4H Continuum Mechanics and Elasticity
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Modern physical theories say that on the microscopic scale matter is discontinuous. It consists of molecules, atoms and even small particles. However, in everyday life, in almost all engineering applications of Mechanics and in many applications in Physics, we usually have to deal with pieces of matter which are very large compared with the small particle which matter is made of. In such cases we are not concerned with the motion of individual atoms and molecules, but only with their behaviour in some average sense. In principle, if we knew enough about the behaviour of matter on the microscopic scale it would be possible to calculate the way in which material behaves on the macroscopic scale. In practice, such calculations are extremely difficult to solve even for simple material systems. Consequently, the actual knowledge of the mechanical behaviour of materials is almost entirely based on observations and experimental tests of their behaviour on a macroscopic scale.

Continuum Mechanics is concerned with the mechanical behaviour of solids and fluids on the microscopic scale. It ignores the discrete nature of matter, and treats material as uniformly distributed throughout regions of space. It is then possible to define quantities such as density, displacement, velocity, and so on, as continuous (or at least piecewise continuous) functions of position. This procedure is found to be satisfactory provided that we deal with bodies whose dimensions are large compared with the characteristic lengths on the microscopic scale (for example, interatomic spacings in a crystal, or mean free path in a gas).

Mechanics is the Science which deals with the interaction between force and motion. Consequently, the variables which occur in Continuum Mechanics are, on the one hand, variables related to forces (usually force per unit area or per unit volume, rather than force itself) and, on the other hand, kinematic variables such as displacement, velocity and acceleration.

The equations of Continuum Mechanics are of two main kinds. Firstly, there are equations which apply to all materials. They describe universal physical laws, such as the conservation of mass and energy. Secondly, there are equations which describe the mechanical behaviour of particular materials; these are known as constitutive equations.

The problems of Continuum Mechanics are of two main kinds. The first is the formulation of constitutive equations which are adequate to describe the mechanical behaviour of various materials. This formulation is essentially a matter for experimental determination, but a theoretical framework is needed in order to devise suitable experiments and to interpret experimental results. The second problem is to solve the constitutive equations, in conjunction with the general equations of Continuum Mechanics, to confirm the validity of the constitutive equations and to predict and describe the behaviour of materials in situations which are of practical interest. At this problem-solving stage the different branches of Continuum Mechanics diverge. For the sake of illustration, in this course we shall focus on the classical and mathematically elegant theory of Elasticity.

Aim of the course

To provide an introduction to the basic theoretical ideas in Continuum Mechanics and Elasticity theory using the theory of Cartesian tensors.

Course contents

1. Introduction to the theory of Cartesian Tensors.
2. Kinematics of deformation and motion.
3. Conservation laws and the stress tensor.
4. Equations of motion and equilibrium, Cauchy's theorem.
5. Constitutive laws: general principles and material symmetry.
6. Isotropic functions of tensors and isotropic materials.
7. Incompressibility.
8. Constitutive laws in linear and nonlinear elasticity.

Bibliography

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4. An introduction to Continuum Mechanics, M.E. Gurtin: Academic Press (1981).
5. An introduction to the Theory of Elasticity, R.J. Atkin, N. Fox: Dover publications (2005).