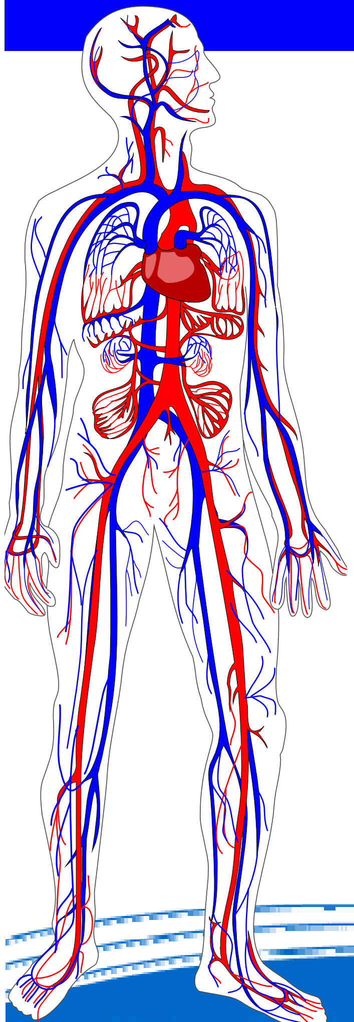


Part II – 3D





Part I One-dimensional fluid-structure interaction for systemic circulation



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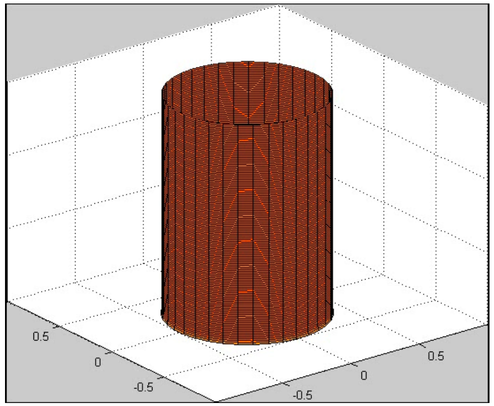

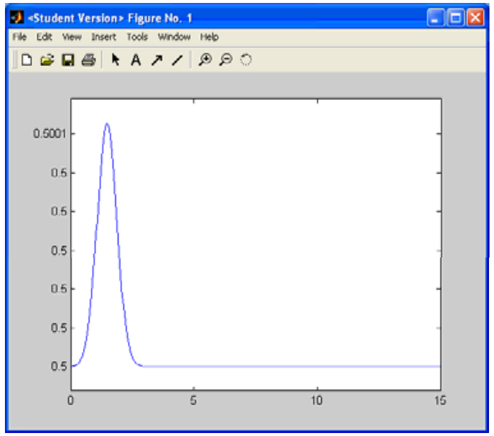

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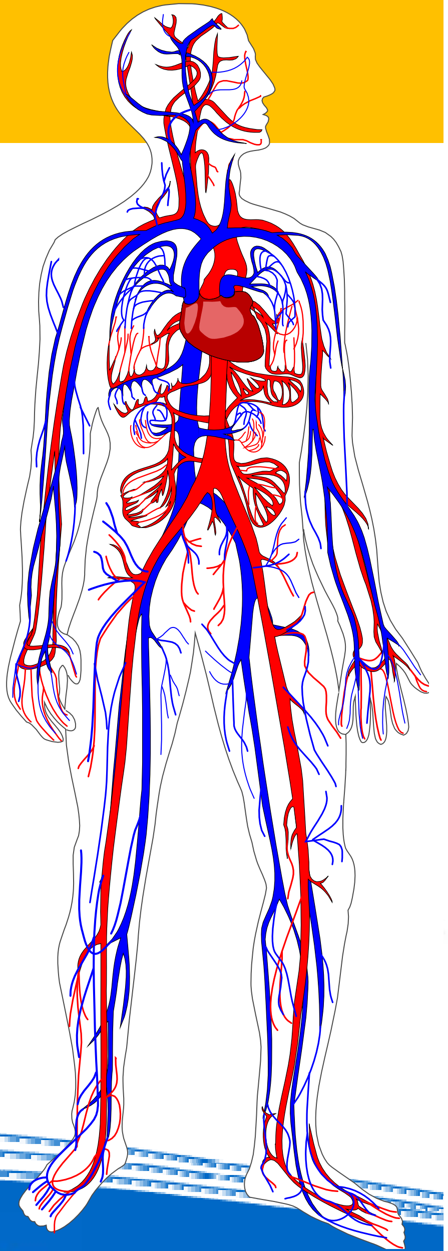
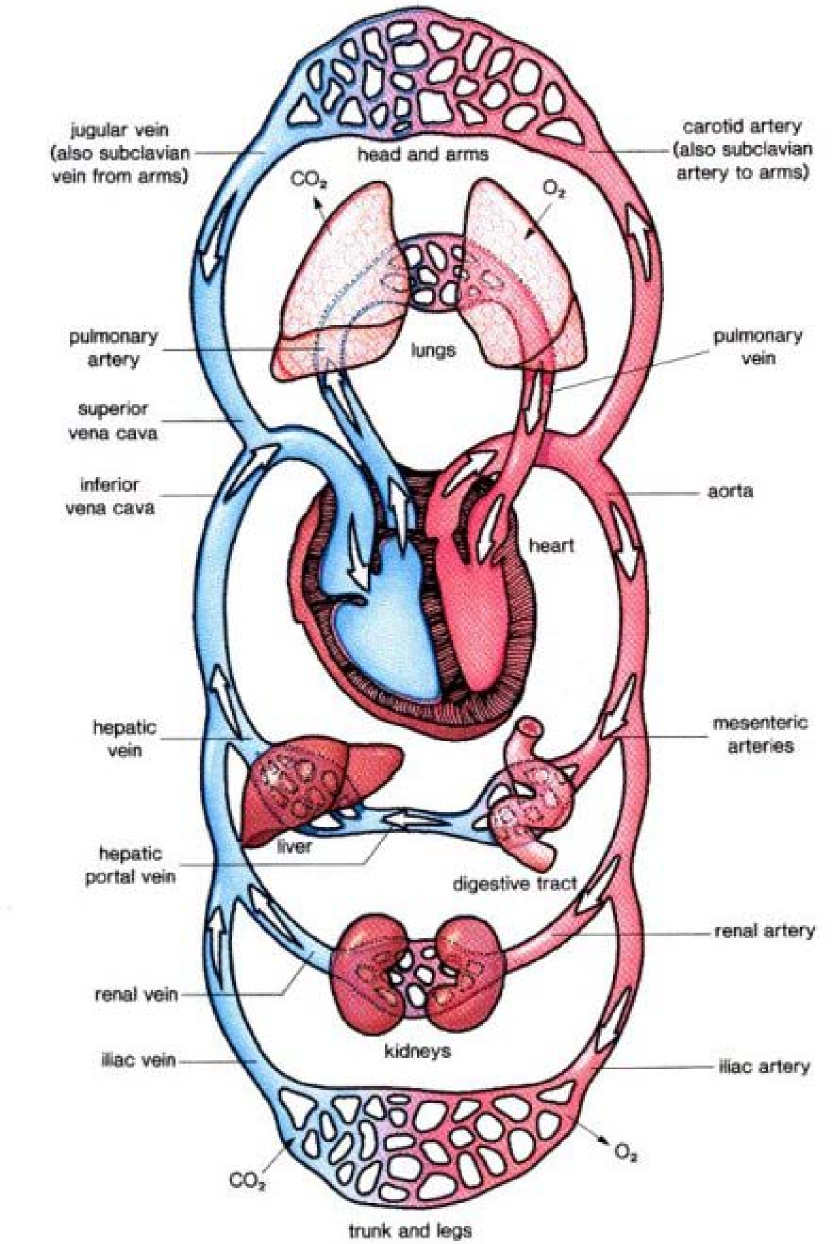
Anatomy

Principles

Circulatory system

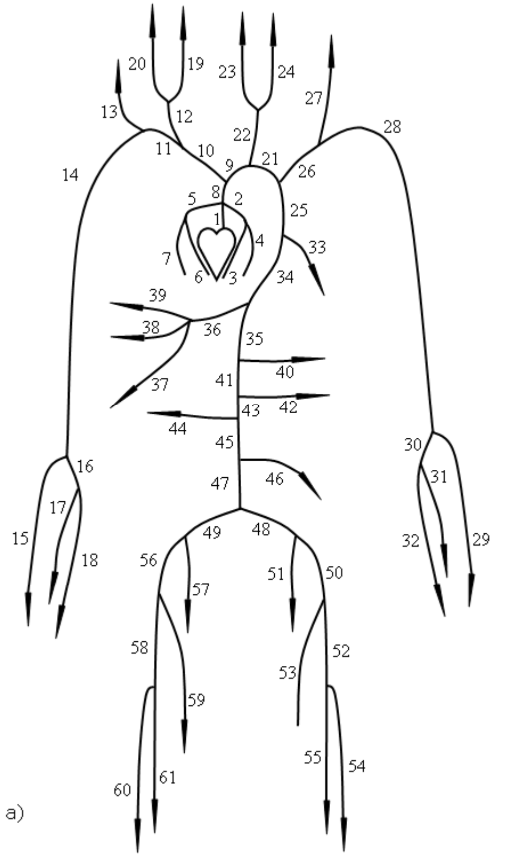


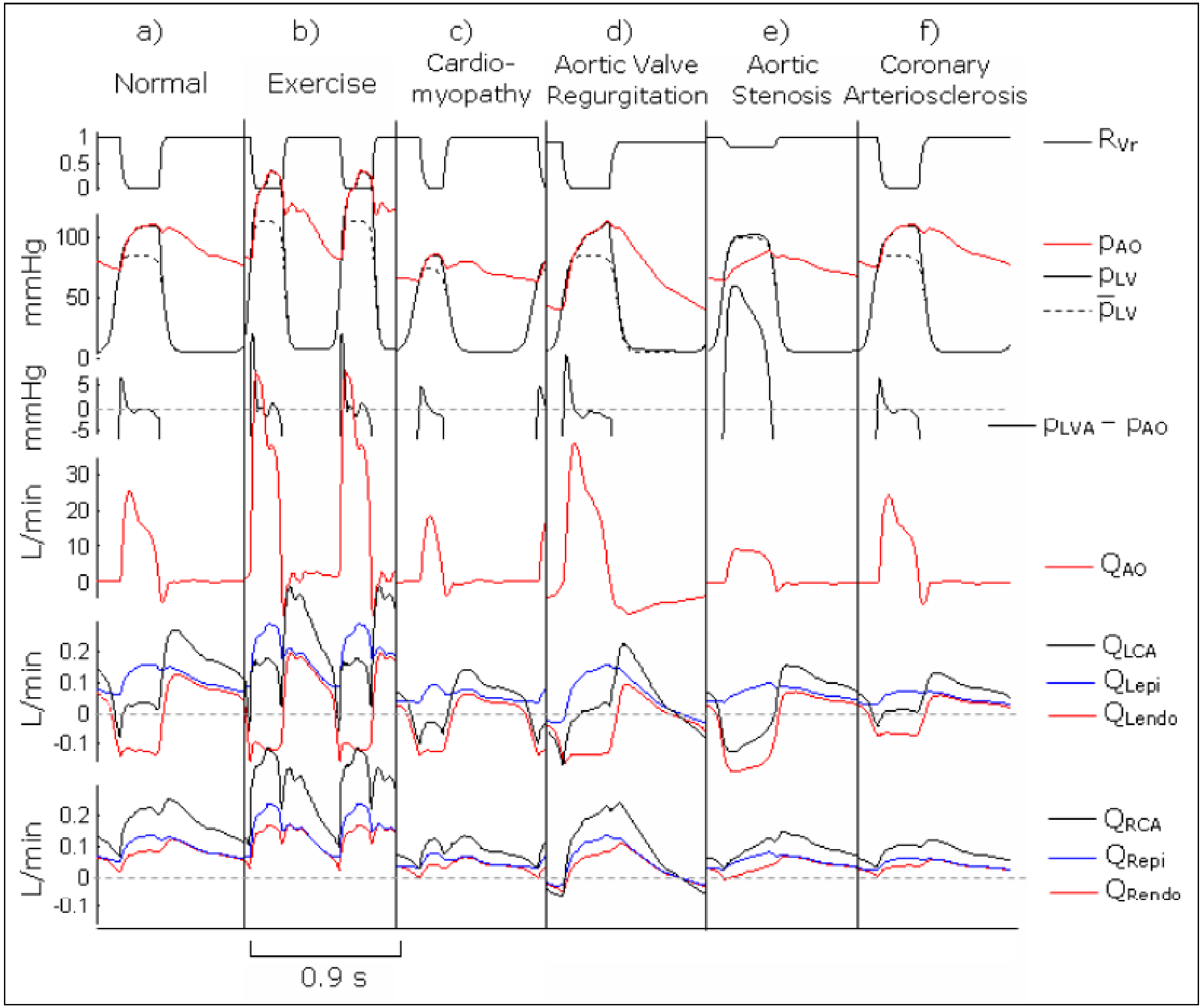


Red – arteries (Oxygenated)

Blue – veins (De-oxygenated)

Disease state representation using 1D

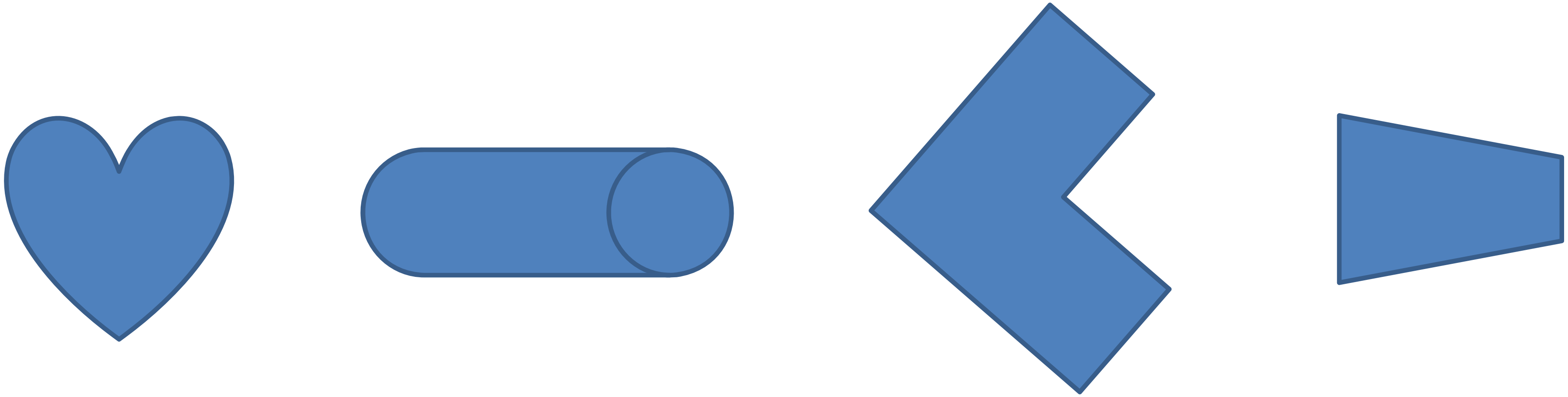




Model construction – Systemic Circulation - OVERVIEW

- Wave length is much longer than diameters.
  1D approximation is valid.
- Representing arteries, ventricle, valves, bifurcation, curvature and micro-circulation.
- Multiple solution at bifurcations.
- Curvature inclusion is required but not included in this lecture.
- Valves may be included in a rudimentary fashion.
- Left ventricle produces a pressure pulse similar to fused sigmoid functions.
- Lumped models or tapered vessels represent micro-circulation.
- Boundary conditions are determined by characteristic waves.

Model construction – Systemic Circulation – CORE SYSTEM

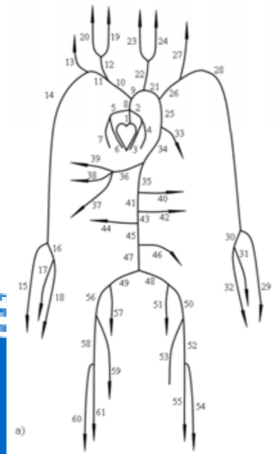


- ✓Left ventricle
- ✓Aortic valve
- ✓ Coronary flow

✓Straight vessels

✓Bifurcations

✓Tapering vessels



- ➢ No organs
- ➢ No venous system

Model construction – Systemic Circulation

Continuity Equation
$$\frac{\partial A}{\partial t} + \frac{\partial (Au)}{\partial x} = 0$$

Momentum Equation
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + f = 0$$

Constitutive Relation
$$p = p_{ext} + \beta\left(\sqrt{A} - \sqrt{A_0}\right)$$

✓Straight vessels

Wall property function

h = *wall thickness*

E = *Young's Modulus*

σ = *Poisson's ratio*

$$\beta = \frac{\sqrt{\pi} h E}{A_0(1-\sigma)^2}$$

u = ***mean velocity over a cross-section***

A = ***cross-sectional area*** (A_0 = ***'unstressed' area***)

p = ***internal pressure*** (p_{ext} = ***external pressure***)

ρ = ***blood density (constant)***

f = ***friction term***

β = ***material property function of the vessel wall***

Equations

$$p = p_{ext} + \beta\left(\sqrt{A} - \sqrt{A_0}\right)$$

Replace pressure derivative

$$\frac{\partial p}{\partial x} = \frac{\partial p_{ext}}{\partial x} + \frac{\beta}{2\sqrt{A}}\frac{\partial A}{\partial x} - \frac{\beta}{2\sqrt{A_0}}\frac{\partial A_0}{\partial x} + \left(\sqrt{A} - \sqrt{A_0}\right)\frac{\partial \beta}{\partial x}$$

The system

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{H}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{C}$$

where

$$\mathbf{U} = \begin{bmatrix} A \\ u \end{bmatrix}, \mathbf{H} = \begin{bmatrix} u & A \\ \frac{\beta}{2\rho\sqrt{A}} & u \end{bmatrix},$$

$$\mathbf{C} = -\frac{1}{\rho}\begin{bmatrix} 0 \\ 8\pi\mu\frac{u}{A} + \frac{\partial p_{ext}}{\partial x} - \frac{\beta}{2\sqrt{A_0}}\frac{\partial A_0}{\partial x} + \left(\sqrt{A} - \sqrt{A_0}\right)\frac{\partial \beta}{\partial x} \end{bmatrix}$$

Poiseuille flow assumption

Equations

The characteristics

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = u \pm \sqrt{\frac{\beta\sqrt{A}}{2\rho}} = \begin{bmatrix} u + c \\ u - c \end{bmatrix}$$

Characteristic variables

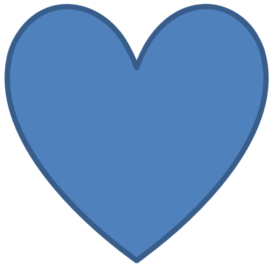
$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} u + 4c \\ u - 4c \end{bmatrix}$$

The flow variables may be written as

$$A = \frac{(w_1 - w_2)^4}{1024} \left(\frac{\rho}{\beta}\right)^2$$

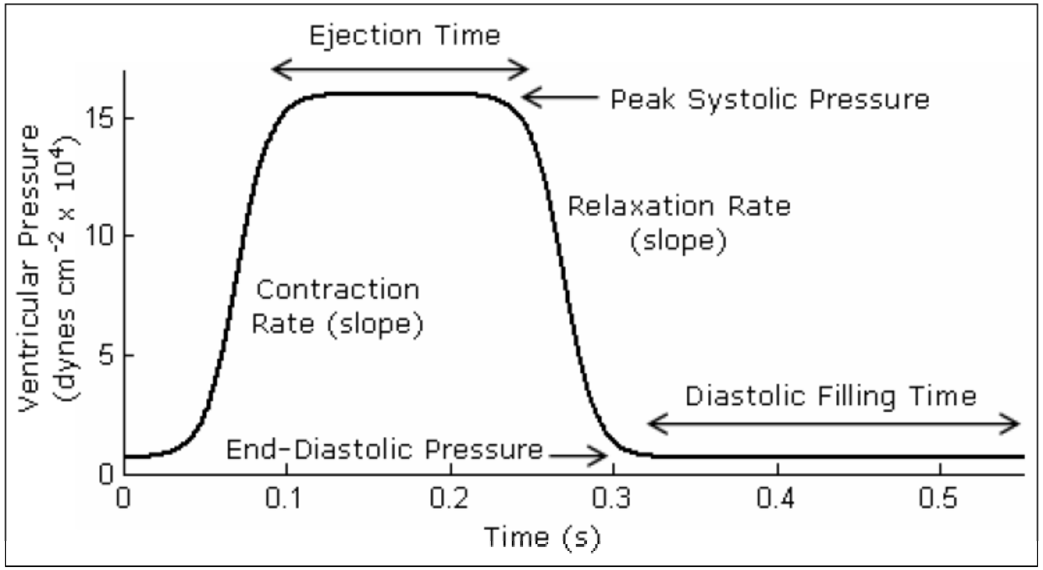
$$u = \frac{1}{2}(w_1 + w_2)$$

Model construction – Systemic Circulation



✓Left ventricle

Two sigmoid function fused mid-ejection



$$p_{sig}(t) = a_1 + \frac{(a_2 - a_1)}{1 + e^{(a_3 - t)/a_4}}$$

$$a_1 = p_{ed} - 9.11 \times 10^{-4} p_{peak}$$

$$a_2 = p_{peak}$$

$$a_3 = 7t_c$$

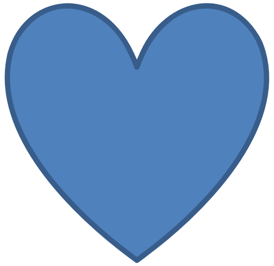
$$a_4 = t_c$$

p_{ed} is end-diastolic pressure

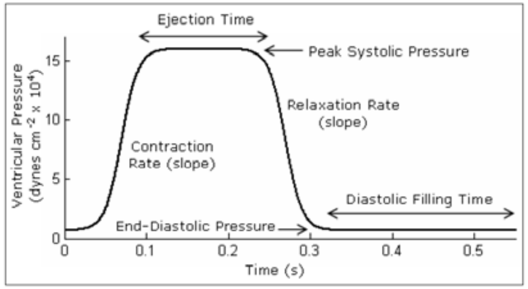
p_{peak} is peak pressure

t_c is a time constant

Model construction – Systemic Circulation



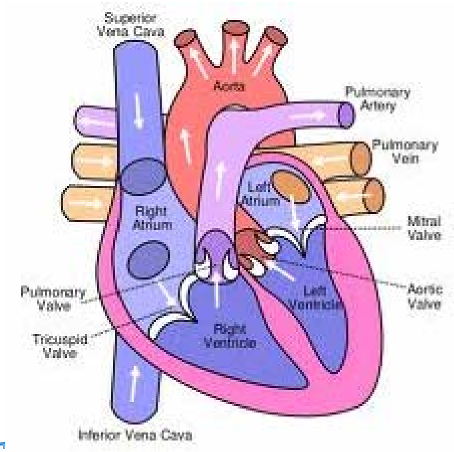
$$w_{1\text{in}}^{n+1} = w_2^0 + 4\sqrt{\frac{2}{\rho}}\sqrt{(\bar{p}^{n+1} - p_{\text{ext}}) + \beta\sqrt{A_0}}$$



$$w_{1\text{in}} = \Delta w_{1p} + \Delta w_{1r} + w_1^0$$

✓**Aortic valve**

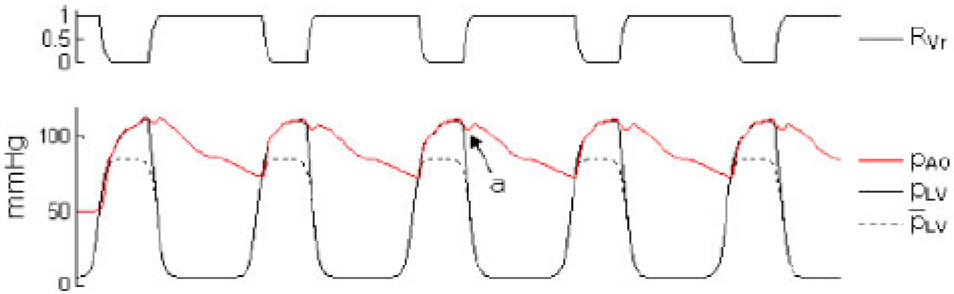
$$\Delta w_{1p} = T_{\text{Vp}}(t)\Delta w_1 \qquad \Delta w_{1r} = R_{\text{Vr}}(t)\Delta w_2$$



$$R_t = -\frac{\Delta w_2}{\Delta w_1} = -\frac{w_2^{n+1} - w_2^0}{w_1^{n+1} - w_1^0}$$

$$T_{\text{Vp}} = 1 - R_{\text{Vr}}(t)$$

***w_1* – incoming characteristic wave**
***w_2* – outgoing characteristic wave**



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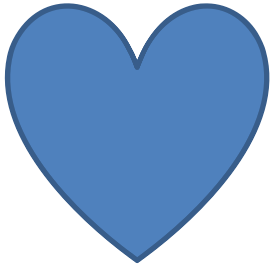
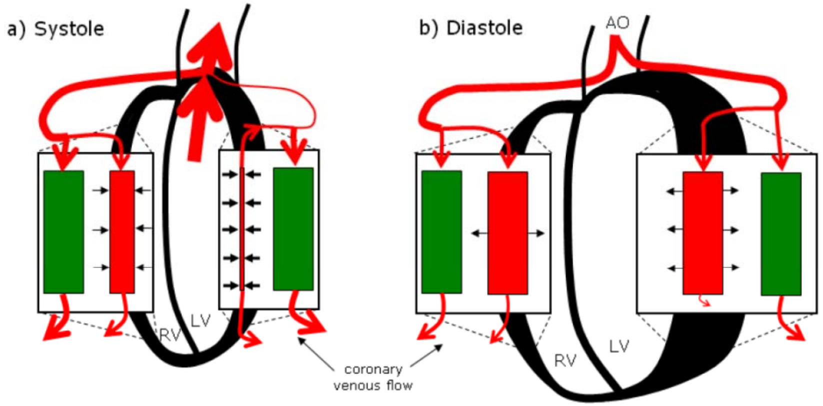
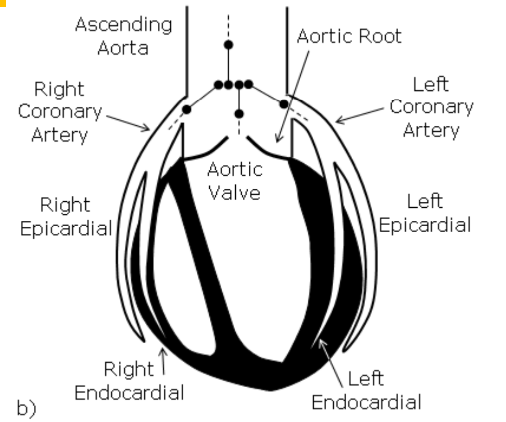
Model construction – Systemic Circulation



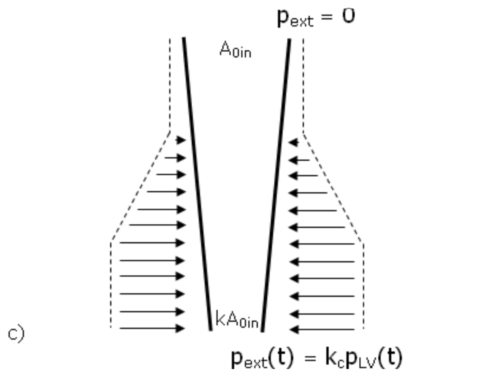
✓Coronary arteries





Pressure on subendocordial vessel

$$p_{ext}(x,t) = \begin{cases} 0 & x < \frac{L_c}{3} \\ k_c p_{LV}(t)\left(\frac{3x}{L_c} - 1\right) & \frac{L_c}{3} \leq x \leq \frac{2L_c}{3} \\ k_c p_{LV}(t) & x > \frac{2L_c}{3} \end{cases}$$



$p_{LV}(t)$ is the time-varying LV pressure

$L_c = 7$ is the length

k_c = 1 left subendocordial vessel

k_c = 0.2 right subendocordial vesses l

Model construction – Systemic Circulation

Vessel branching

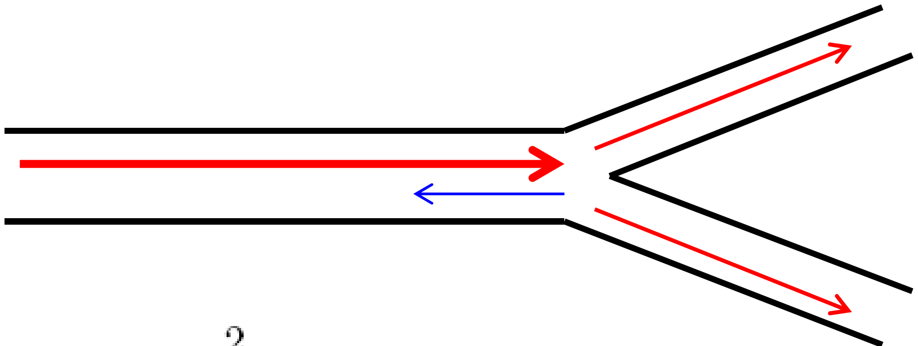
Parent vessel

$$w_{1p} = u_p + 4A_p^{1/4}\sqrt{\frac{\beta_p}{2\rho}}$$

Daughter vessels

$$w_{2i} = u_i - 4A_i^{1/4}\sqrt{\frac{\beta_i}{2\rho}}$$

✓**Bifurcations**



Mass conservation

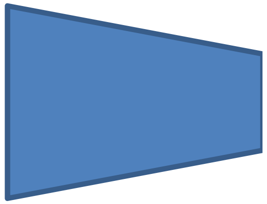
$$Q = A_p u_p = \sum_{i=1}^{N} A_i u_i$$

Pressure continuity

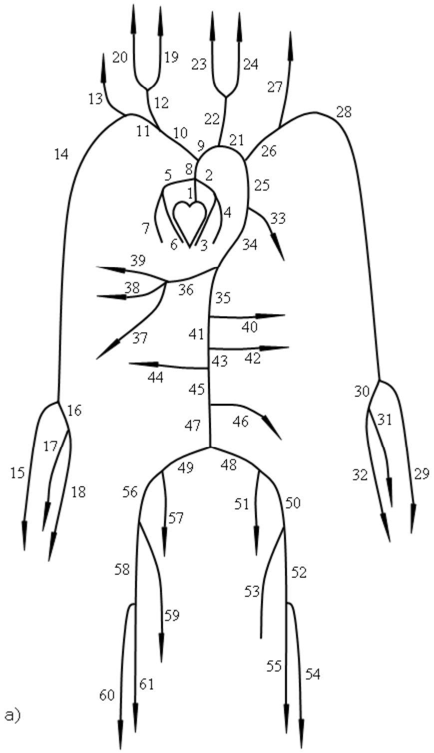
$$\frac{\rho u_p^2}{2} + p_{ext(p)} + \beta\left(\sqrt{A_p} - \sqrt{A_{p0}}\right) = \frac{\rho u_i^2}{2} + p_{ext(i)} + \beta\left(\sqrt{A_i} - \sqrt{A_{i0}}\right)$$

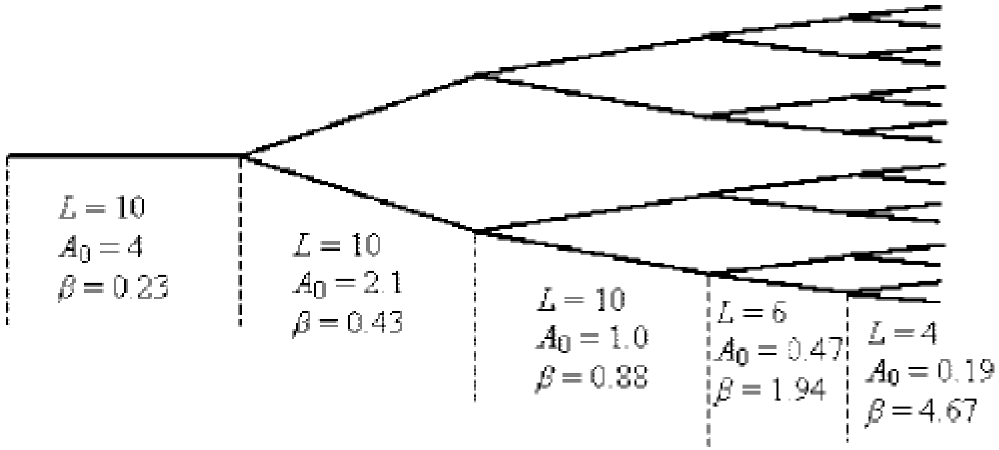
Total number of equations = 2(N + 1); where N – number of daughter vessels

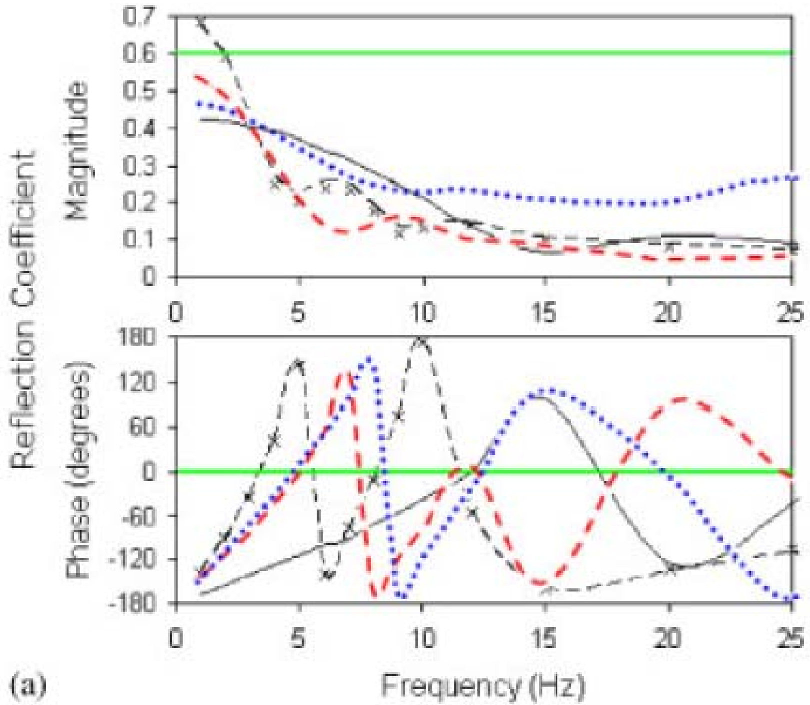
Model construction – Systemic Circulation



✓**Tapering vesse**

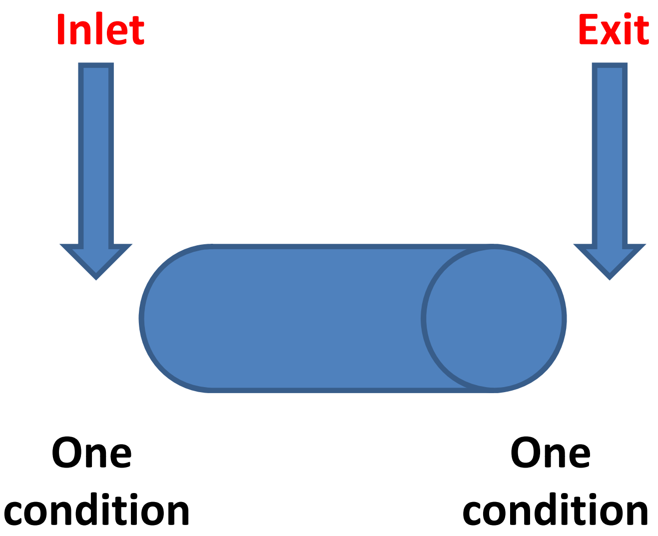






Green – Simple resistance
Others – Tapering vessels

Model construction – Boundary conditions



$$w_1^{n+1}\big|_{x=x_L} = w_1^n\big|_{x=x_L-\lambda_1^n \Delta t}$$

$$w_2^{n+1}\big|_{x=x_0} = w_2^n\big|_{x=x_0-\lambda_2^n \Delta t}$$

$$w_{1\mathrm{in}}^{n+1} = w_2^0 + 8(\bar{A}^{n+1})^{1/4}\sqrt{\frac{\beta}{2\rho}}$$

$$w_{1\mathrm{in}}^{n+1} = w_2^0 + 4\sqrt{\frac{2}{\rho}}\sqrt{(\bar{p}^{n+1} - p_{\mathrm{ext}}) + \beta\sqrt{A_0}}$$

$$w_{1\mathrm{in}}^{n+1} = 2\bar{u}^{n+1} - w_2^0$$

w_1 – incoming characteristic wave
w_2 – outgoing characteristic wave

Solution Method – Taylor Galerkin – Time Discretisation

The System

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{S} - \frac{\partial \mathbf{F}}{\partial x}$$

The Second Derivative

$$\frac{\partial^2 \mathbf{U}}{\partial t^2} = \mathbf{S}_{\mathbf{U}} \frac{\partial \mathbf{U}}{\partial t} - \frac{\partial}{\partial x}\left(\mathbf{F}_{\mathbf{U}} \frac{\partial \mathbf{U}}{\partial t}\right)$$

$$\frac{\partial^2 \mathbf{U}}{\partial t^2} = \mathbf{S}_{\mathbf{U}} \left(\mathbf{S} - \frac{\partial \mathbf{F}}{\partial x}\right) - \frac{\partial \left(\mathbf{F}_{\mathbf{U}} \mathbf{S}\right)}{\partial x} + \frac{\partial}{\partial x}\left(\mathbf{F}_{\mathbf{U}} \frac{\partial \mathbf{F}}{\partial x}\right)$$

The Semi-Discrete Form

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \Delta t \frac{\partial \mathbf{U}^n}{\partial t} + \frac{\Delta t^2}{2} \frac{\partial^2 \mathbf{U}^n}{\partial t^2} + O\left(\Delta t^3\right)$$

$$\frac{\mathbf{U}^{n+1} - \mathbf{U}^n}{\Delta t} = \mathbf{S}^n - \frac{\partial \mathbf{F}^n}{\partial x} - \frac{\Delta t}{2}\left[\frac{\partial}{\partial x}\left(\mathbf{F}_{\mathbf{U}}^n \mathbf{S}^n - \mathbf{F}_{\mathbf{U}}^n \frac{\partial \mathbf{F}^n}{\partial x}\right) - \mathbf{S}_{\mathbf{U}}^n \frac{\partial \mathbf{F}^n}{\partial x} - \mathbf{S}_{\mathbf{U}}^n \mathbf{S}^n\right]$$

Solution Method – Spatial DIscretisation

$$[\mathbf{M}_e]\{\Delta\mathbf{U}\} = \Delta t\left([\mathbf{K}_e]\{\mathbf{F}\}^n + [\mathbf{L}_e]\{\mathbf{S}\}^n + \{\mathbf{f}_{\Gamma_e}\}^n\right)$$

where

$$[\mathbf{M}_e] = \frac{l_e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Lumped

$$[\mathbf{M}_e] = \frac{l_e}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

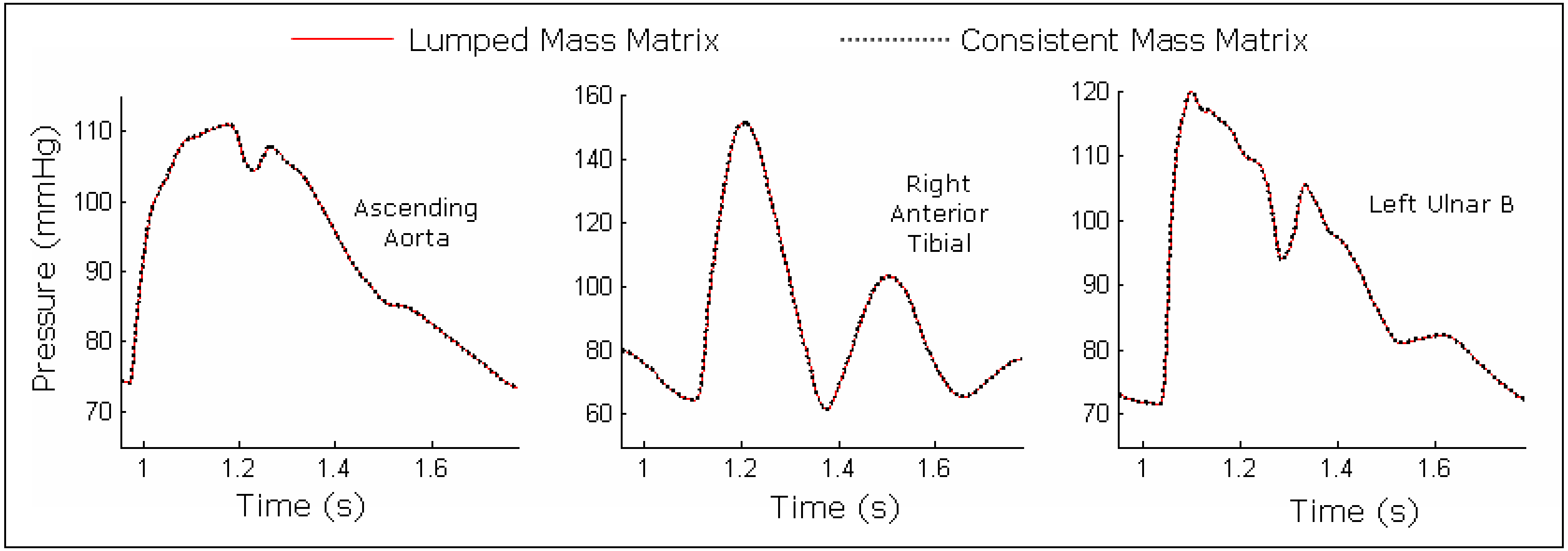
$$-\int_{\Omega_e} \mathbf{N}^T \frac{\partial \hat{\mathbf{F}}^n}{\partial x} d\Omega_e = \int_{\Omega_e} \frac{\partial \mathbf{N}^T}{\partial x}\hat{\mathbf{F}}^n d\Omega_e - \int_{\Gamma_e} \mathbf{N}^T \hat{\hat{\mathbf{F}}}^n \mathbf{n} d\Gamma_e$$

Time step $\Delta t_{max} = \dfrac{\Delta x_{min}}{c_{max}}$

Post processed flux

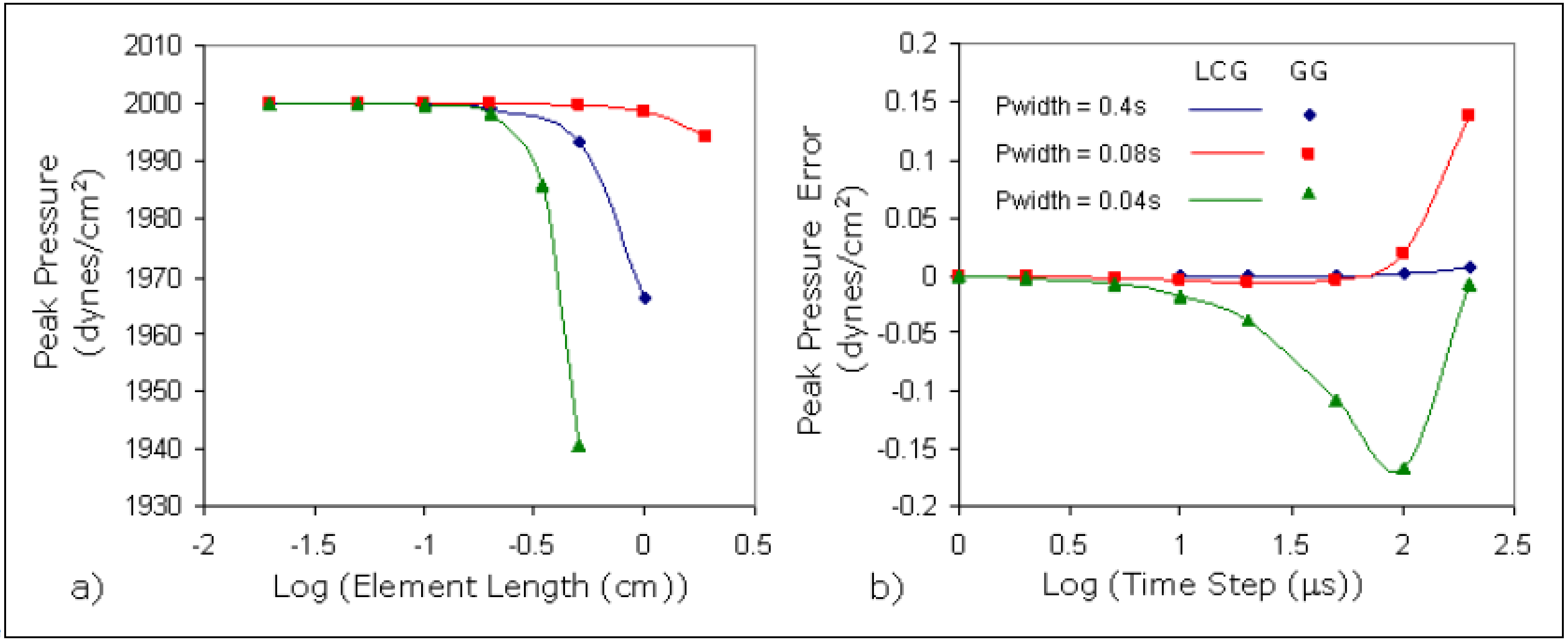
Results

Consistent and Lumped Mass



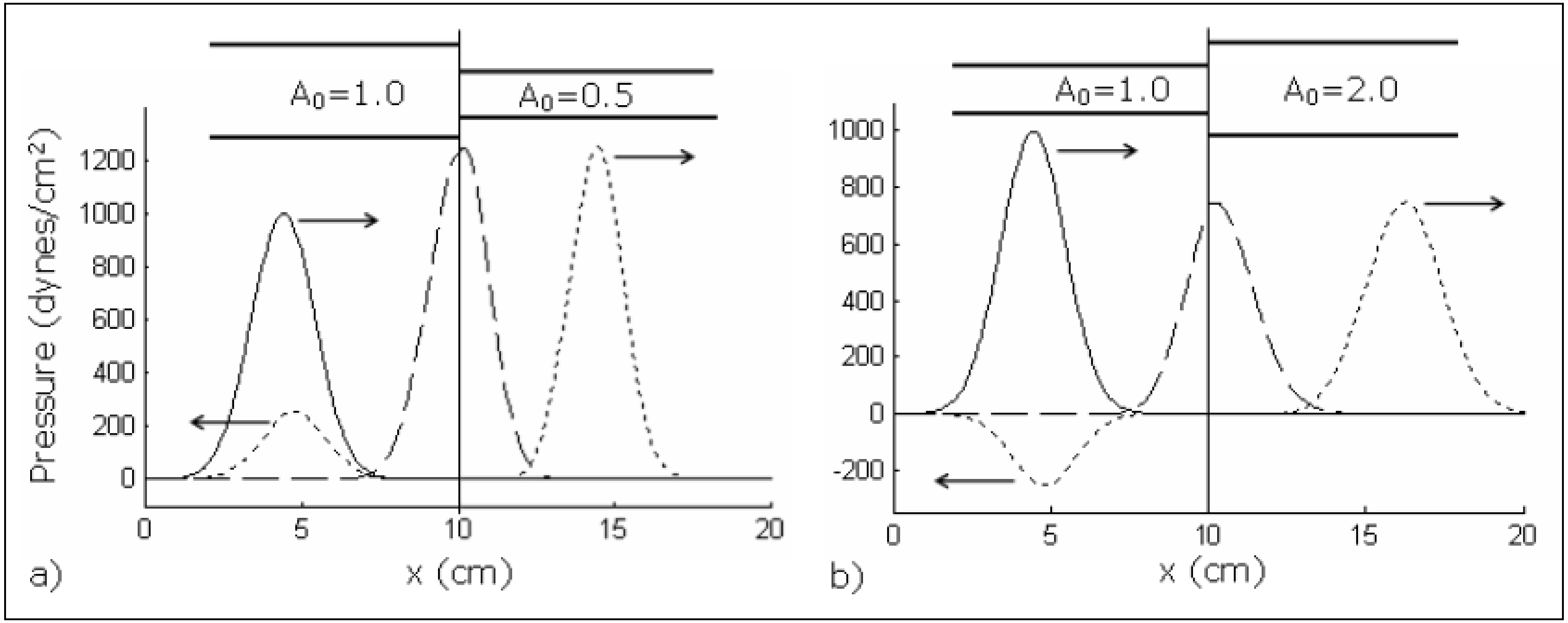
Results

Comparison Between LCG and GG



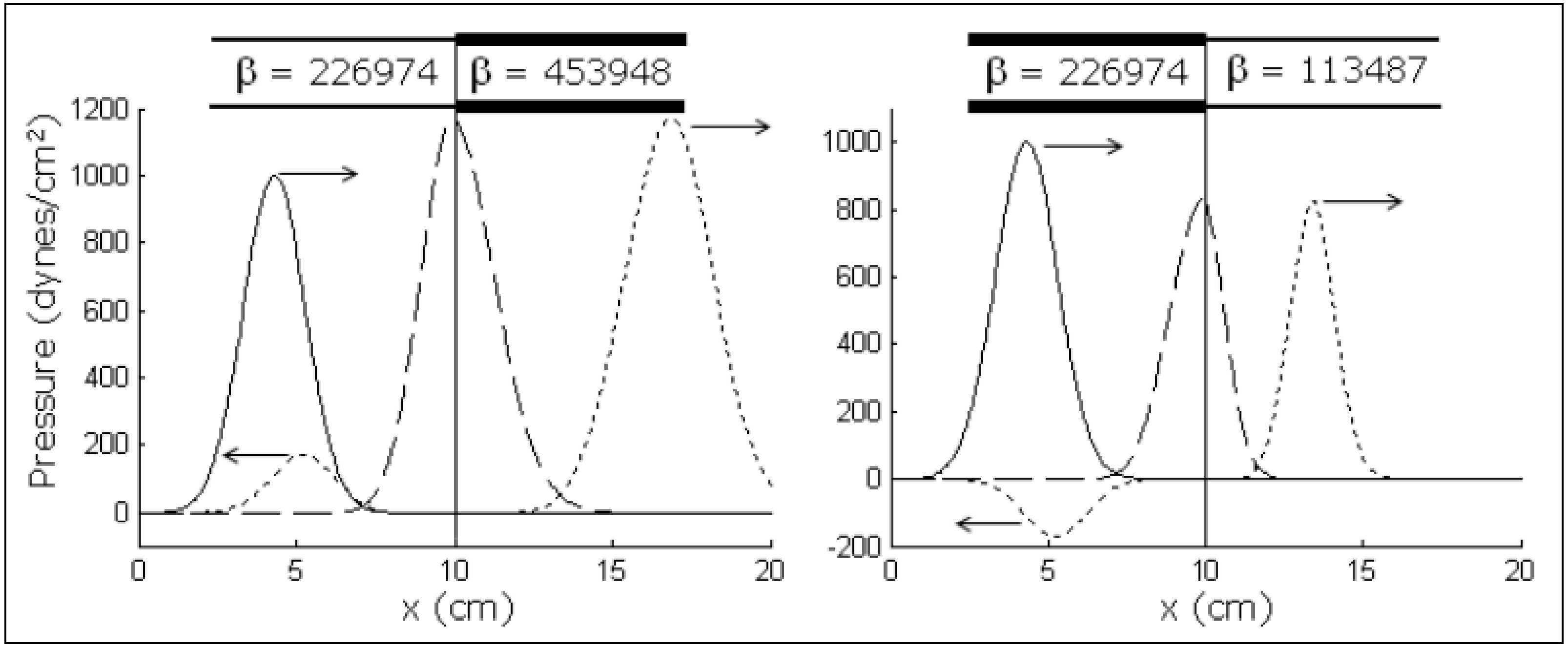
Results

Single Vessel – Step increase and decrease in A



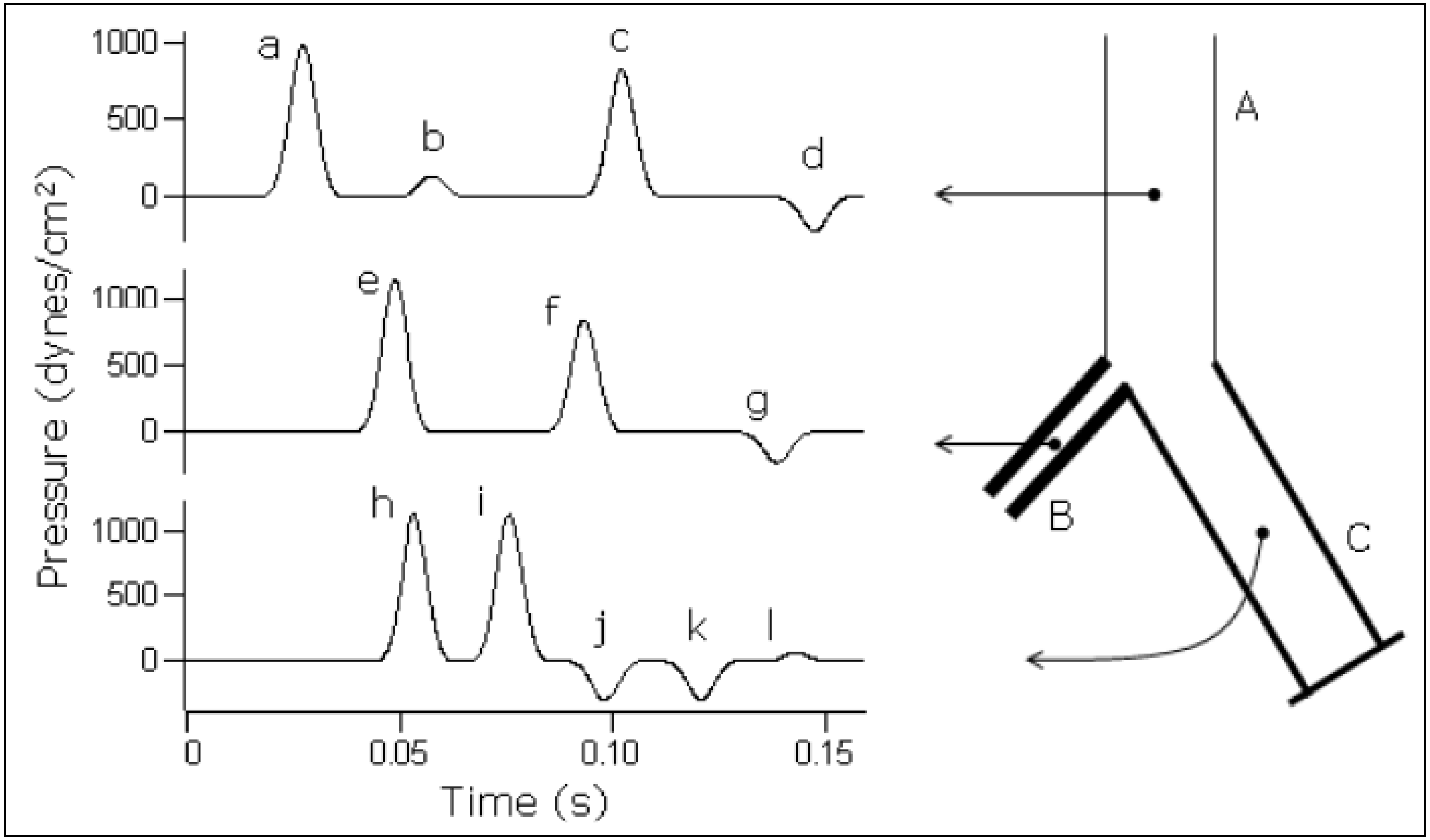
Results

Single Vessel – Step increase and decrease in Property Function

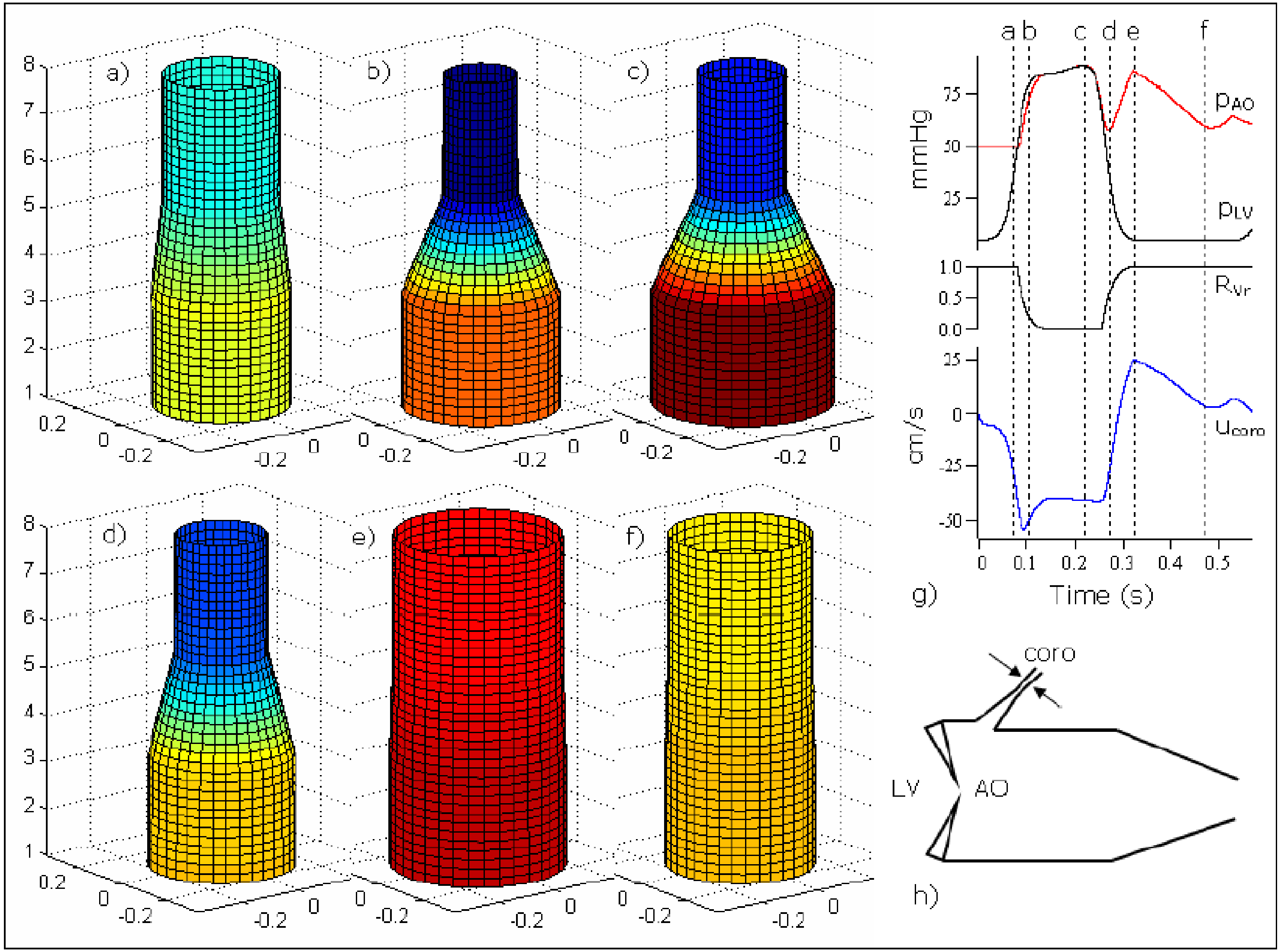


Results

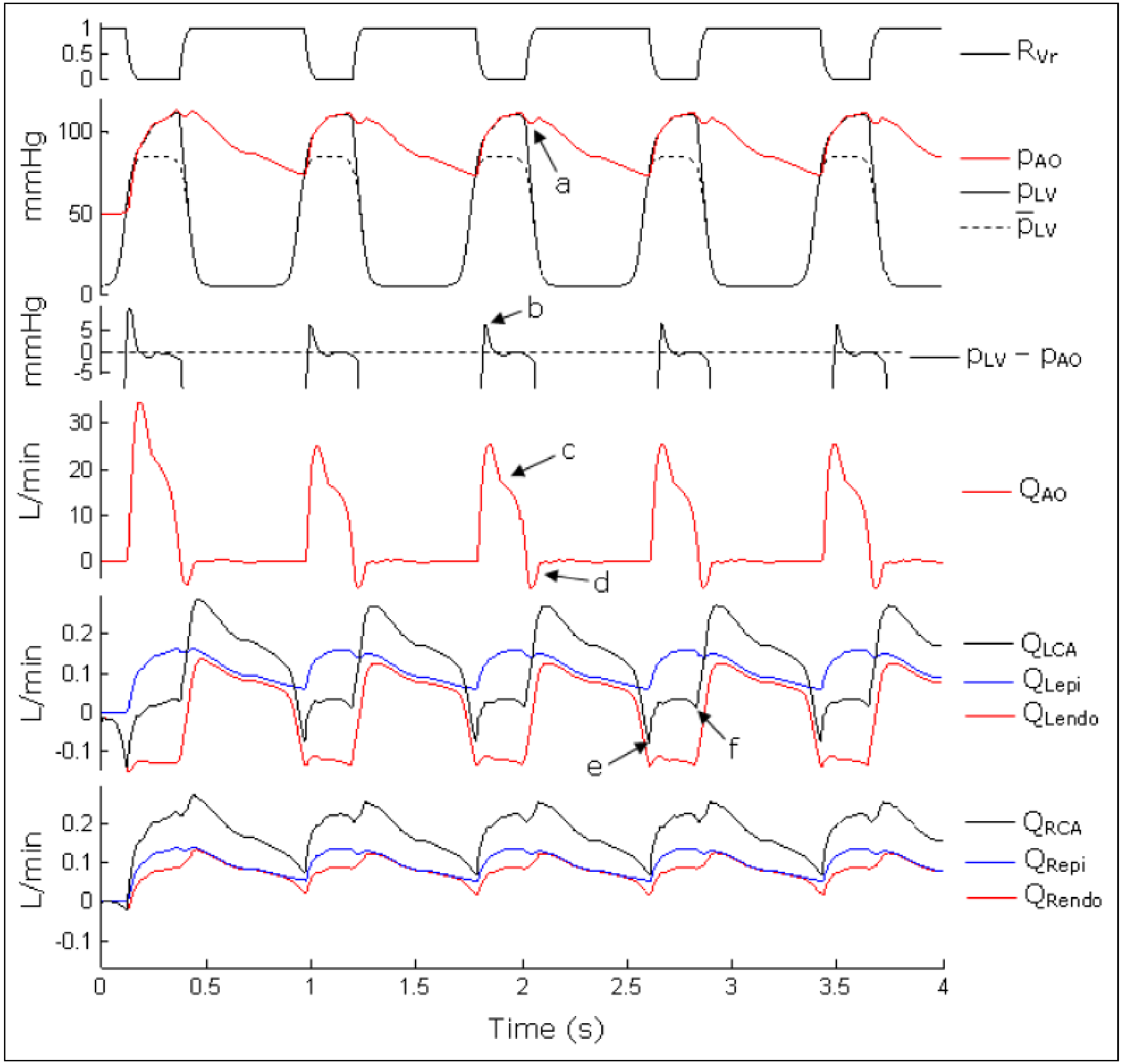
Single Bifurcation



Results – Coronary artery

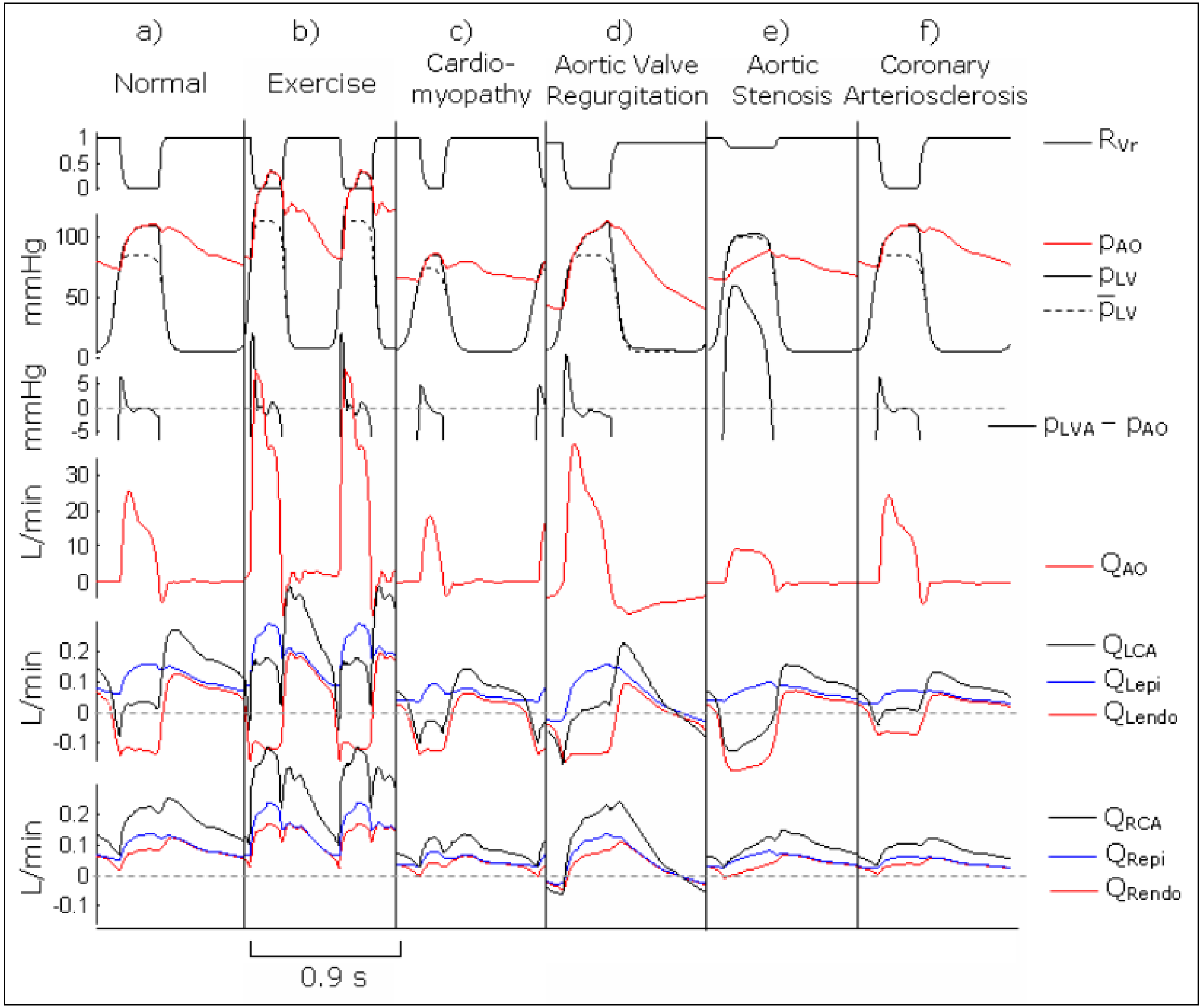


Results



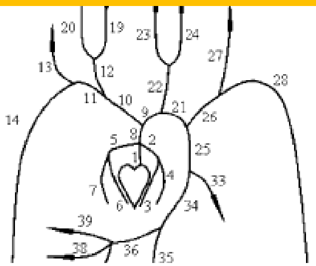
Rvr
mmHg
pAO
pLV
p̄LV
a
b
mmHg
pLV − pAO
c
L/min
QAO
d
L/min
QLCA
QLepi
QLendo
e
f
L/min
QRCA
QRepi
QRendo
Time (s)

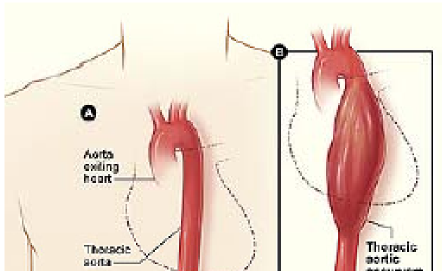
Results

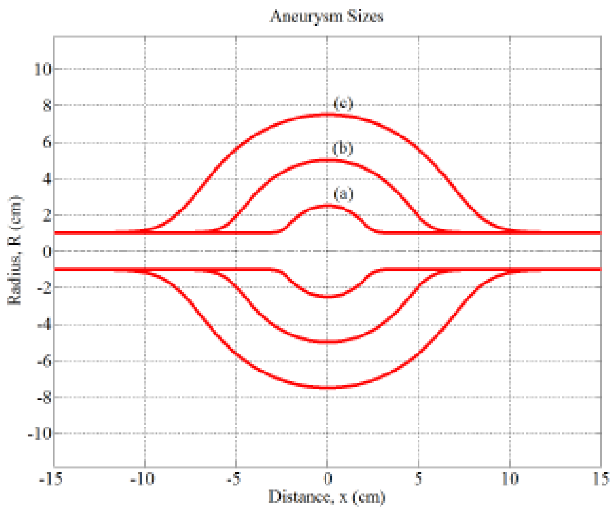


a) Normal
b) Exercise
c) Cardio-myopathy
d) Aortic Valve Regurgitation
e) Aortic Stenosis
f) Coronary Arteriosclerosis
mmHg
L/min
0.9 s
RVr
pAO
pLV
pLVA − pAO
QAO
QLCA
QLepi
QLendo
QRCA
QRepi
QRendo

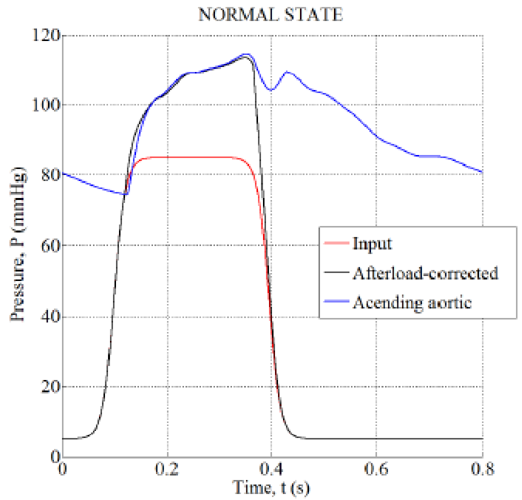
Results – thoracic aneurysms

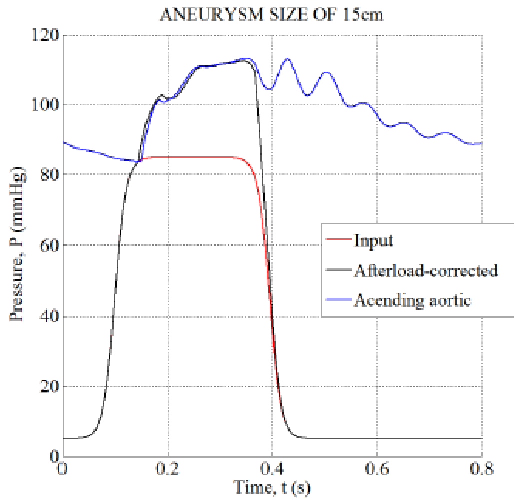




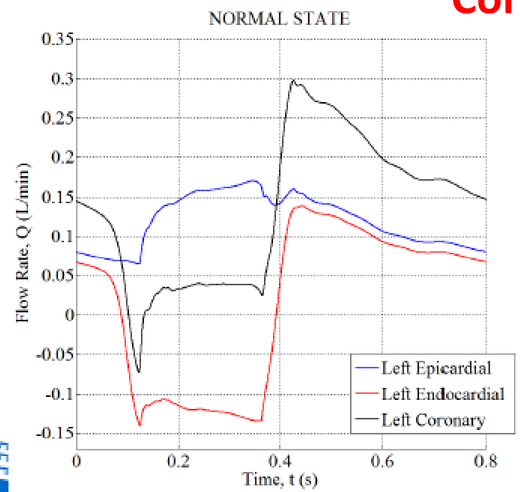


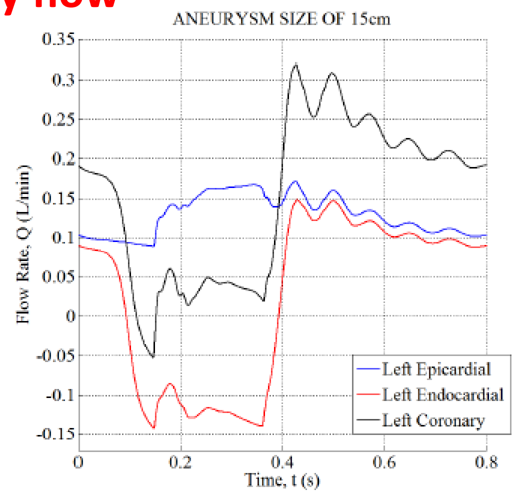
Aortic flow





Coronary flow





Part I – 1D flow conclusions

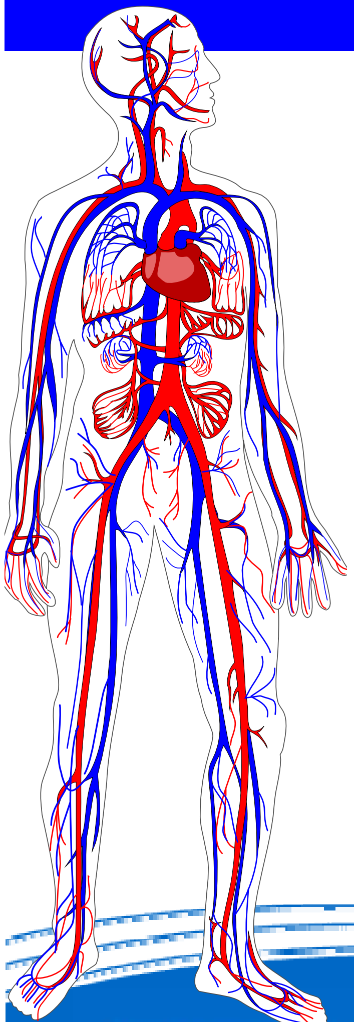
You have learned:

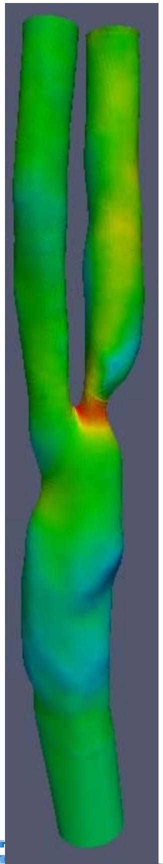
- One-dimensional systemic circulation model development
- One method of solution
- How to obtain results

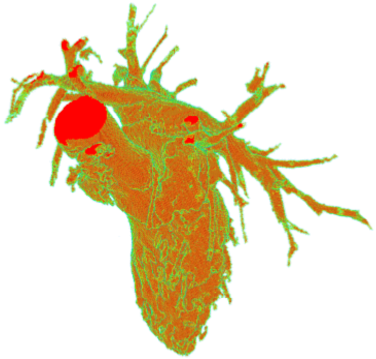
For further details:

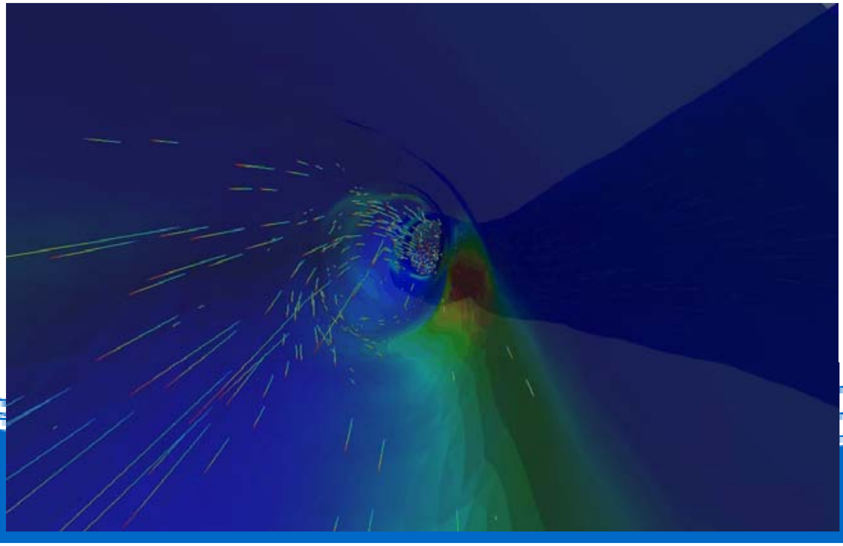
Mynard and Nithiarasu, 2008, Communications in Numerical Methods in Engineering.

Part II Subject-Specific 3D Modelling

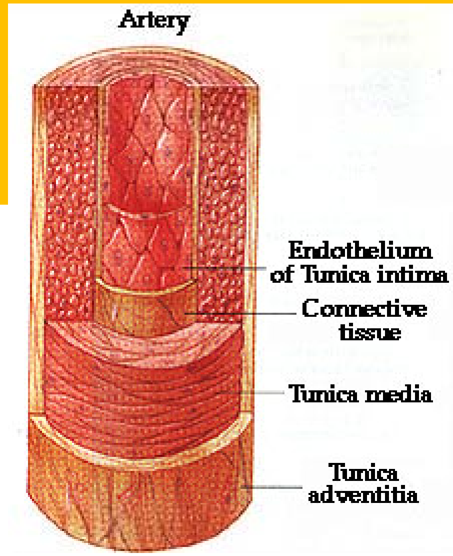








Carotid Artery



Carotid Artery Disease

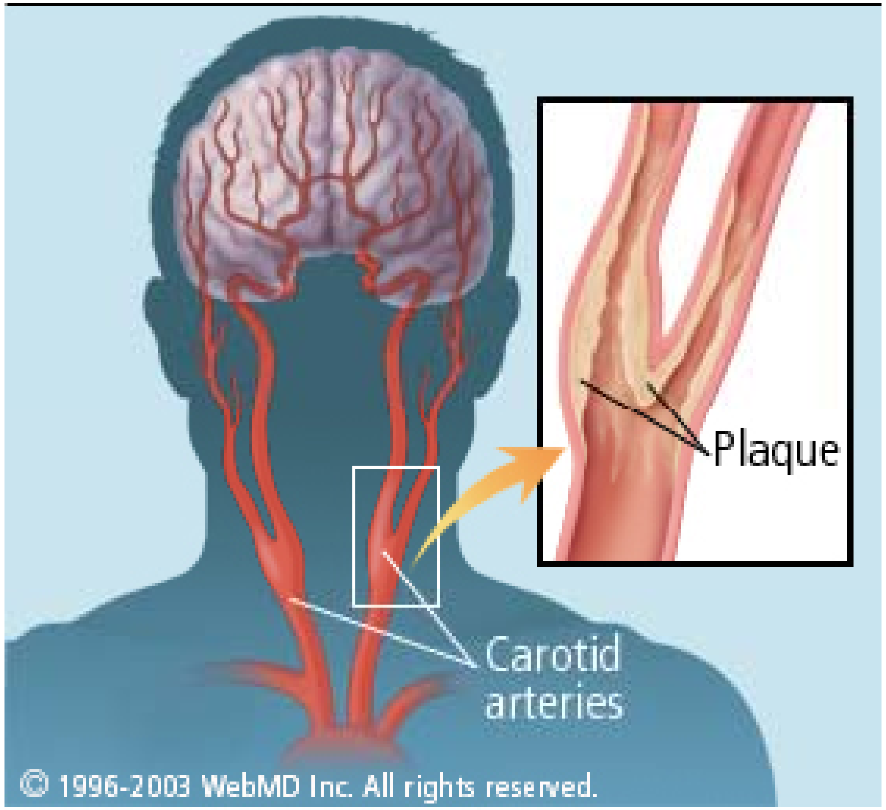
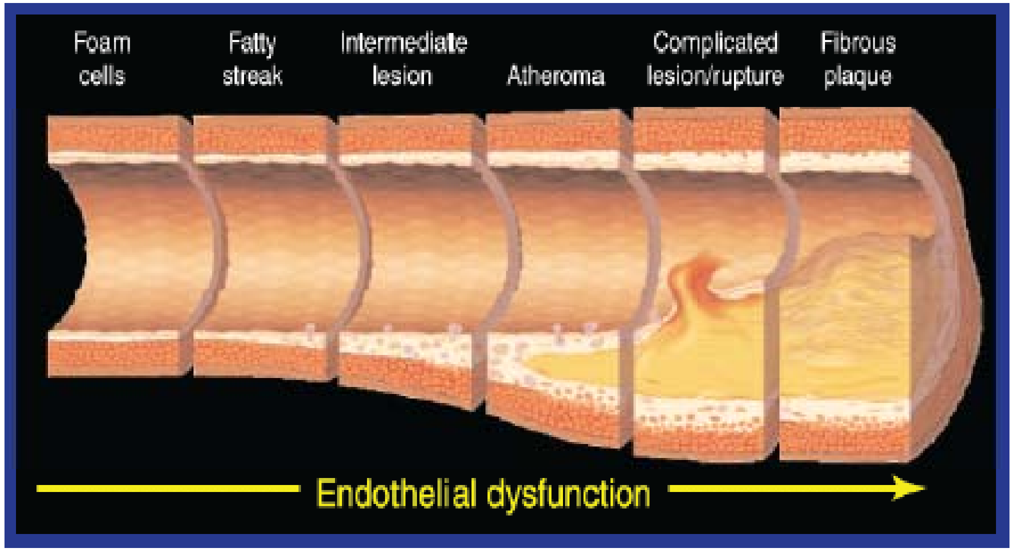


Illustration of Carotid Artery Stenosis



Courtesy of Prof Julian Halcox, Cardiff University

Motivation

- ❑ **Better and faster diagnosis.**
- ❑ **Understanding plaque build-up and atherosclerosis.**

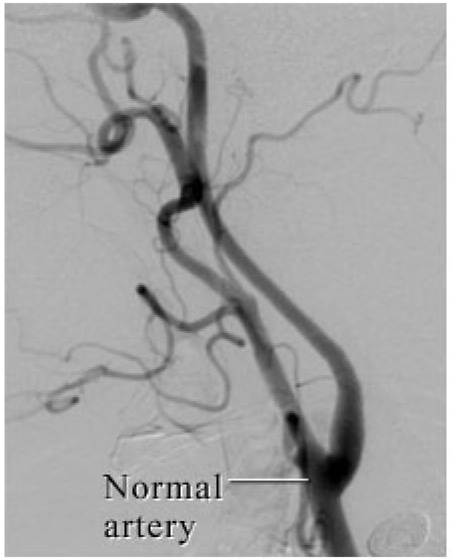


Figure 1

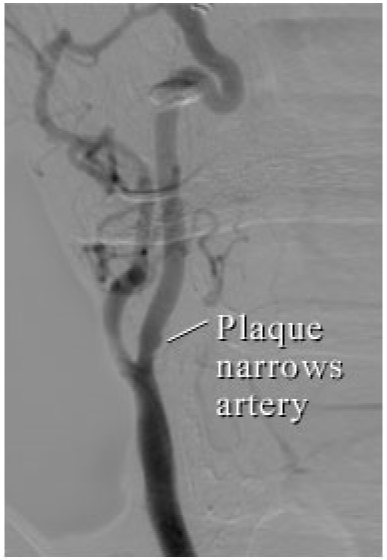
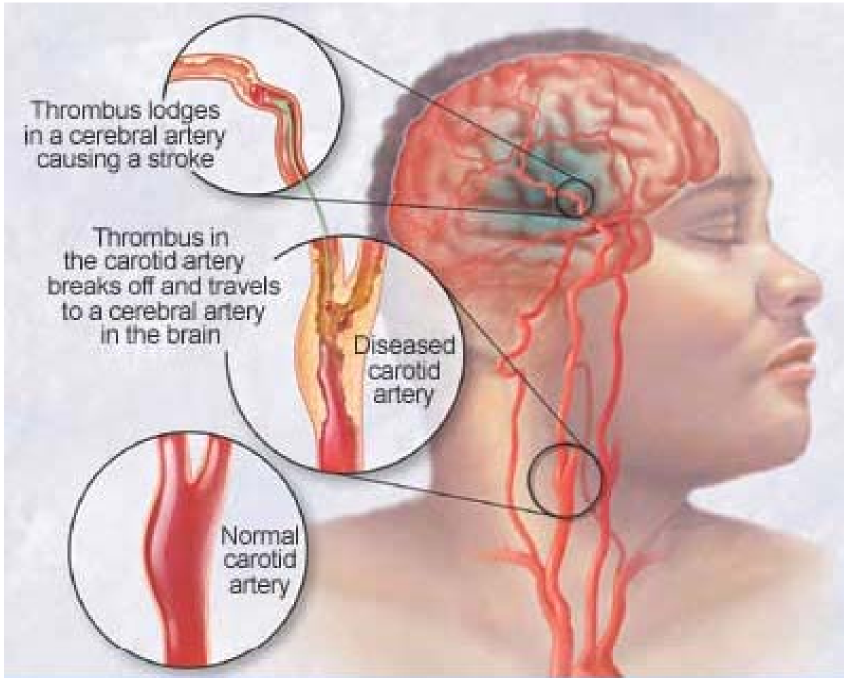


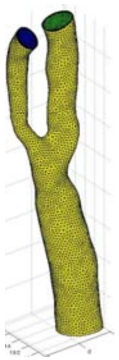
Figure 2

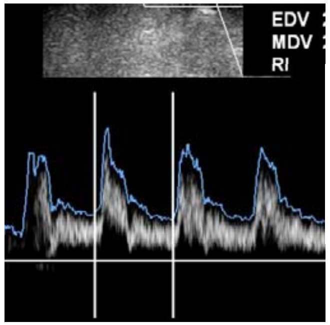


Better and faster diagnosis

➢ Reduction in waiting list/stroke

UK statistics
- 120,000 cerebrovascular event per year
- 10,000 patients eligible for endartrectomy
- Only 4500 endartrectomy performed every year
- 2000 patients face stroke while waiting for treatment
- Only 20% patients are treated within 2 weeks (after TIA)





TIA to objective selection
- Duplex scan
- CT angiogram
- Radiologist time
- Together take between 6 and 12 weeks

Question:
- Multiple scan essential?

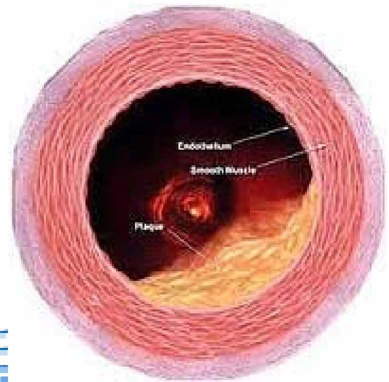
Answering:
- Clinical, image processing, CFD to establish the correlation between scanning modalities.
- If successful, translate to clinic

Plaque Build-up/atherosclerosis

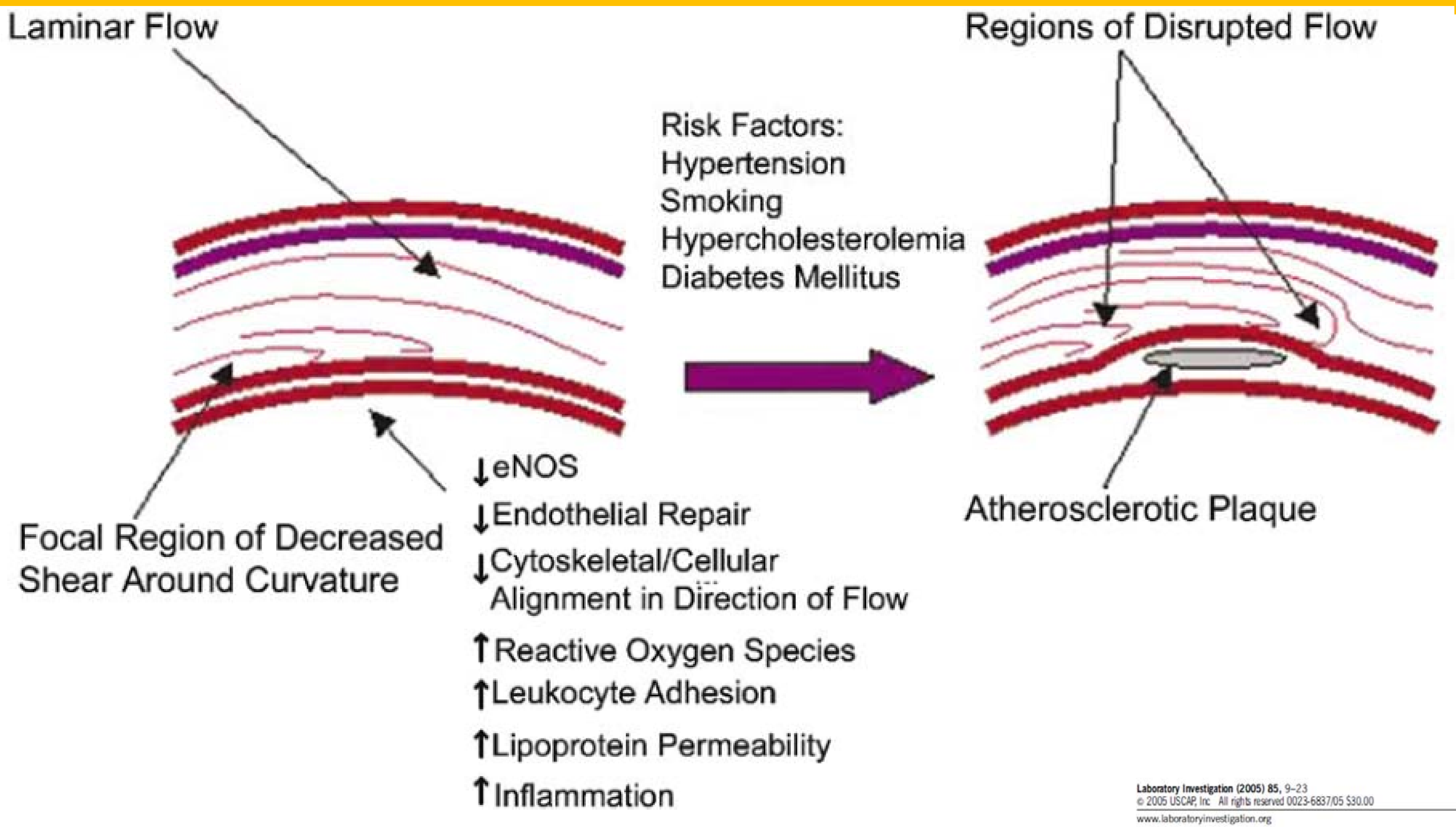
- As plaque deposits grow, a condition called <u>atherosclerosis</u> results. This condition causes the arteries to narrow and harden.

- Although experts don't know for sure what starts atherosclerosis, the process seems to stem from <u>damage to the arterial</u> wall.

- Thus <u>wall forces</u> play an important role.



Wall Shear Stress



Laboratory Investigation (2005) 85, 9–23
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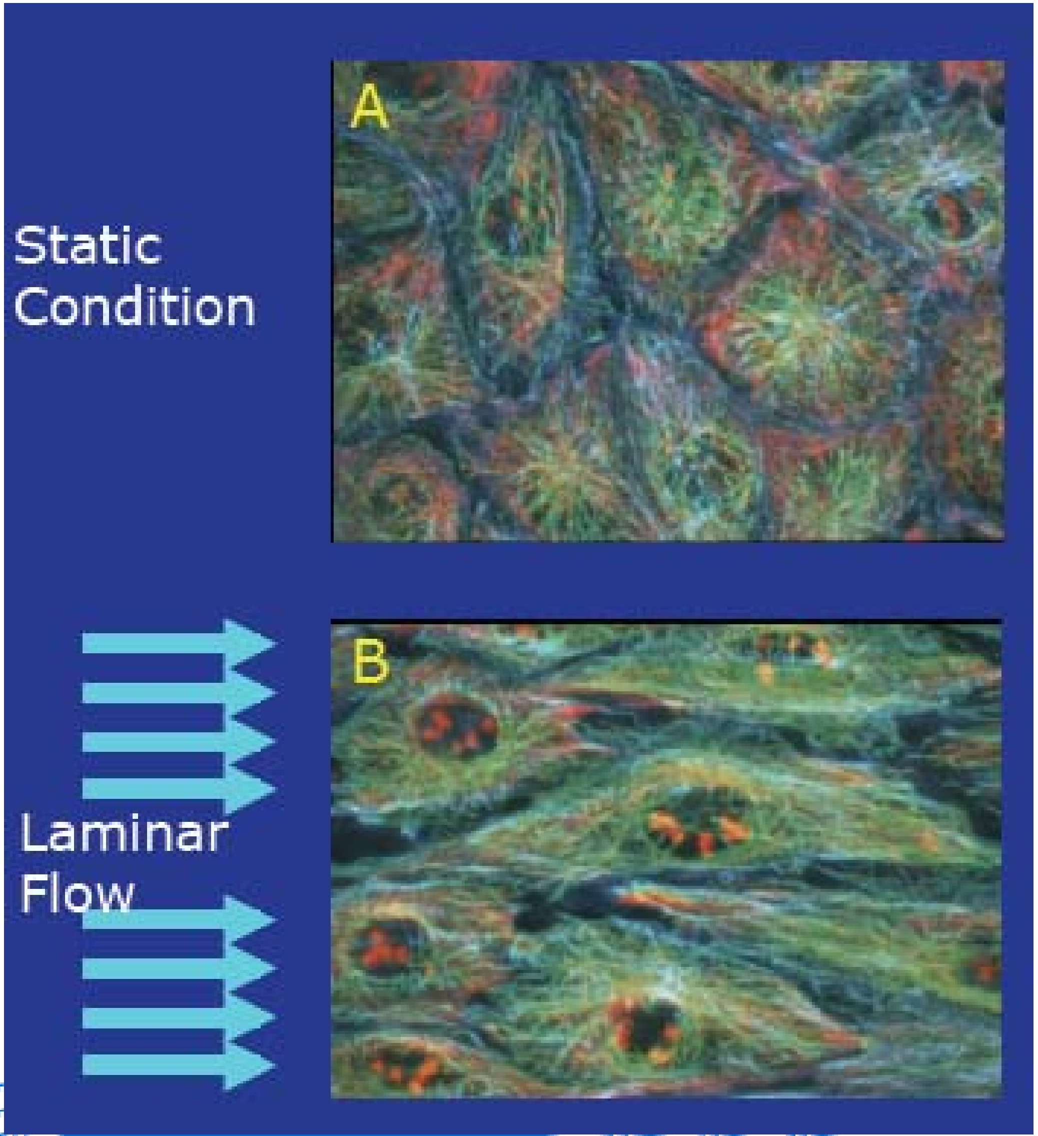
www.laboratoryinvestigation.org

The role of shear stress in the pathogenesis of atherosclerosis

Kristopher S Cunningham[1,2] and Avrum I Gotlieb[1,2]

[1]*Department of Pathology, Toronto General Research Institute, University Health Network, Canada and*
[2]*Department of Laboratory Medicine and Pathobiology, University of Toronto, Toronto, Ontario, Canada*

Wall Shear Stress



Cells Elongate in
Response to Shear

Fibre Cytoskeleton
Aligns PARALLEL
To flow

Chien AJP(H) 2007;292:1209

Subject-specific modelling

- A robust subject-specific modelling framework

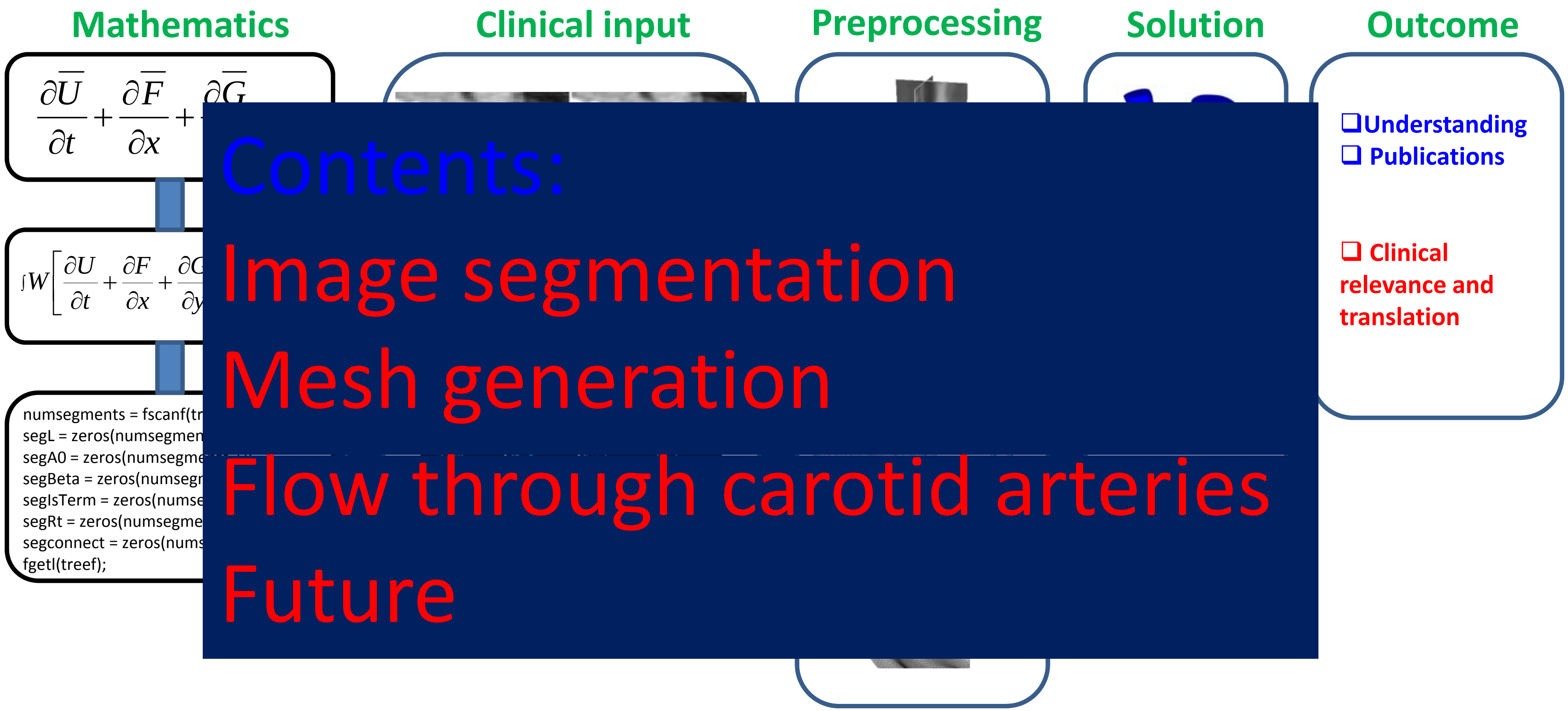
Subject-specific modelling framework

Pre-processing
Data preparation, image processing, meshing , boundary conditions etc.

Modelling
Material properties, turbulence, non-Newtonian, FSI, mutliscale, etc.

Clinical translation
Understanding, establishing new diagnostic and treatment protocols, surgical simulation, etc.

Subject-Specific Modelling Pipeline - Swansea Model



Contents:

Image segmentation

Mesh generation

Flow through carotid arteries

Future

Image Segmentation

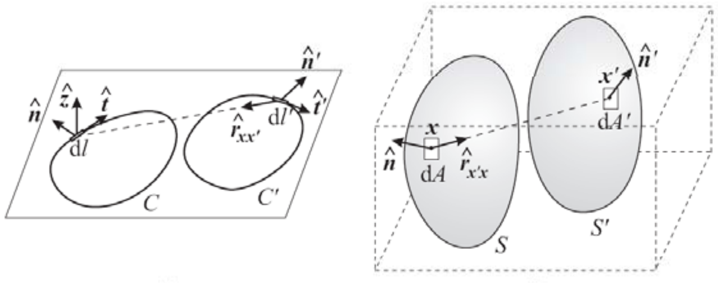
- Level set representation given as

$$\frac{\partial \Phi}{\partial t} = \alpha\, g(\boldsymbol{x})\, \kappa(\boldsymbol{x}, t)\, \|\nabla \Phi\| - (1-\alpha)(\boldsymbol{F}(\boldsymbol{x}) \cdot \nabla \Phi)$$

where α is a tuning parameter, $g(\boldsymbol{x}) = 1/(1 + \|\nabla I\|)$ is the edge stopping function, $\kappa(\boldsymbol{x}, t) = \nabla \cdot (\nabla\Phi/\|\nabla\Phi\|)$ is the curvature of the surface $\Phi = const$, $\boldsymbol{F}(\boldsymbol{x}) = [F_x, F_y, F_z]^T$ is the flow function determined by image I.

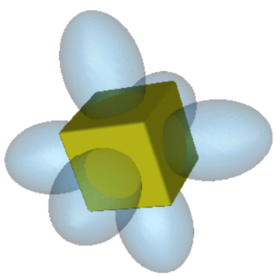
$$\boldsymbol{F}(\boldsymbol{x}) = \pm \frac{\nabla \Phi(\boldsymbol{x})}{\|\nabla \Phi(\boldsymbol{x})\|} G(\boldsymbol{x})$$

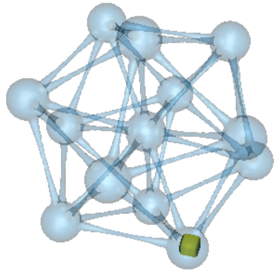
$$G(\boldsymbol{x}) = P.V. \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|^{\lambda+1}} * \nabla I(\boldsymbol{x}) = P.V. \iint_{\boldsymbol{x}' \in \mathcal{D}} \frac{\boldsymbol{x} - \boldsymbol{x}'}{\|\boldsymbol{x} - \boldsymbol{x}'\|^{\lambda+1}} \cdot \nabla I(\boldsymbol{x}')\, d\boldsymbol{x}'.$$



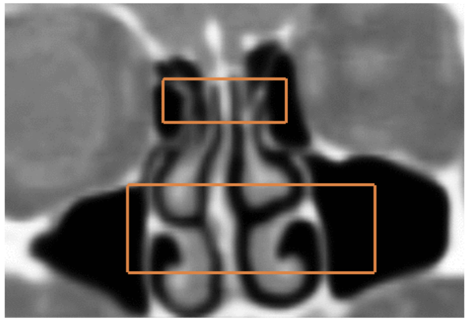
Relative position and orientation between geometries in 2D (a) and 3D (b).

Image Segmentation

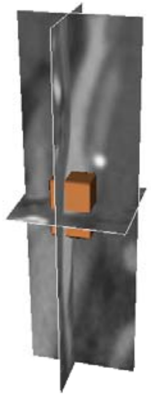




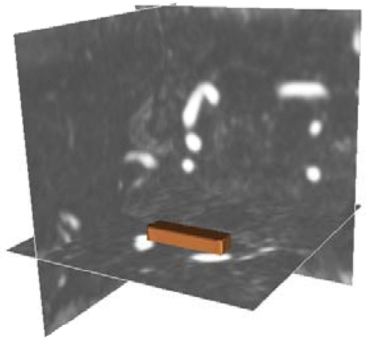
Nose

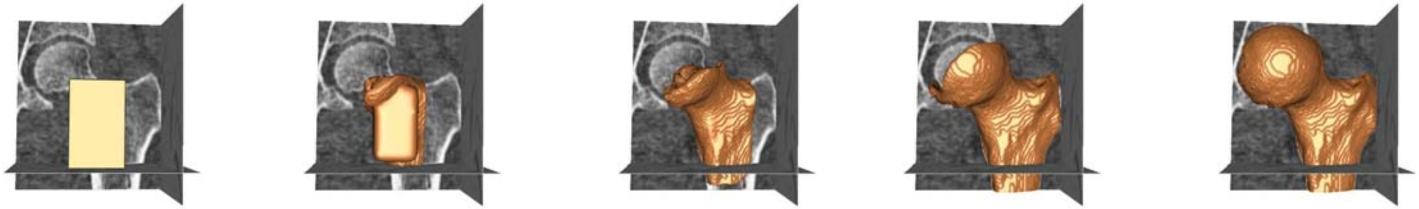


Carotid



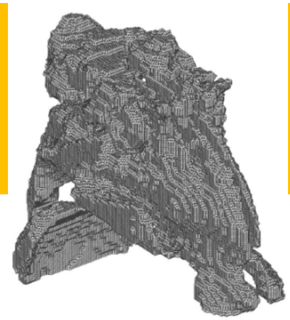
Circle of Willis



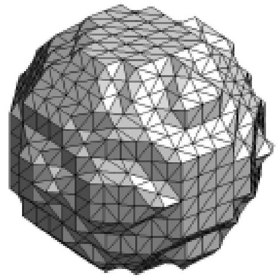


Femur or thigh bone

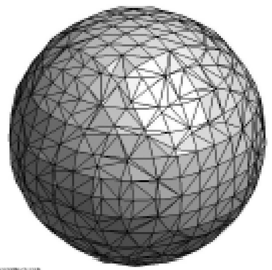
Semi-Automatic Meshing



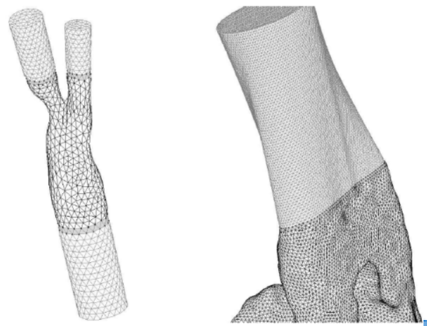
- Image processor gives 3D binary file



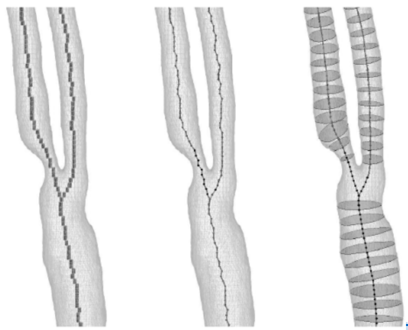
- Initial surface mesh
- (advanced marching cube)



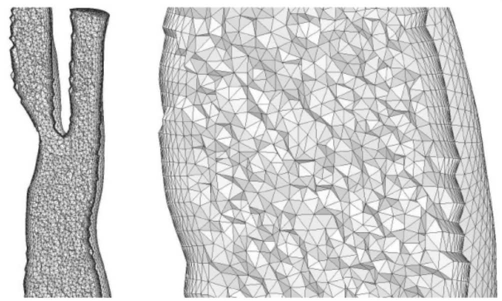
- Construct boundary layer mesh
- Volume mesh



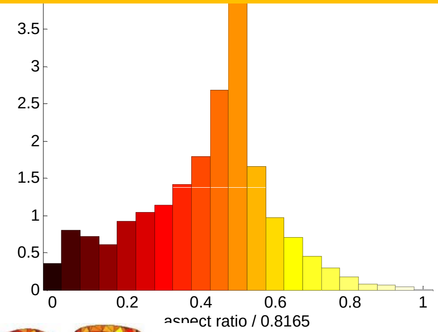
Boundary attachment



skeleton



Taubing smoothing (restricted)

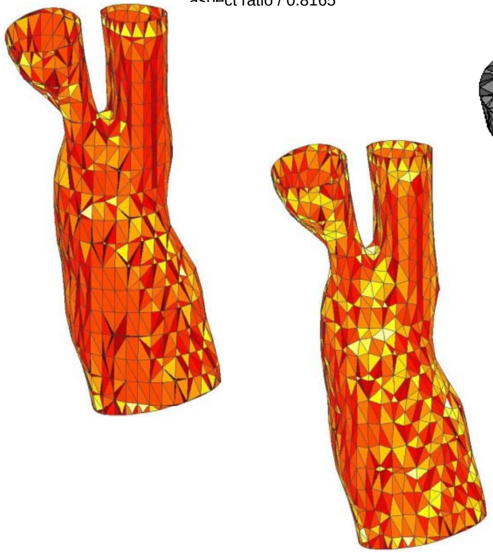


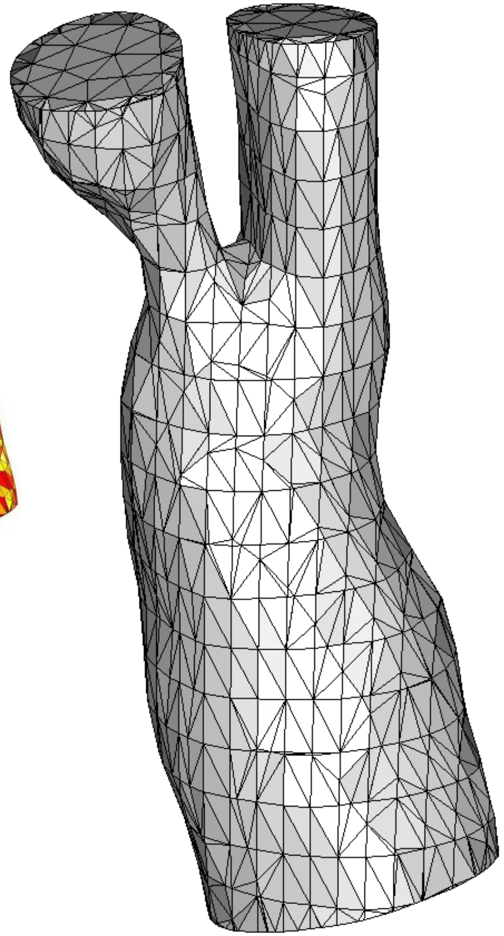
$$p^{\mathrm{inter}} = (1-\lambda)\,p + \lambda\,\frac{1}{d_p}\sum_{i\in\mathcal{P}_p} p_i$$

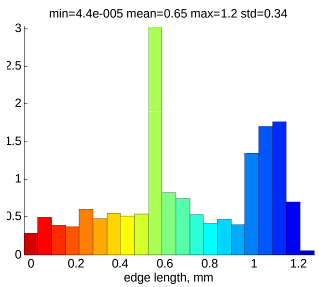
$$p^{\mathrm{new}} = (1-\mu)\,p^{\mathrm{inter}} + \mu\,\frac{1}{d_p}\sum_{i\in\mathcal{P}_p} p_i^{\mathrm{inter}}$$

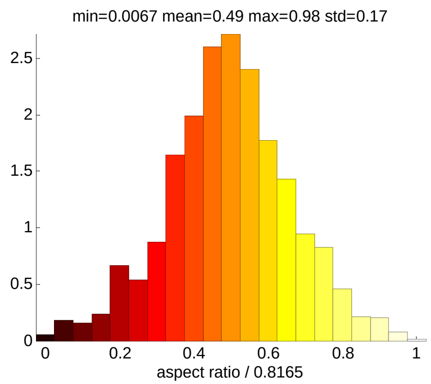
$\lambda>0,\ \mu<0$

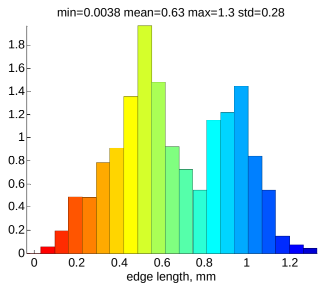
$$p^{\mathrm{new,constr.}} = p + h_{\max}\frac{p^{\mathrm{new}}-p}{\|p^{\mathrm{new}}-p\|}, \qquad \text{if } \|p^{\mathrm{new}}-p\| > h_{\max}$$



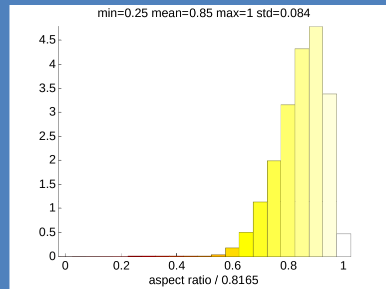


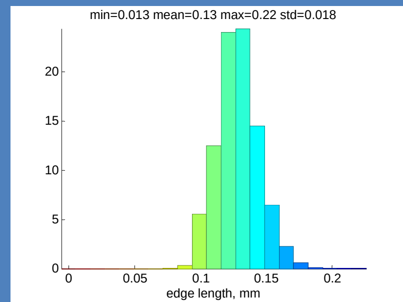






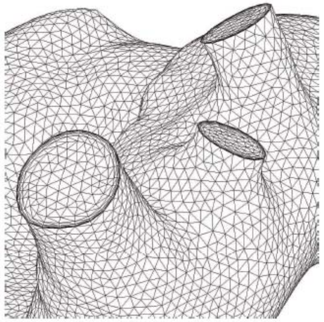
Final mesh



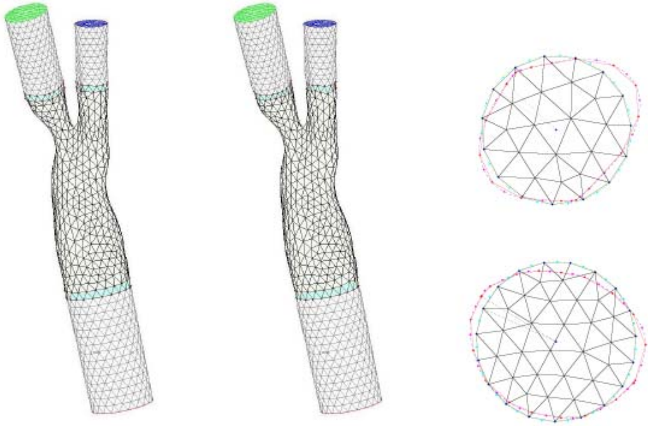


Semi-Automatic Meshing

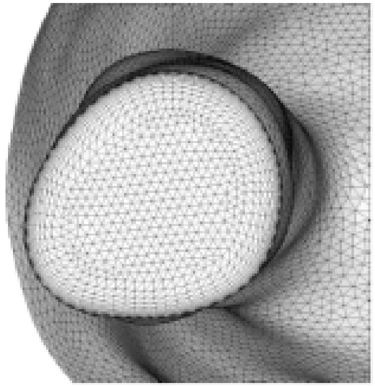
Aorta

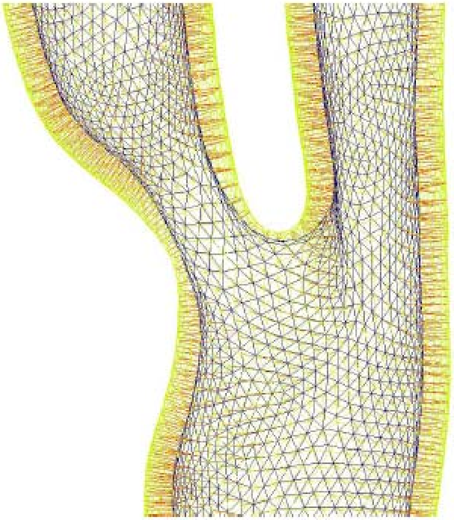


Carotid with extension tubes

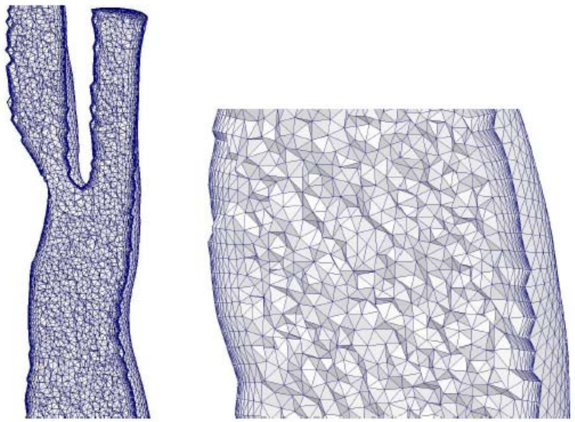


Carotid with boundary layer mesh

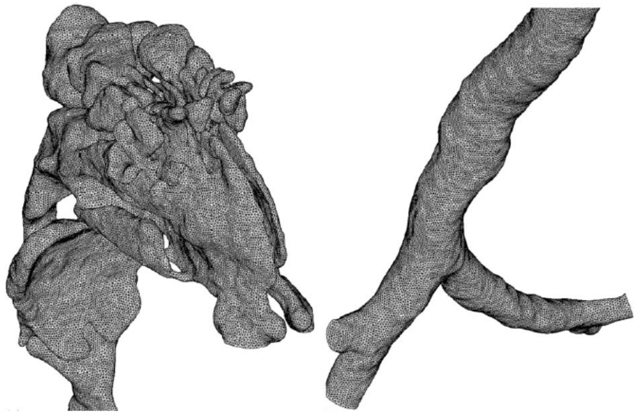




Carotid surface normal

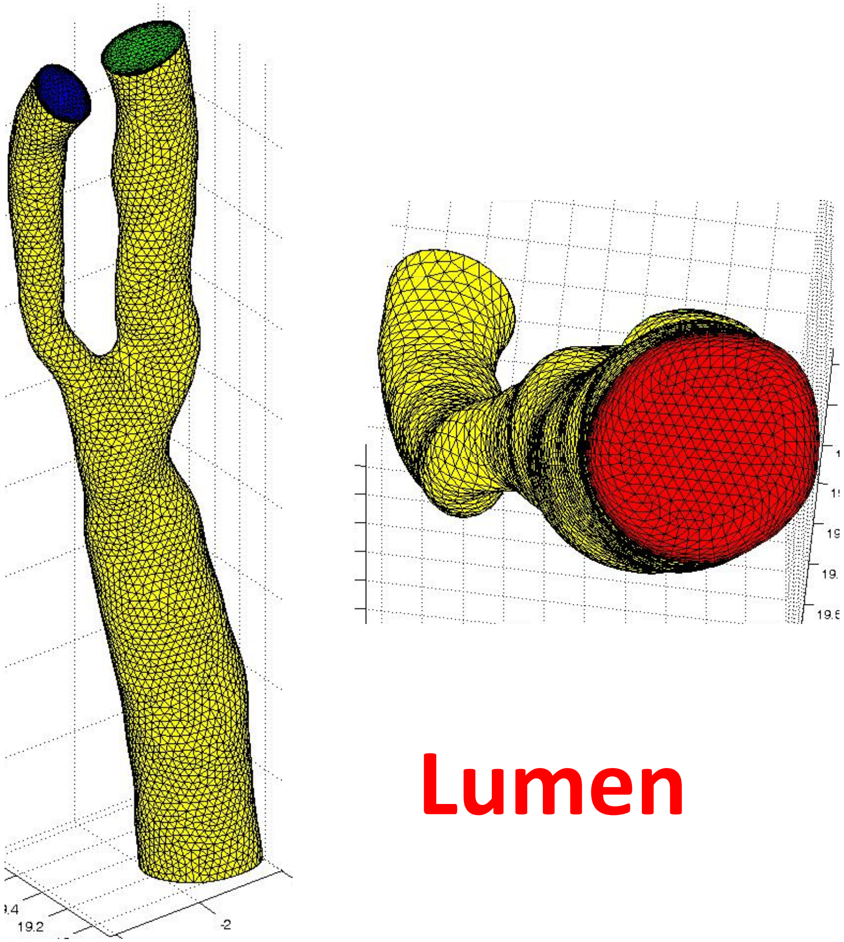


Carotid volume mesh

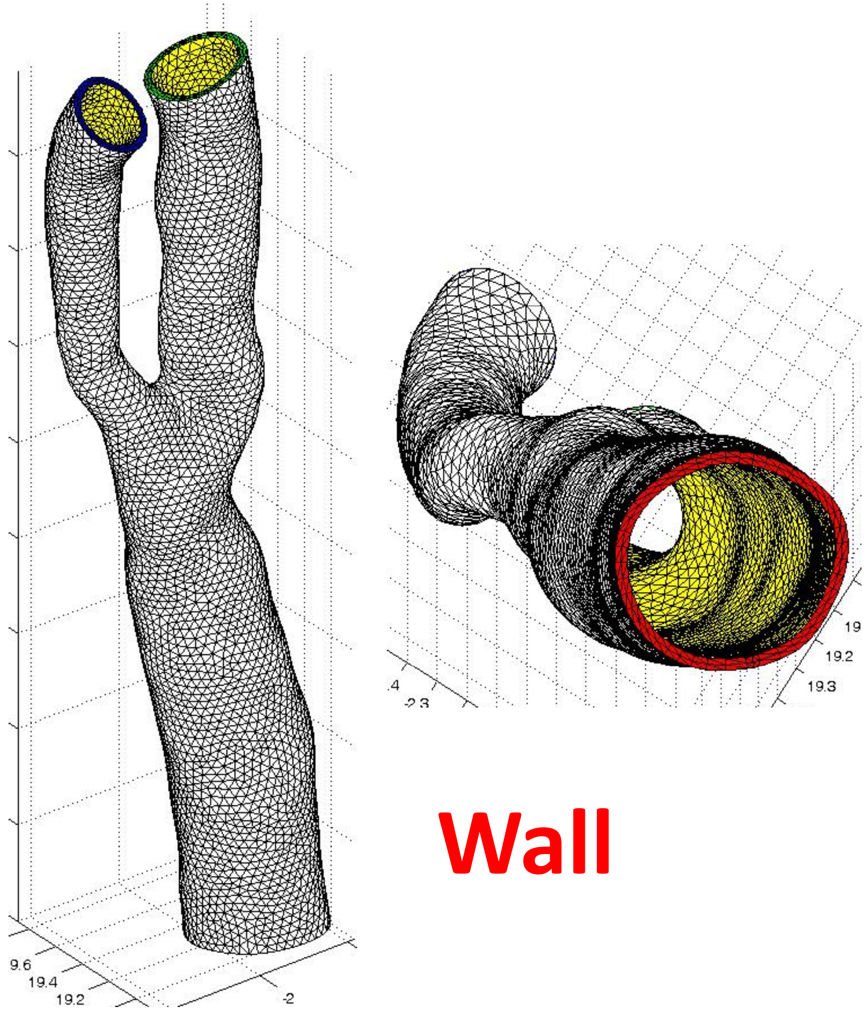


Human upper airway

Fluid/Solid Meshing



Lumen



Wall

CFD - Fractional Step Method

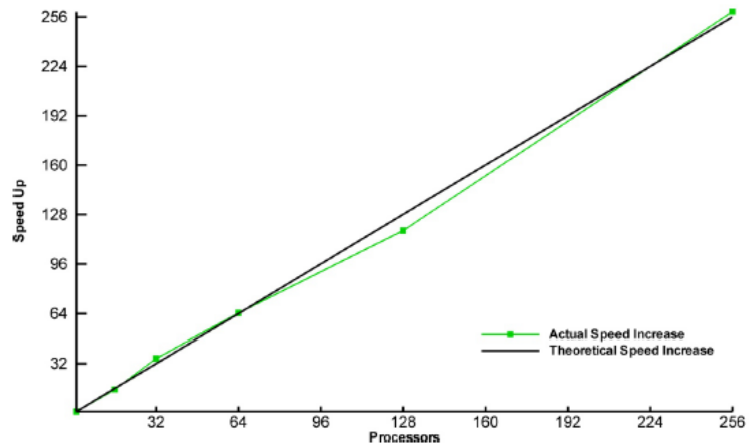
- The first step of the CBS scheme is

$$\frac{u_i^{\dagger} - u_i^{n}}{\Delta t} = -\left(\frac{\partial F_{ij}}{\partial x_j}\right)^n + \frac{\Delta t}{2} u_k \frac{\partial}{\partial x_k}\left(\frac{\partial F_{ij}}{\partial x_j}\right)^n$$

where $F_{ij} = \left(u_j u_i - \frac{1}{Re}\frac{\partial u_i}{\partial x_j}\right)$

- The second step is defined as

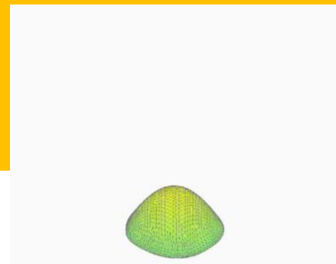
$$\frac{1}{\beta^2}\frac{\Delta p}{\Delta t} = -\rho \frac{\partial}{\partial x_i}\left(u_i^{\dagger} - \Delta t\left(\frac{\partial p}{\partial x_i}\right)^n\right)$$



- The third step is

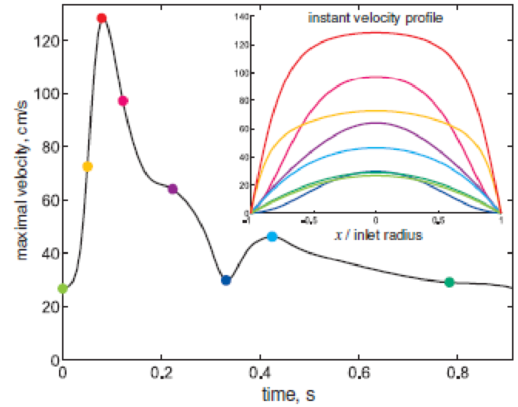
$$u_i^{n+1} = u_i^{\dagger} - \Delta t\left(\frac{\partial p}{\partial x_i}\right)^n + \frac{\Delta t^2}{2} u_k \frac{\partial}{\partial x_k}\left(\frac{\partial p}{\partial x_i}\right)^n$$

Flow Boundary Conditions



Compute complex amplitude:

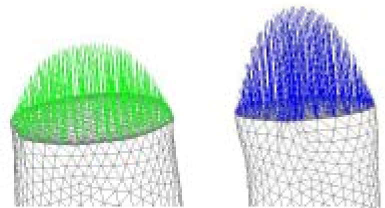
$$\tilde{U}_n = \int_0^T U(t) e^{-i\omega_n t}, \quad \omega_n = \frac{2\pi}{T}, \quad n = 0, ..., N$$



Solve

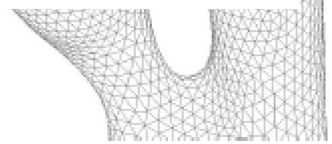
$$\begin{aligned} \nabla_\perp^2 \tilde{u}_n + k_n^2 \tilde{u}_n &= -1, \qquad \{x, y\} \in \Omega \\ \tilde{u}_n &= 0, \qquad \{x, y\} \in \partial\Omega. \end{aligned}$$

$$k_n = \sqrt{-i\omega_n / \nu}$$

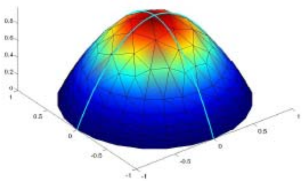


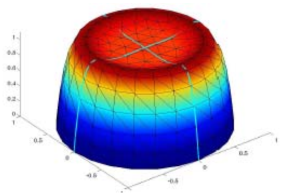
Compute velocity profile

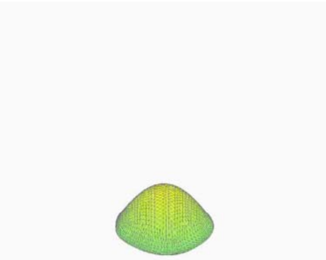
$$u(x, y, t) = \sum_{n=-N}^{N} \tilde{U}_n \, \tilde{v}_n(x, y) \; e^{i\omega_n t} = U_0 \tilde{v}_0(x, y) + 2 \sum_{n=1}^{N} \mathrm{Re} \left[\tilde{U}_n \, \tilde{v}_n(x, y) \; e^{i\omega_n t} \right]$$

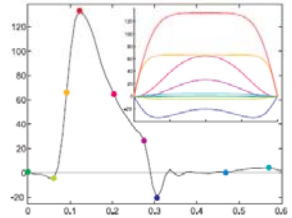


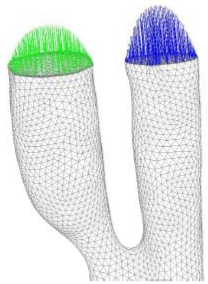
Flow Solver

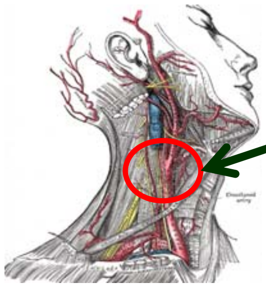


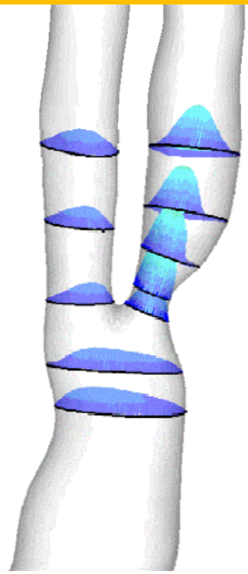


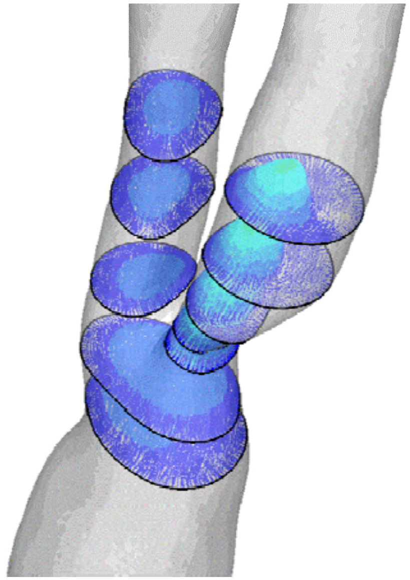


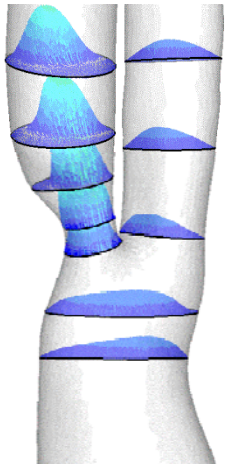










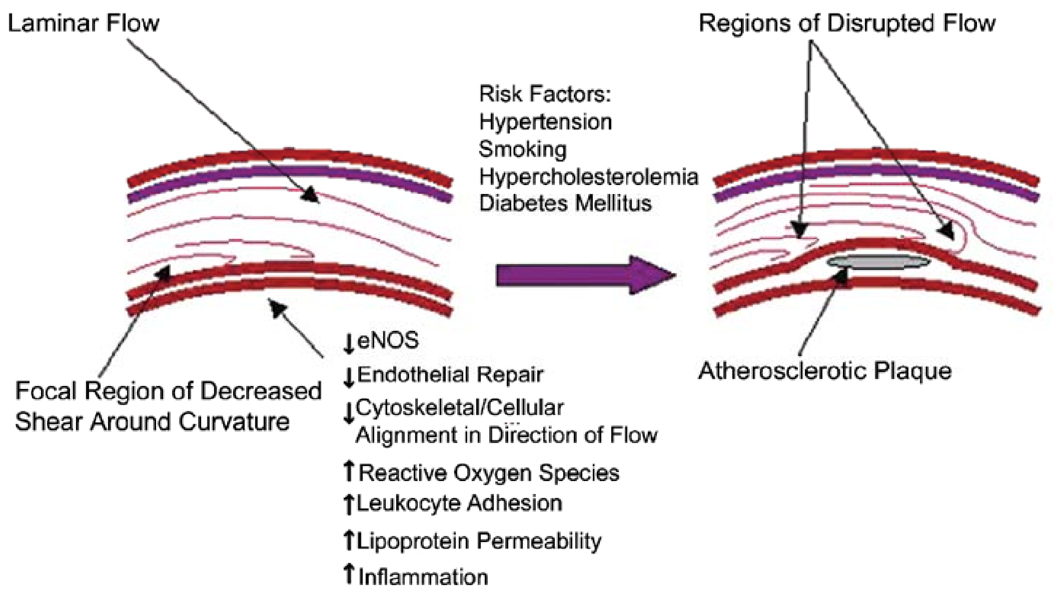


Boundary conditions

Wall Parameters

Mean wall shear stress:

$$\tau_{\text{mean}} = \left\| \frac{1}{T} \int_0^T \mathbf{t}_{\mathbf{s}} \, \mathrm{d}t \right\|$$



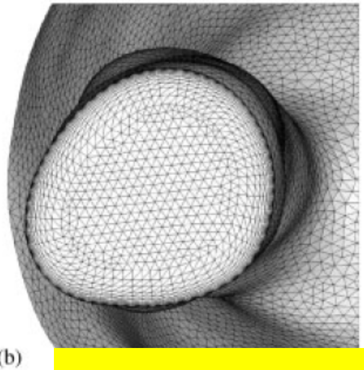
Oscillatory shear index:

$$OSI = \frac{1}{2}\left(1 - \frac{\tau_{\text{mean}}}{\tau_{\text{abs}}}\right)$$

Wall shear stress angle deviation:

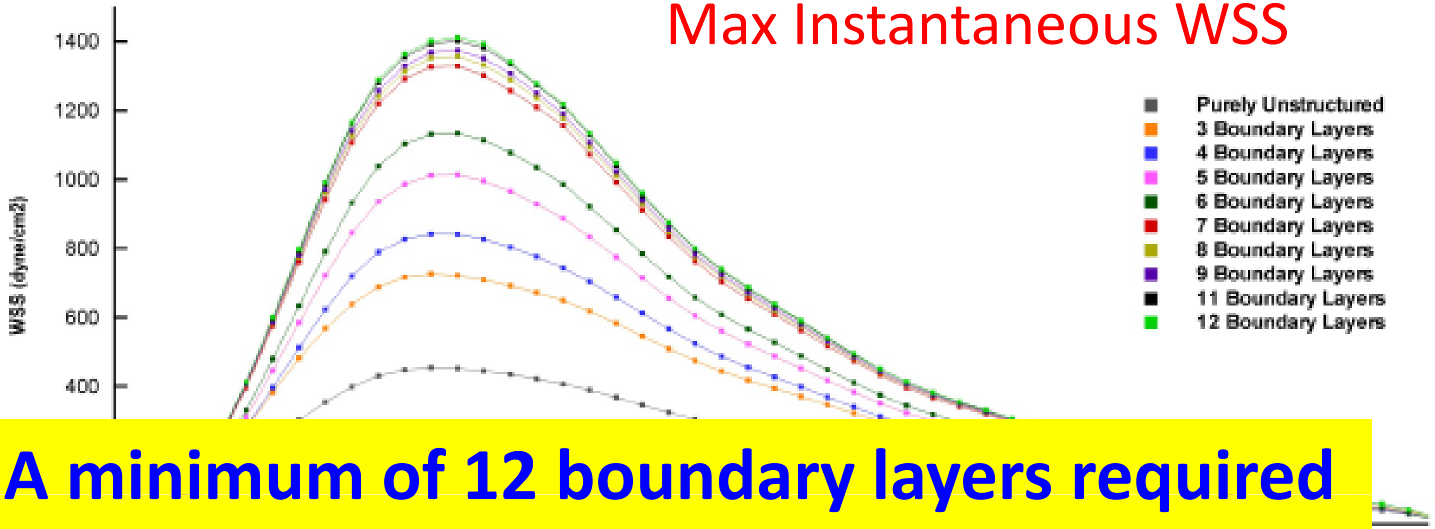
$$\text{WSSAD} = \frac{1}{T}\int_0^T \left(\frac{1}{A_i}\int_S \phi_i \, \mathrm{d}A_i\right)\mathrm{d}t \, ; \quad \phi_i = \arccos\left(\frac{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j}{\|\boldsymbol{\tau}_i\| \cdot \|\boldsymbol{\tau}_j\|}\right)$$

WSS Convergence

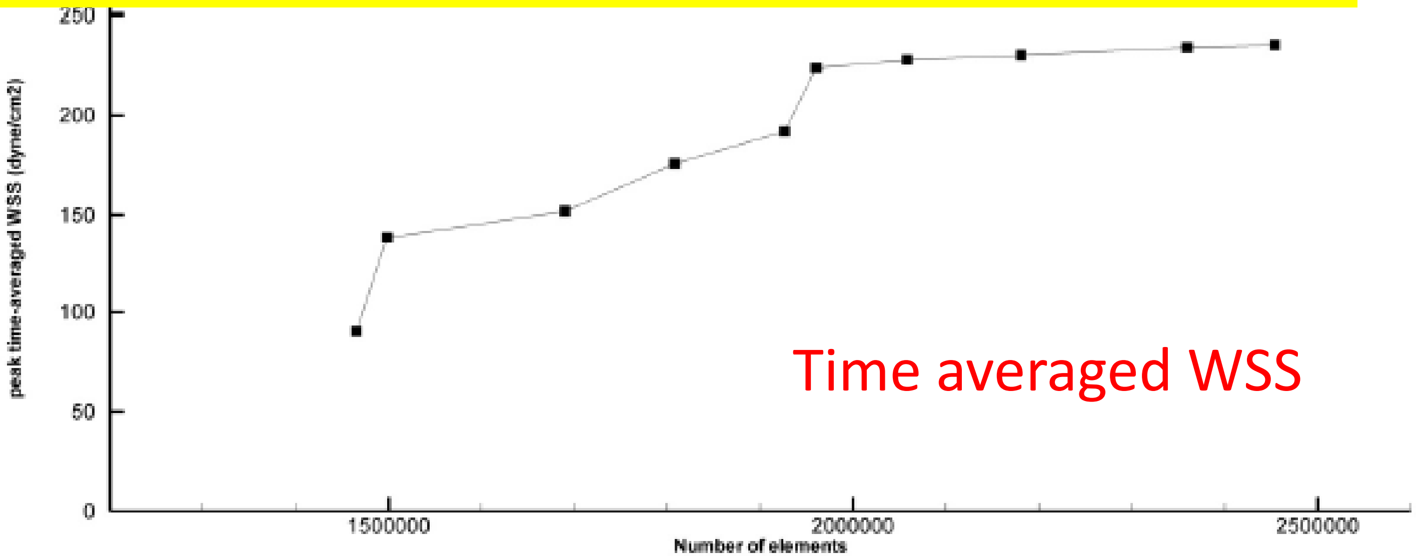


(b)

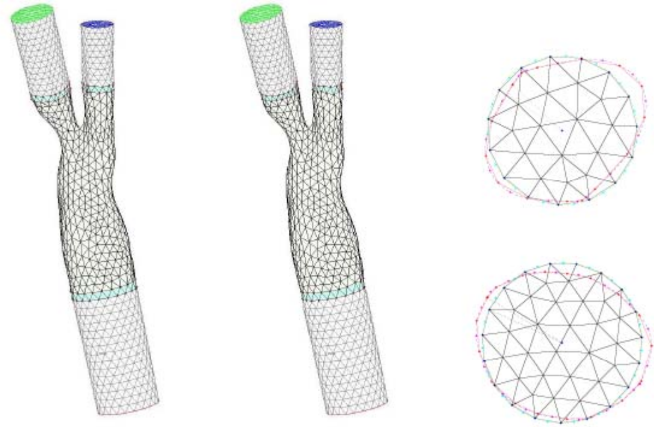
(a)



Conclusion: A minimum of 12 boundary layers required with geometric progression



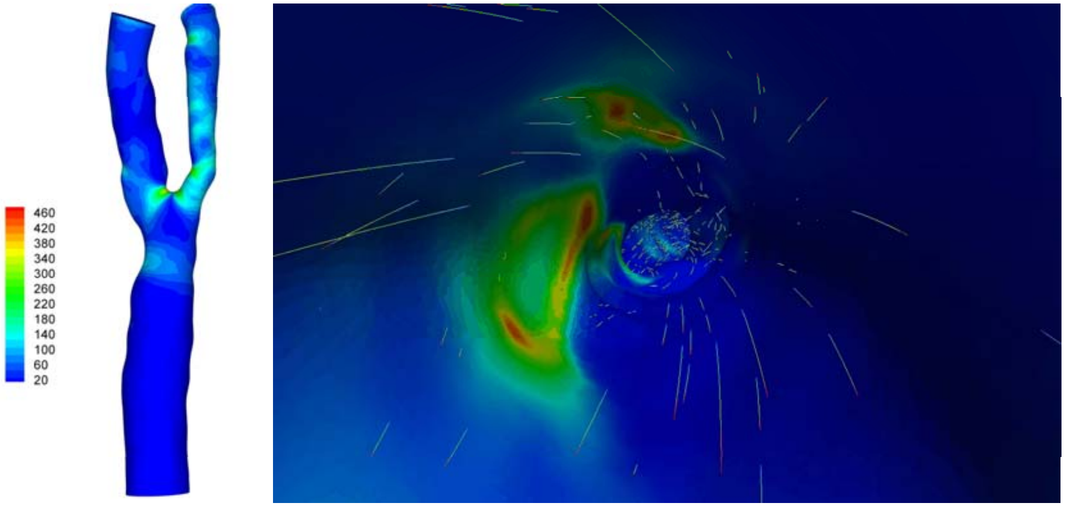
Effect of Inlet Extensions on WSS, dyne/cm^2

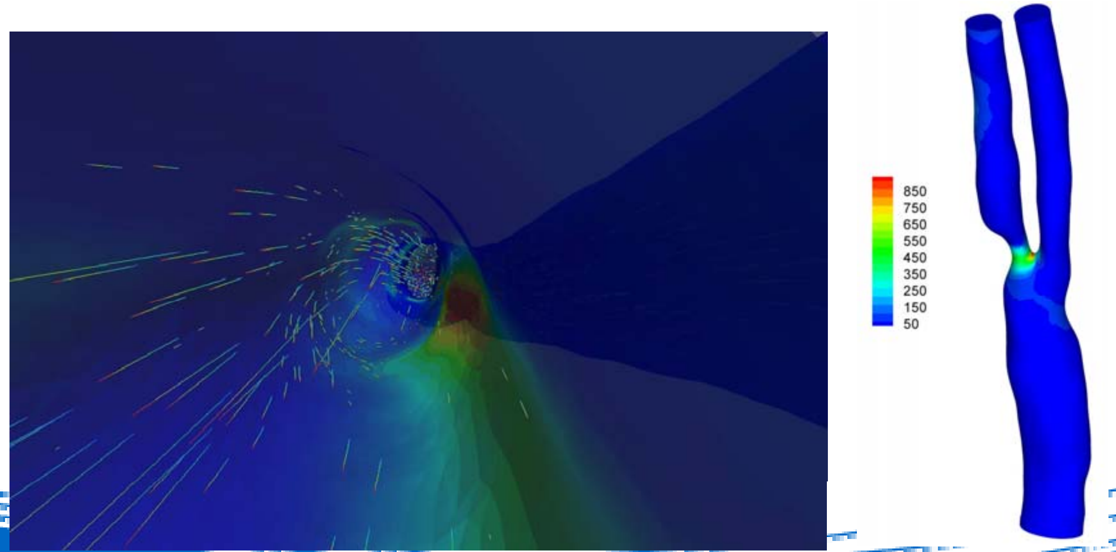


Inlet extension	Peak Time averaged WSS	Max WSS	Minimum WSS
[illegible] extension	[illegible]	[illegible]	[illegible]
1.5D extension	748.48	3930.08	181.024
3.0D extension	748.70	4046.81	168.361

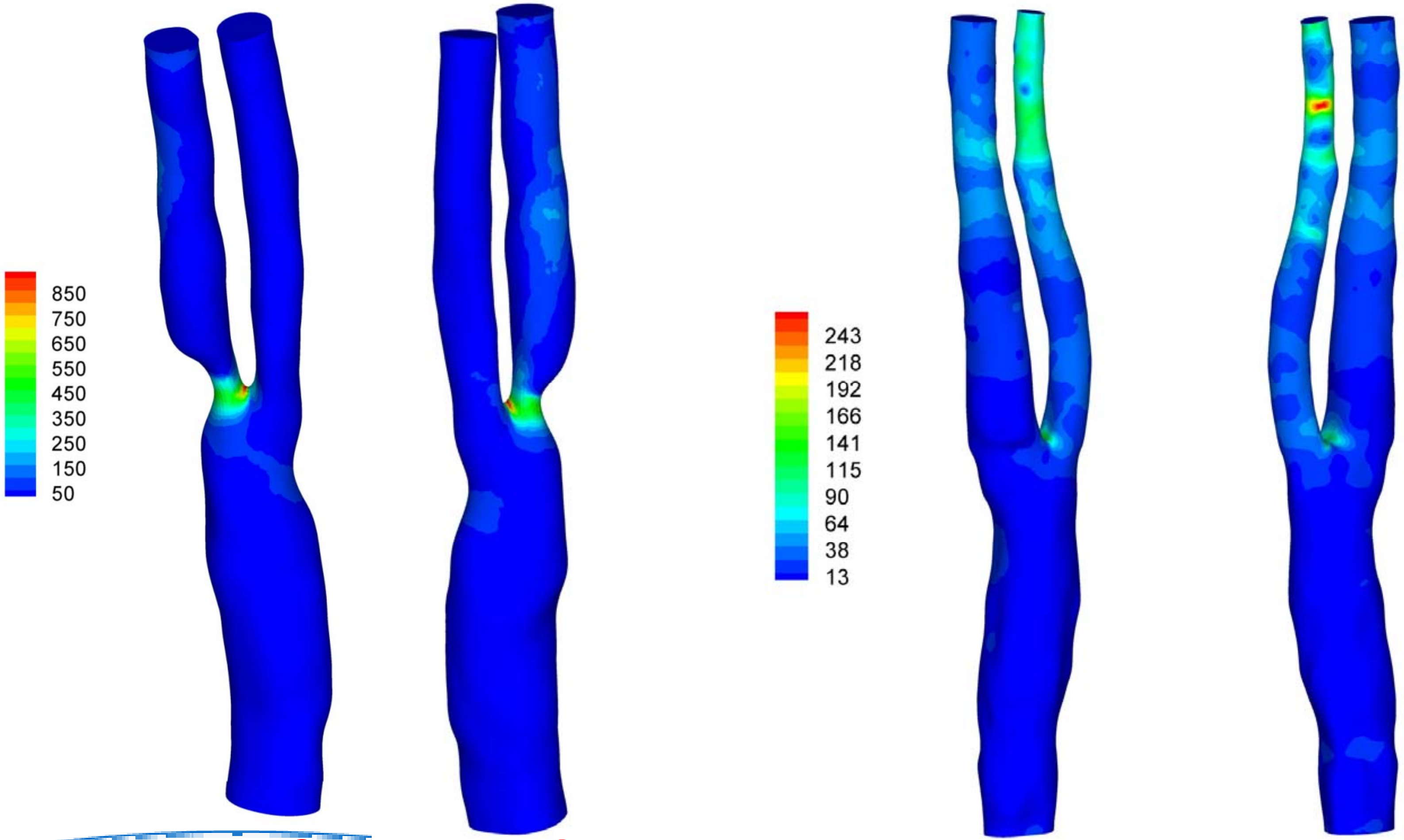
Conclusion: Inlet/outlet extensions have only moderate influence on WSS.

Flow Pattern





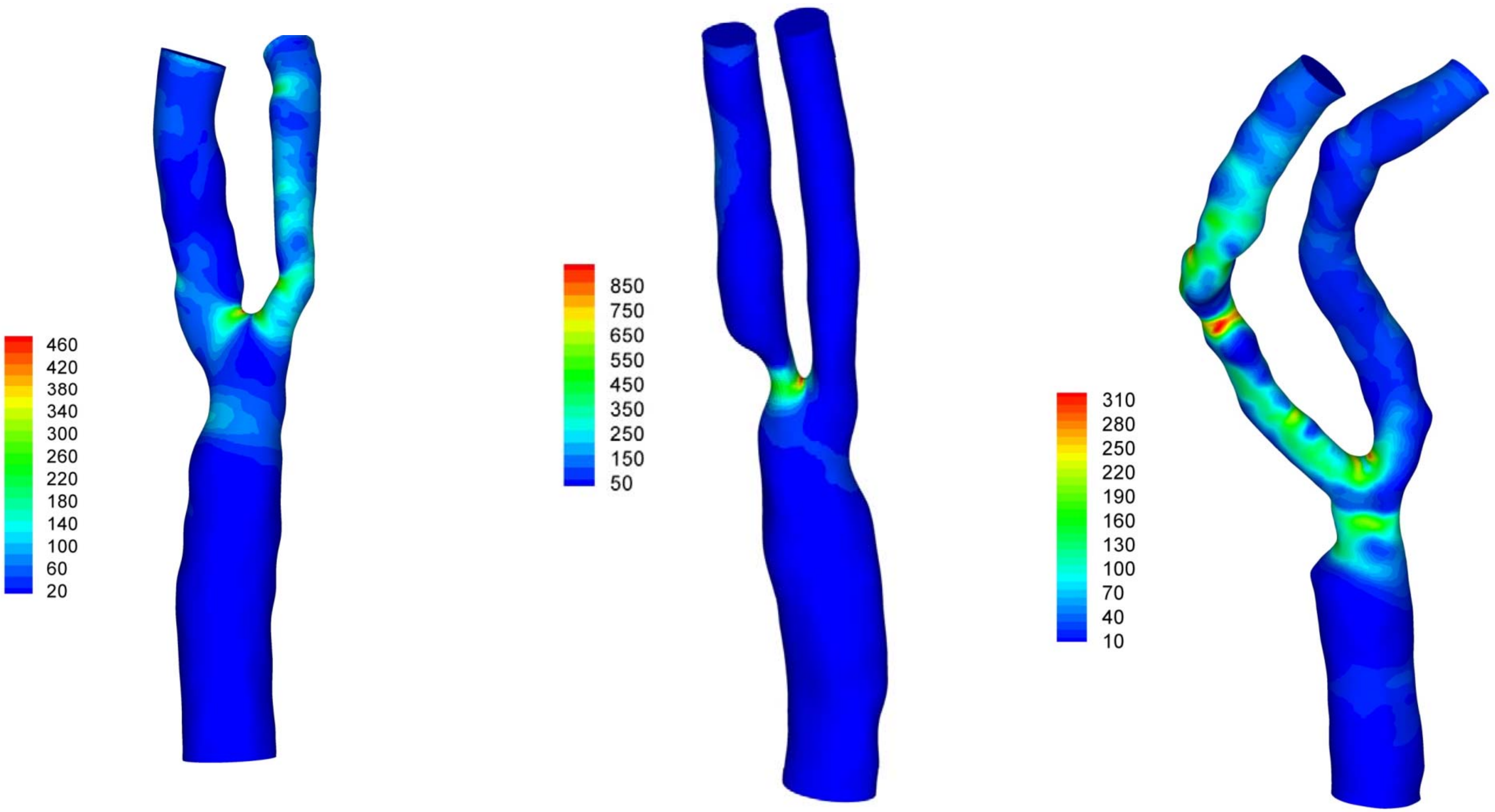
Abnormal and Normal Carotid Arteries of a Patient – Time averaged WSS



Left carotid

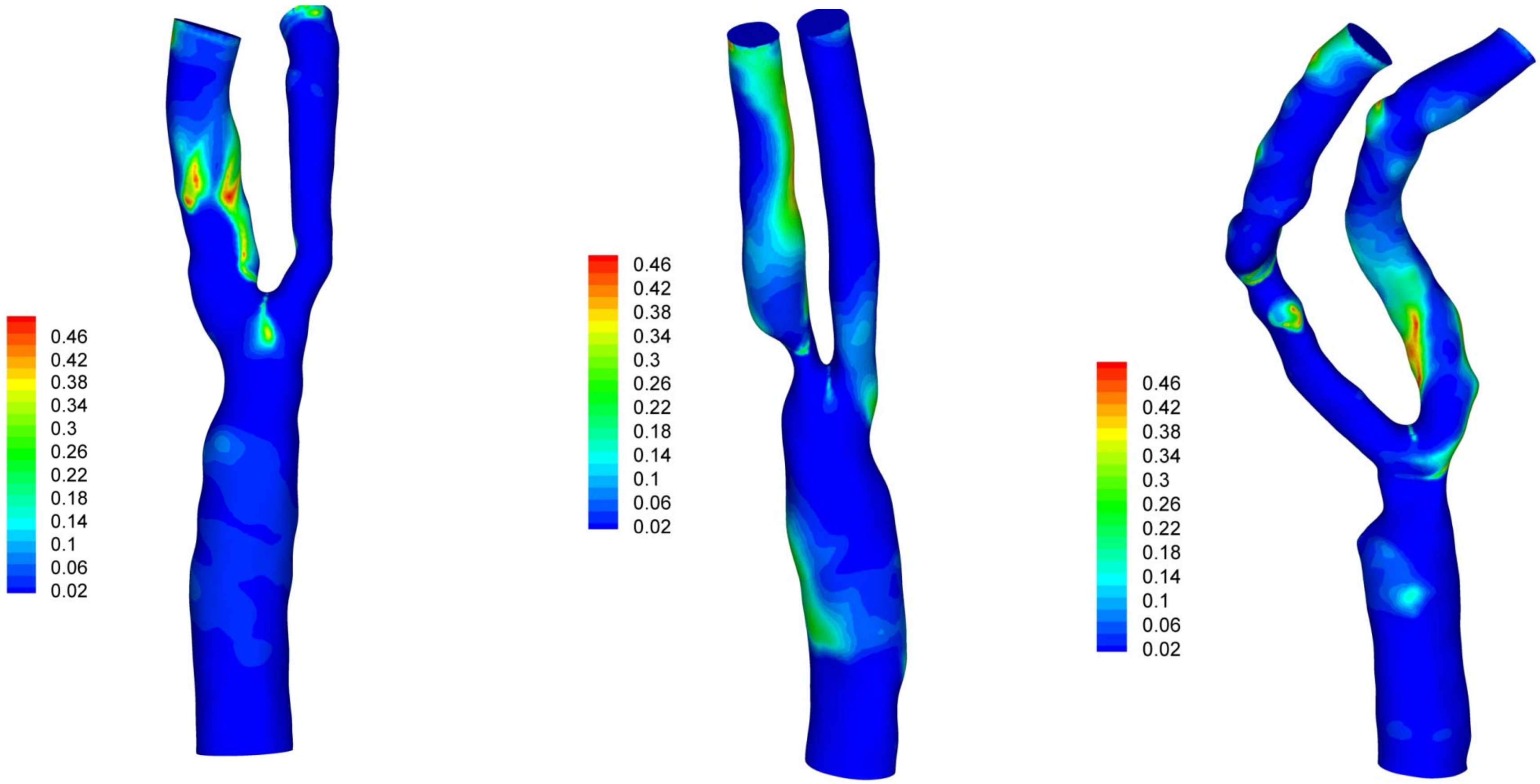
Right carotid

Abnormal Carotid Arteries of Different Patients – Time Averaged WSS



P1 - Right P3 - Left P4 - Left

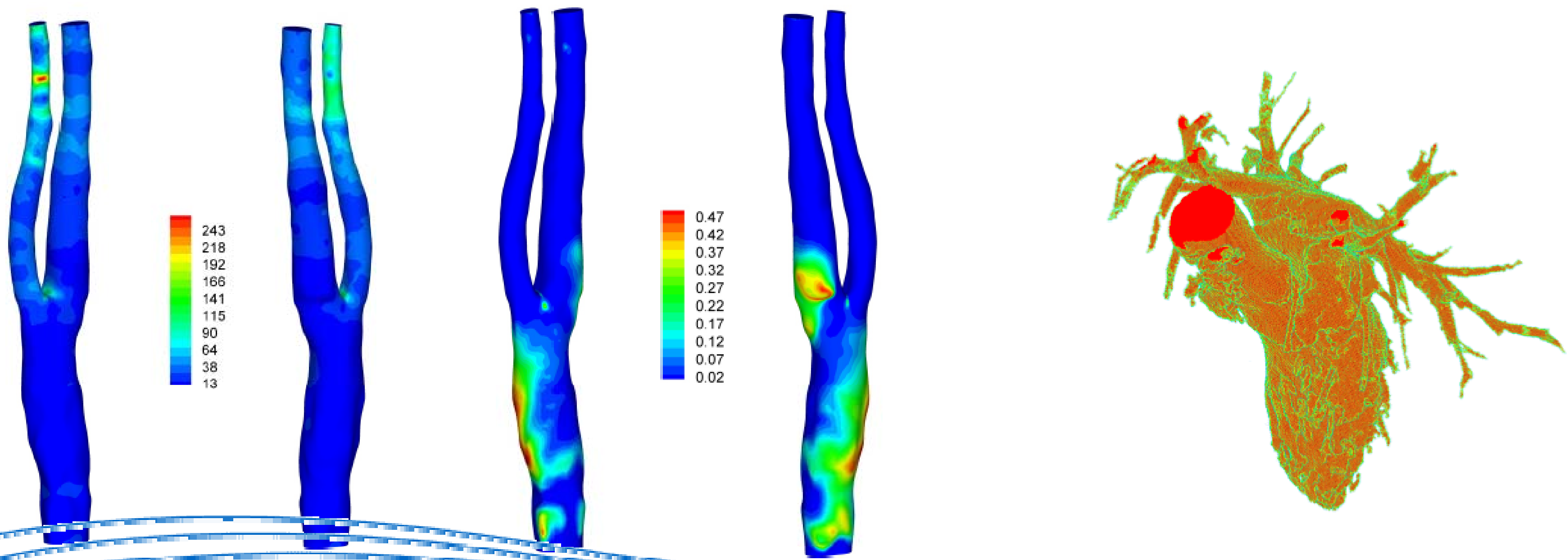
Abnormal Carotid Arteries of Different Patients – OSI



P1 - Right **P3 - Left** **P4 - Left**

Wall Motion

- Without wall motion, false stenosis locations predicted
- Wall motion measurements, image registration and integration should be part of a pipeline.



Part II 3D Conclusions

You have learned:

- Important parts of a subject-specific modelling pipeline
- Learned about image segmentation, meshing and solution
- Learned about result generation
- Learned about the uncertainties and drawbacks of existing models

For further details:

Sazonov, Yeo, Bevan, Xie, van Loon and Nithiarasu, 2011,
International Journal for Numerical Methods in Biomedical Engineering,
DOI 10.1002/cnm.1446