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Plasticity of sandwich beam with metallic foam core

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Research Topics in My Group

Mechanics of inhomogeneous solids

- Damage and ductile fracture mechanics, 1988 --
- Deformation, fatigue and fracture of polymers, 1997 --
- Bending, buckling and thermal stress of functionally graded structures, 2001 --
- Waves in layered piezoelectric structures, 2001 -
- Mechanics of nanomaterials and structures, 2006--
- Plasticity of lightweight sandwich structures, 2006 --
- Strength theory and fracture in TBC system, 2007 --

Damage and ductile fracture mechanics, 1988 --

- **T.J. Wang and K. Kishimoto.** Higher order fields for damaged nonlinear antiplane shear notch, crack and inclusion problems. *Euro. J. Mechanics*, A/Solids, 1999,18(6): 963-986
- **T.J. Wang and Z.B. Kuang.** Stress, deformation and damage fields near the tip of a crack in a damaged nonlinear material. *Int. J. Fracture*, 1996, 79(1):1-26.
- **T.J. Wang.** Unified CDM model and local criterion for ductile fracture -I. *Eng. Fracture Mech.*, 1992, 42(1): 177-183
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- **T.J. Wang and Z.W. Lou.** A continuum damage model for weld heat affected zone under low cycle fatigue loading. *Eng. Fracture Mech.*, 1990, 37(4): 825-829

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- **Z.N. Yin and T.J. Wang,** **Deformation response and constitutive modeling of PC, ABS and PC/ABS alloys under impact tensile loading. *Materials Science and Engineering* A, 2010, 527: 1461-1468**
- **Z.N. Yin and T.J. Wang,** **Deformation of PC/ABS alloys at elevated temperatures and high strain rates. *Materials Science and Engineering* A, 2008, 494: 304-313**
- **Q.Z. Fang, T.J. Wang, H.G. Beom, H.P. Zhao,** **Rate-dependent large deformation behavior of PC/ABS alloy. *Polymer*, 2009, 50: 296-304**
- **Y.J. Jin and T.J. Wang,** **Three-dimensional numerical modeling of the damage mechanism of amorphous polymer network. *Computational Materials Science*, 2009, 46: 632-638**
- **H.M. Li, G.F. Wang and T.J. Wang,** **Effect of crack-tip shape on the near tip field in glassy polymer. *Int. J. Solids & Structures*, 2008, 45: 1087-1100**

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- **Q.Z. Fang, T.J. Wang and H.M. Li, Overload induced retardation of the fatigue crack propagation of polycarbonate. *Int. J. Fatigue*, 2008, 30: 1419-1429**
- **Q.Z. Fang, T.J. Wang and H.M. Li, 'Tail' phenomenon and fatigue crack propagation of PC/ABS alloy. *Polymer Degradation and Stability*, 2008, 93: 281-290**
- **Q.Z. Fang, T.J. Wang and H.M. Li, Overload effect on the fatigue crack propagation of PC/ABS alloy. *Polymer*, 2007, 48: 6691-6706**
- **Q.Z. Fang, T.J. Wang and H.M. Li, Large tensile deformation behavior of PC/ABS alloy. *Polymer*, 2006, 47: 5174-5181.**
- **T.J. Wang, Kishimoto K, Notomi M, Effect of triaxial stress constraint on the deformation and fracture of polymers. *Acta Mech Sin*, 2002, 18(5): 480-493**

Bending, buckling and thermal stress of functionally graded structures, 2001 ---

- **Z.S. Shao, T.J. Wang,** **Transient thermo-mechanical stresses in functionally graded cylindrical panels.** ***AIAA Journal,*** **2007, 45:2487-2496**
- **Z.S. Shao and T.J. Wang,** **Three-dimensional solutions for the stress fields in functionally graded cylindrical panel with finite length.** ***Int. J. Solids & Structures*****, 2006, 43: 3856-3874**
- **L.S. Ma and T.J. Wang.** **Relationships between axisymmetric bending and buckling solutions of FGM circular plates based on third-order plate theory and classical plate theory.** ***Int. J. Solids Structures*****, 2004, 41:85-101**
- **L.S. Ma and T.J. Wang.** **Nonlinear bending and post-buckling of a functionally graded circular plate under mechanical and thermal loadings.** ***Int. J. Solids Structures*****, 2003, 40: 3311-3330**

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- **H. Liu, T.J. Wang, et al.,** Effect of a biasing electric field on the propagation of symmetric Lamb waves in piezoelectric plates. *Int. J. Solids & Structures*, 2002, 39 (7): 2031-2049
- **H. Liu, Z.K. Wang and T.J. Wang.** Effect of initial stress on the propagation behavior of Love wave in a layered piezoelectric half-space. *Int. J. Solids & Structures*, 2001, 38: 37-51
- **F. Jin, Z.K. Wang and T.J. Wang.** The Bleustein-Gulyaev (B-G) wave in a piezoelectric layered half-space. *Int. J. Engineering Science*, 2001, 39:1271-1285

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- **W.X. Zhang, T.J. Wang and X. Chen,** Effect of surface/interface stress on the plastic deformation of nanoporous materials and nanocomposites. *Int. J. Plasticity*, 2010, 26: 957–975
- **W.X. Zhang and T.J. Wang,** Effect of surface energy on the yield strength of nanoporous materials. *Appl Phys Lett*, 2007, 90: 063104
- **Y. Ru, G.F. Wang and T.J. Wang,** Diffractions of elastic waves near a cylindrical nano-inclusion incorporating surface effect. *J. Vib. Acoustics*, 2009, 131: 061011
- **G.F. Wang, T.J. Wang and X.Q. Feng,** Diffraction of plane P-wave by a nanosized circular hole with surface effect. *Appl Phys Lett* , 2006, 89: 231923.
- **Z.Y. Ou, G.F. Wang and T.J. Wang,** Elastic fields around a nanosized spheroidal cavity under arbitrary uniform remote loadings. *Euro. J. Mech.*, A/Solids, 2009, 28: 110-120
- **G.F. Wang, X.Q. Feng, T.J. Wang and W. Gao,** Surface effects on the stresses near a crack tip. *J. Appl. Mech.*, 2008, 75: 011001
- **G.F. Wang and T.J. Wang,** Deformation around a nanosized elliptical hole with surface effect. *Appl Phys Lett* , 2006, 89: 161901.

Plasticity of lightweight sandwich structures, 2006 -

- **Q.H. Qin and T.J. Wang,** Low-velocity heavy-mass impact response of slender metal foam core sandwich beam. *Composite Structures*, 2011, 93:1526-1537
- **W.X. Zhang, Z.M. Xu, T.J. Wang and X. Chen,** Effect of inner pressure on the elasto-plastic behavior of porous materials: Second-order moment micromechanics model. *Int. J. Plasticity*, 2009, 25: 1231-1252
- **Q.H. Qin and T.J. Wang,** An analytical solution for the large deflections of a slender sandwich beam with a metallic foam core under transverse loading by a flat punch. *Composite Structures*, 2009, 88: 509-518
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Plasticity of sandwich beam with metallic foam core *

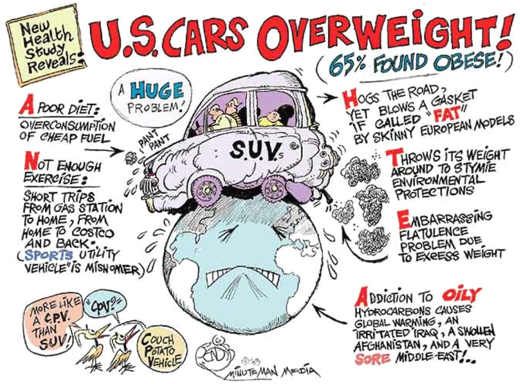
- Background
- Plastic yield criterion of sandwich cross-section
- Static plastic behavior of sandwich beam
- Results and discussion
- Summary

* Q.H. Qin and T.J. Wang, *Composite Structures*, 2009, 88: 509-518

Background

Challenges of Energy and Environment

- **Energy security and environmental protection have been urgent tasks.**
- **Weight reduction and energy saving—lightweight and multifunctional structures, e.g. aircraft, spacecraft, speed train, automobile, ship, etc.**
- **Ultralight and high performance materials in offshore platform, wind power machine, etc.**
- **New materials and structures: multifunctionality.**



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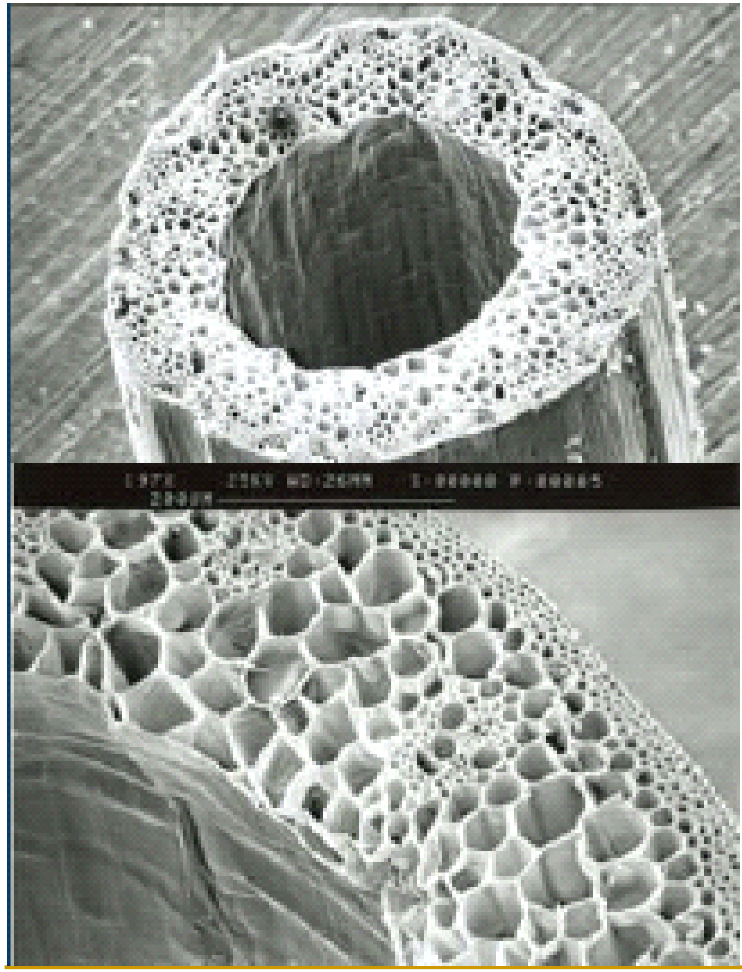
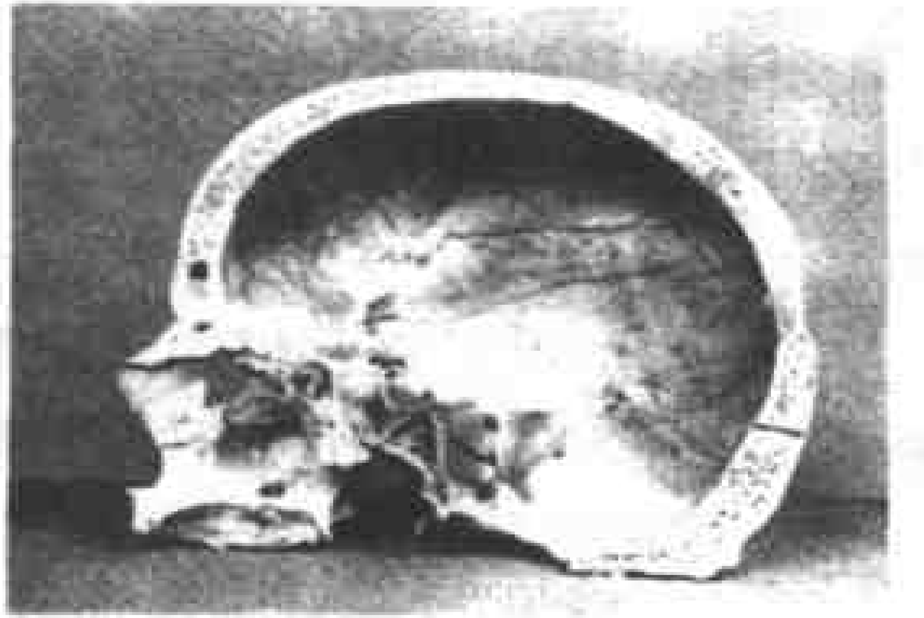
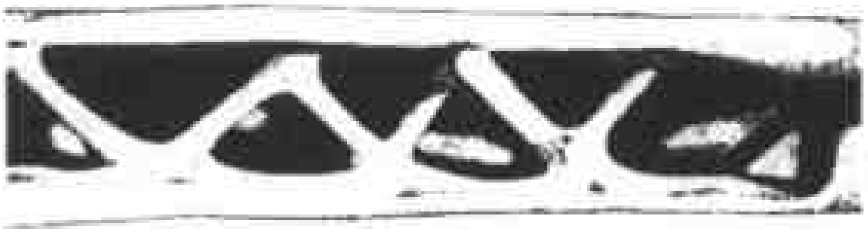
Learn from nature: porous materials and structures



Plant stem

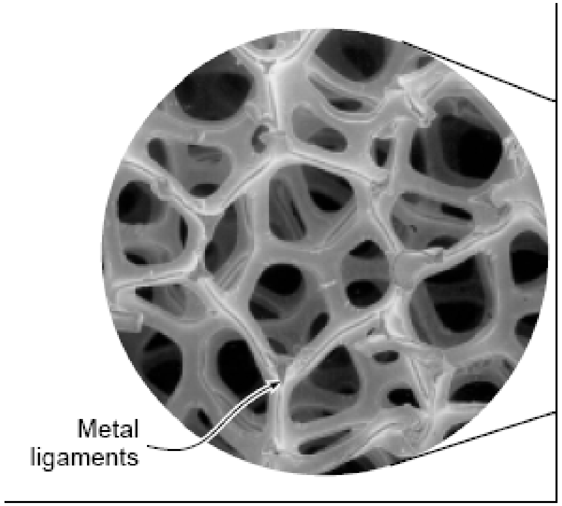


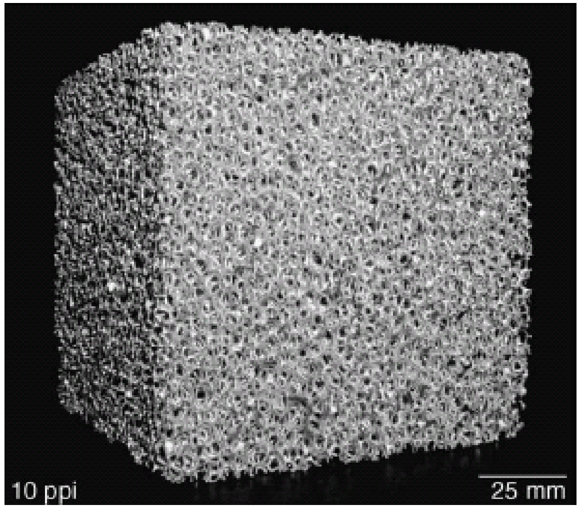
A section through human skull



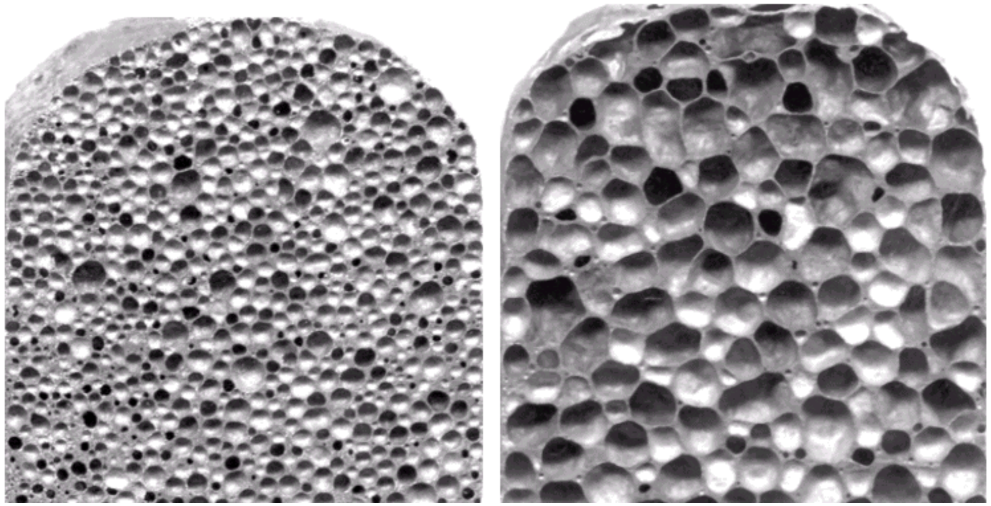
A section through a bird's wing

Ultra-lightweight porous materials – foams





Open-cell



Closed-cell

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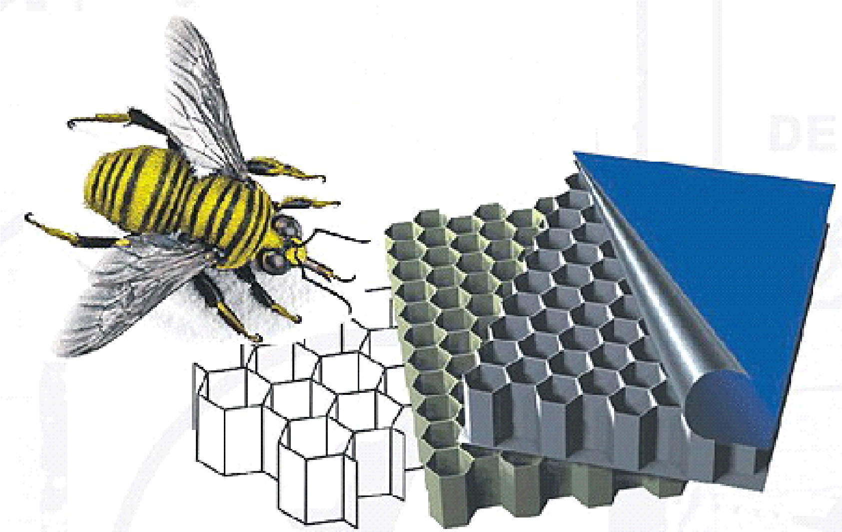
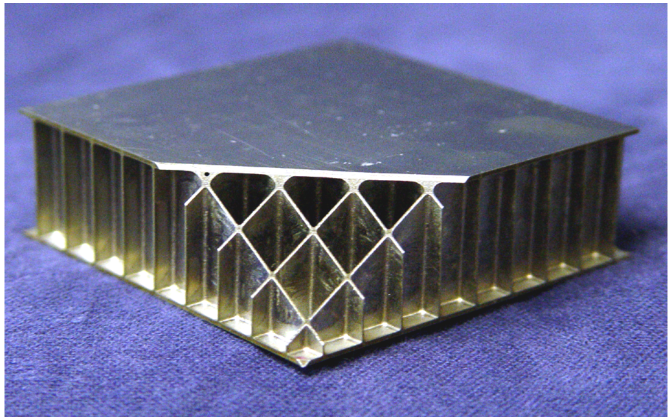
Ultra-lightweight porous materials – Lattices

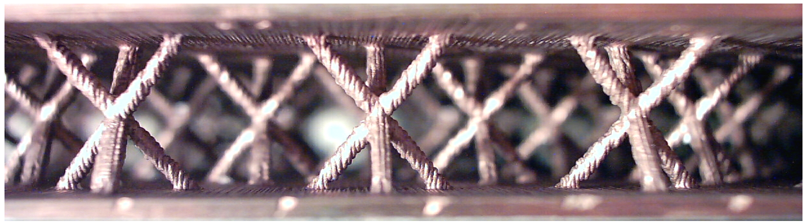


Hexagonal honeycomb core



Square honeycomb core

Pyramidal truss core



Kagome core

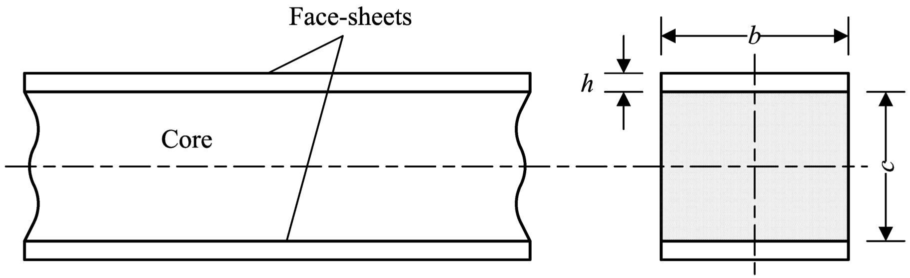
Characteristics of ultra-lightweight porous materials

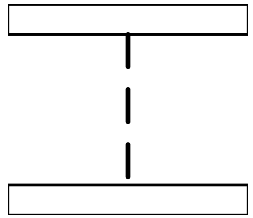
- High porosity (>90%), ultra-lightweight
- High strength and ductility
- High crashworthiness
- High strength to weight ratio
- High stiffness to weight ratio
- Effective heat transfer
- ……

Ultra-lightweight multifunctional materials

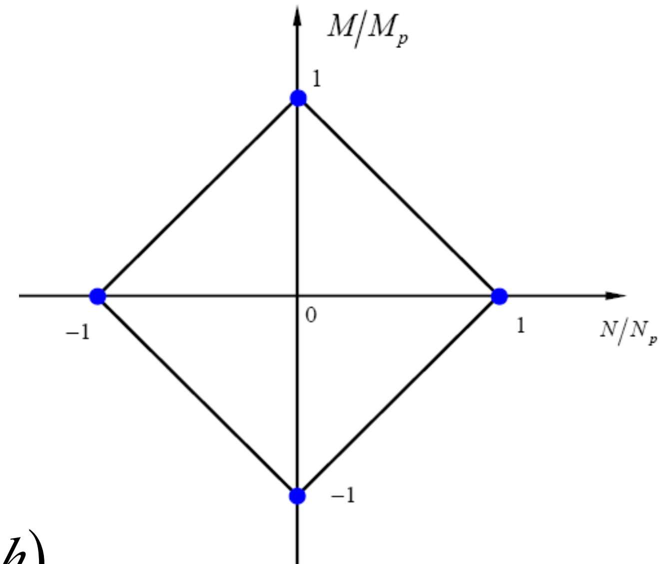
Yield criterion for lightweight sandwich structures

Classical yield criterion for sandwich cross-section





Ideal sandwich cross-section



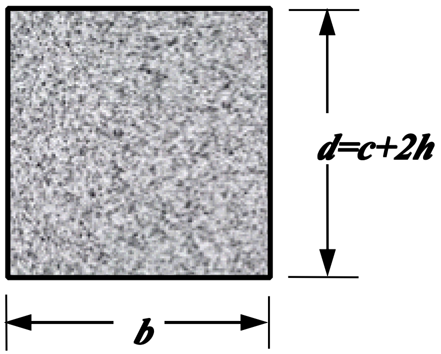
$$\frac{|M|}{M_p} + \frac{|N|}{N_p} = 1, \quad for \quad \frac{h}{c} << 1 \, and \, \frac{\sigma_c}{\sigma_f} << 1$$

Fully plastic axial force and bending moment

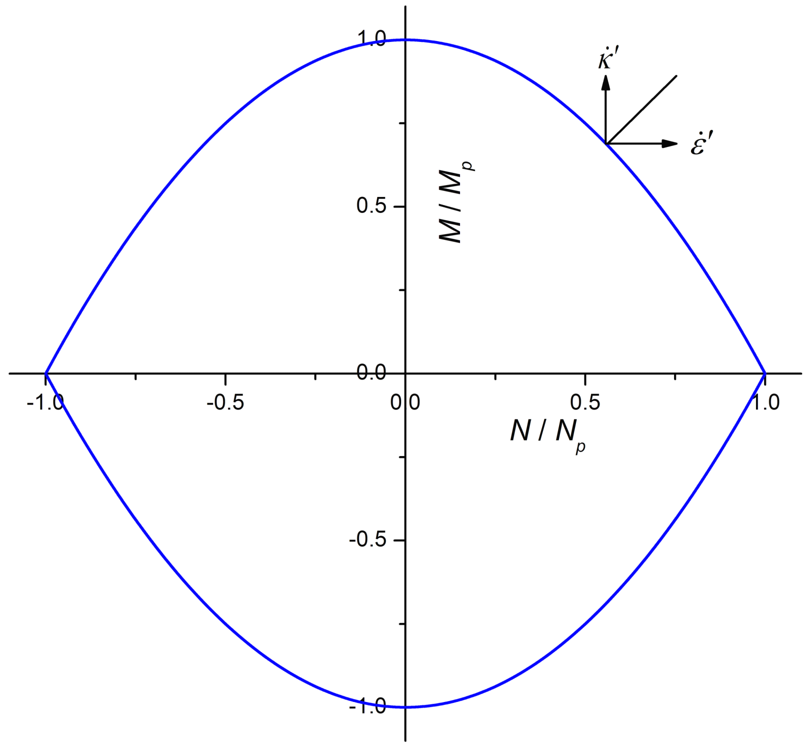
$$N_p = 2bh\sigma_f + \sigma_c bc, \qquad M_p = \sigma_c b\frac{c^2}{4} + \sigma_f bh\left(c+h\right)$$

Fleck and Deshpande, ASME, J. Appl. Mech., 2004, 71:386-401

Yield criterion for monolithic solid cross-section



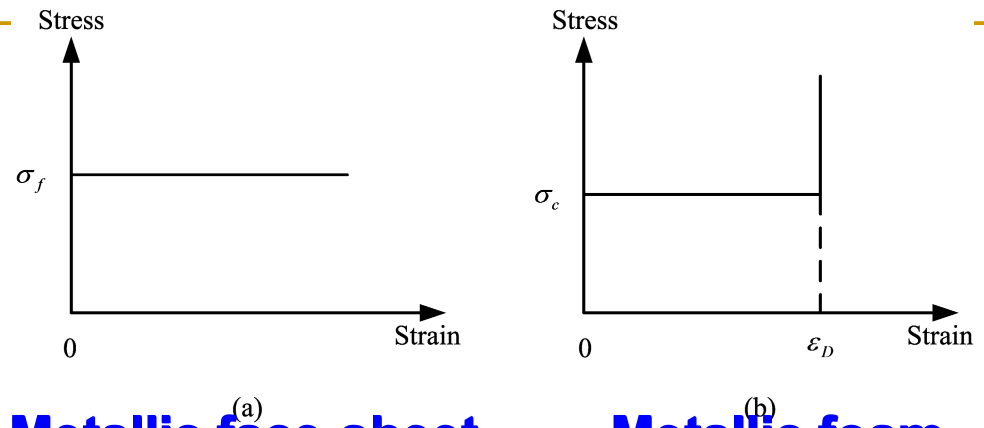
$$\frac{|M|}{M_p} + \left(\frac{|N|}{N_p}\right)^2 = 1$$



Yield criterion for ideal sandwich cross-section

Yield criterion for monolithic solid cross-section

Constitutive behavior



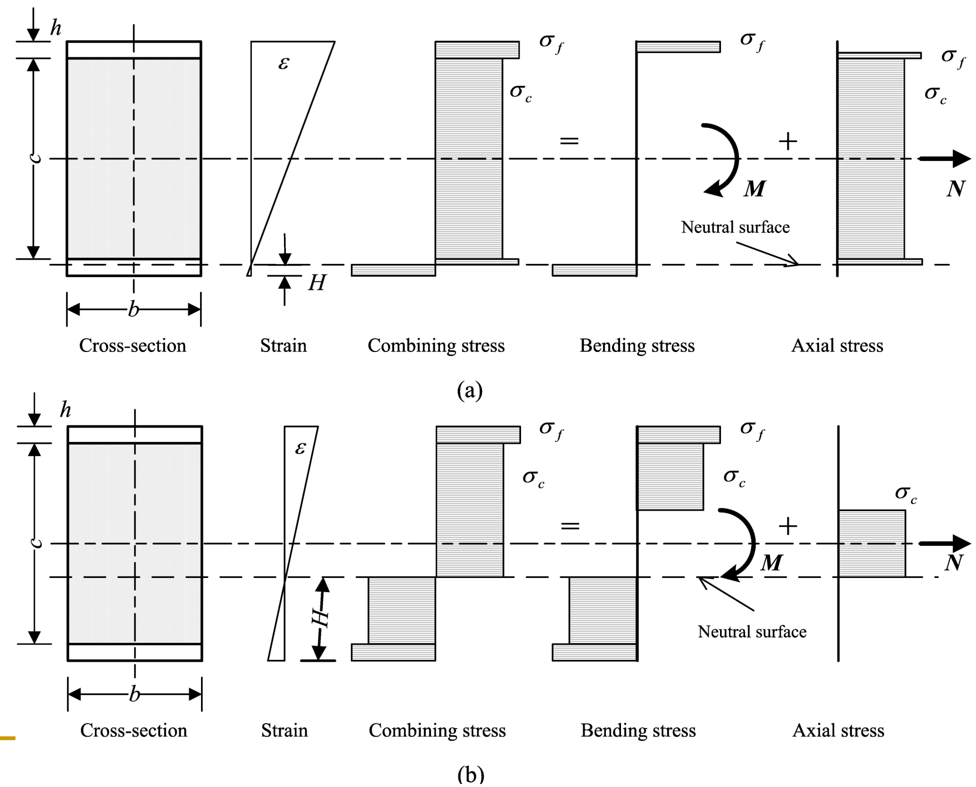
Metallic face-sheet **Metallic foam**

Axial force

$$N=\int_A \sigma\, dA=\begin{cases}\sigma_c bc+2\sigma_f b\left[h-\zeta(c+2h)\right], & 0\le\zeta\le\dfrac{h}{c+2h}\\ \sigma_c b\left[(c+2h)-2\zeta(c+2h)\right], & \dfrac{h}{c+2h}\le\zeta\le\dfrac{1}{2}\end{cases}$$

Bending moment

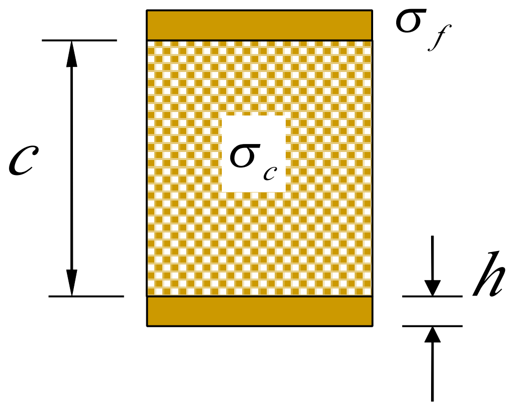
$$M=\int_A \sigma z\, dA=\begin{cases}\sigma_f b(c+2h)^2\zeta(1-\zeta), & 0\le\zeta\le\dfrac{h}{c+2h}\\ \dfrac{b}{4}\left\{\sigma_f\left[(c+2h)^2-c^2\right]+\sigma_c\left[c^2-(c+2h)^2(1-2\zeta)^2\right]\right\}, & \dfrac{h}{c+2h}\le\zeta\le\dfrac{1}{2}\end{cases}$$



Distributions of stress and strain

Yield criterion for sandwich structure

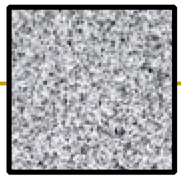
$$\begin{cases} |m| + \dfrac{(\bar{\sigma}+2\bar{h})^2}{4\bar{\sigma}\bar{h}(1+\bar{h})+\bar{\sigma}^2} n^2 = 1, & 0 \le |n| \le \dfrac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}} \\ |m| + \dfrac{(\bar{\sigma}+2\bar{h})\left[(\bar{\sigma}+2\bar{h})|n| + 2\bar{h} - \bar{\sigma} + 2\right](|n|-1)}{4\bar{h}(1+\bar{h})+\bar{\sigma}} = 0, & \dfrac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}} \le |n| \le 1 \end{cases}$$



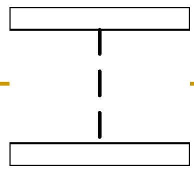
where $m = M/M_p, \quad n = N/N_p, \quad \bar{\sigma} = \sigma_c/\sigma_f, \quad \bar{h} = h/c$

Yield criterion for sandwich structure

$\bar{\sigma} = 1$ → $|m| + n^2 = 1$ → **Monolithic solid cross-section**

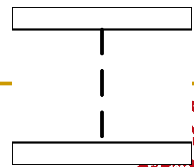


$\bar{\sigma} \ll 1$ → $|m| + (|n|-1)\left[\dfrac{\bar{h}}{1+\bar{h}}|n| + 1\right] = 0$ → **Ideal sandwich cross-section**

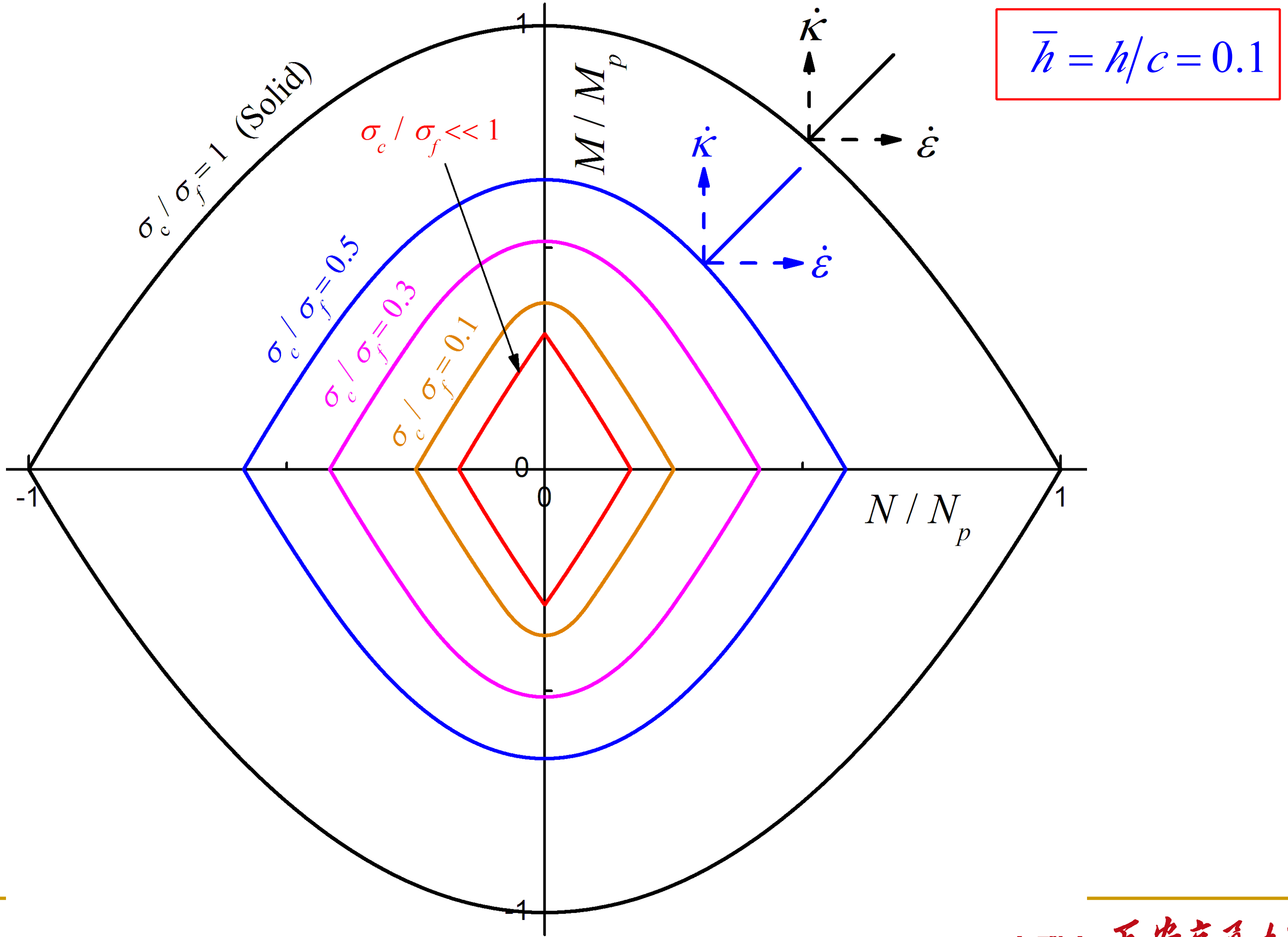


$\bar{h} \ll 1$ → $|m| + |n| = 1$

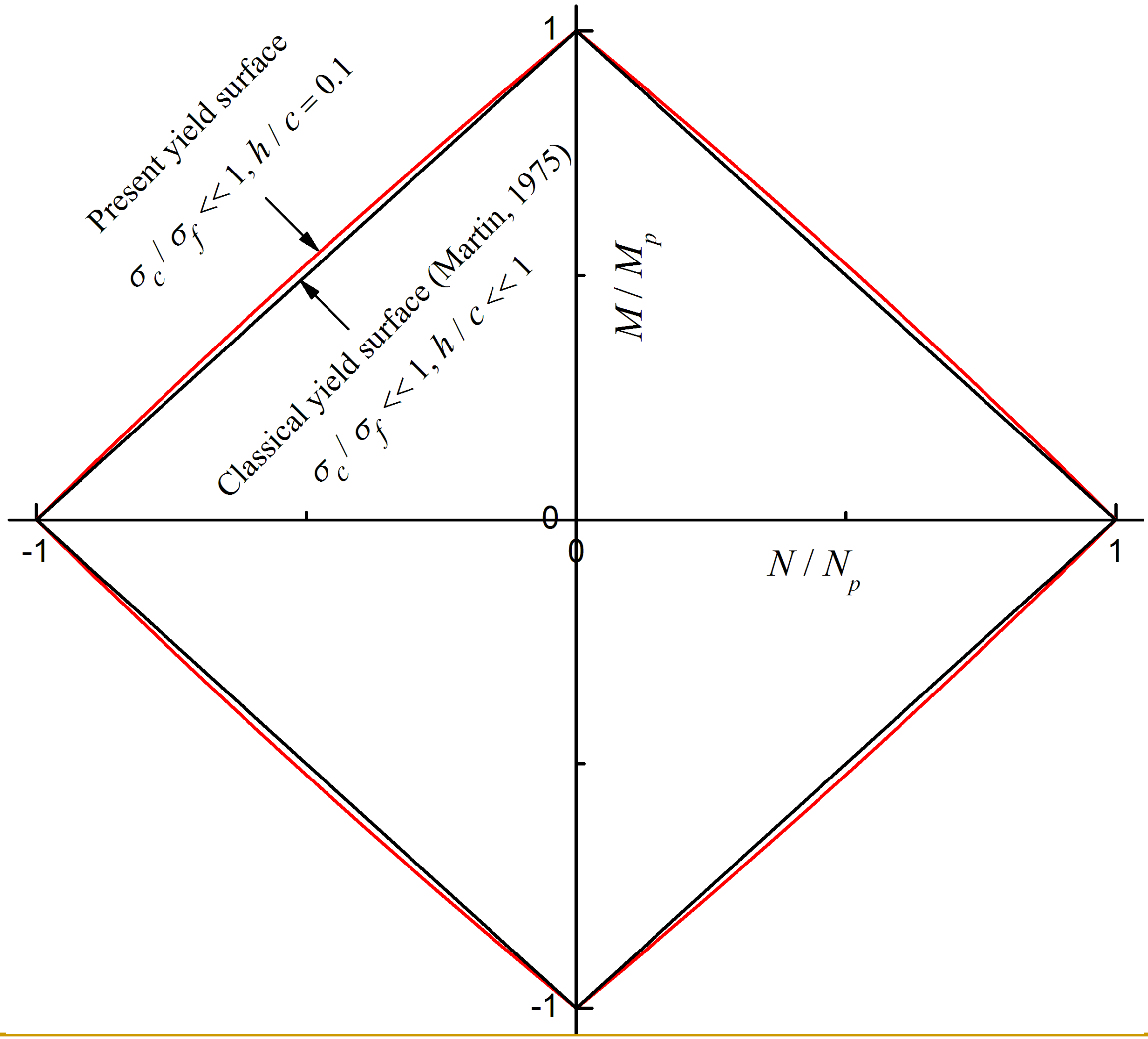
$\bar{\sigma} \ll 1 \quad \bar{h} \ll 1$ → $|m| + |n| = 1$ → **Ideal sandwich cross-section**



Yield criterion for sandwich structure



Yield criterion for sandwich structure



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Yield criterion for sandwich structure with dent

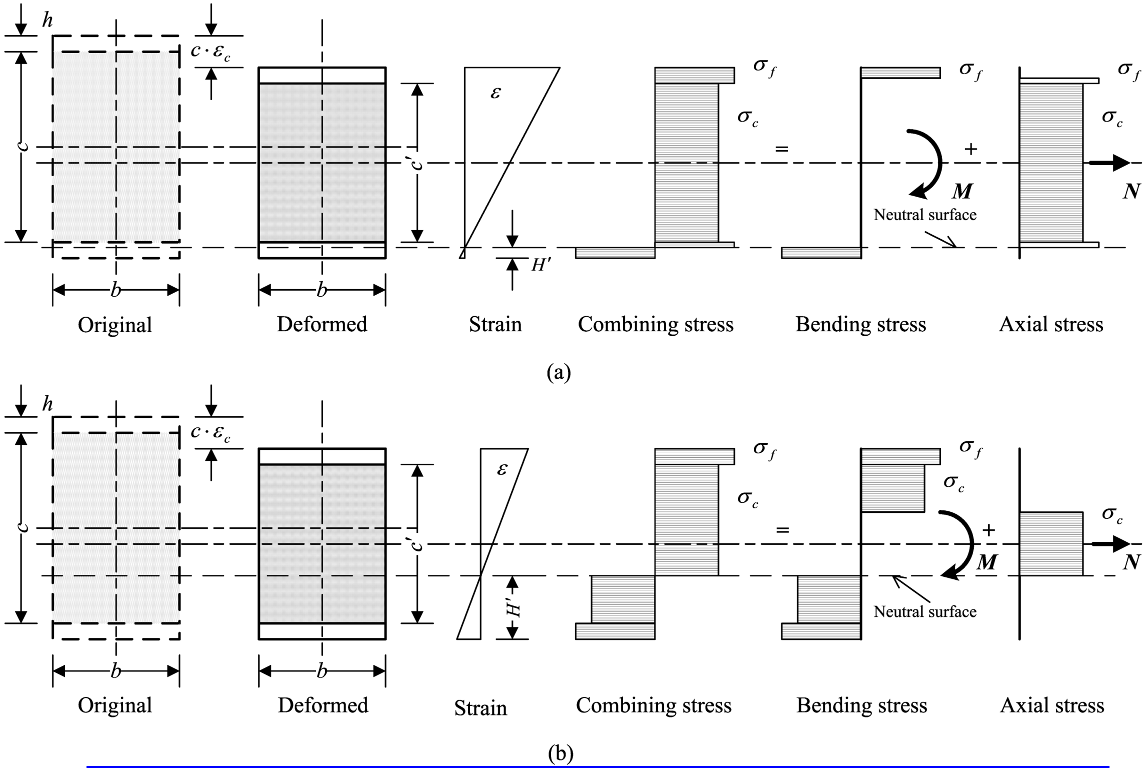
Basic assumption

$$c' = c(1-\varepsilon_c)$$

$$N'_p = N_p = 2bh\sigma_f + \sigma_c bc$$

$$\sigma'_c = \frac{\sigma_c}{1-\varepsilon_c}$$

$$M'_p = \sigma_f b\frac{(c'+2h)^2}{4} - \sigma_f b\frac{c'^2}{4} + \sigma'_c b\frac{c'^2}{4}$$



Distributions of stress and strain for sandwich cross-section with a dent

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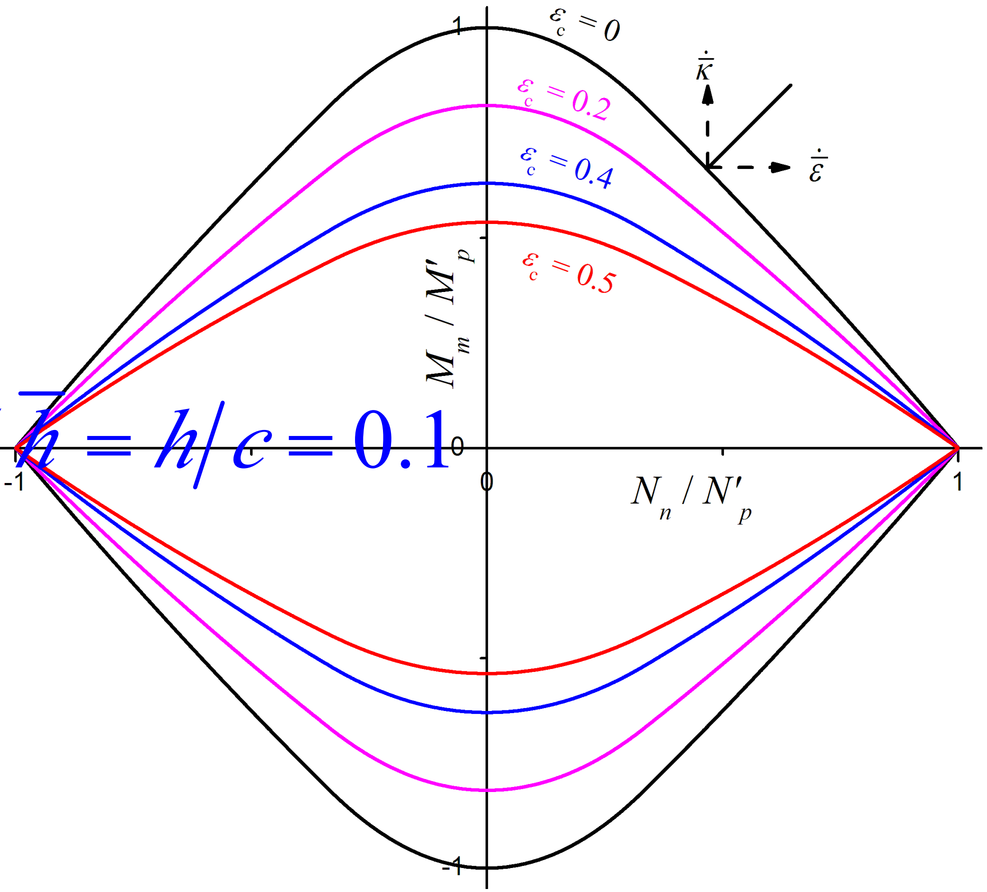
Yield criterion for sandwich structure with dent

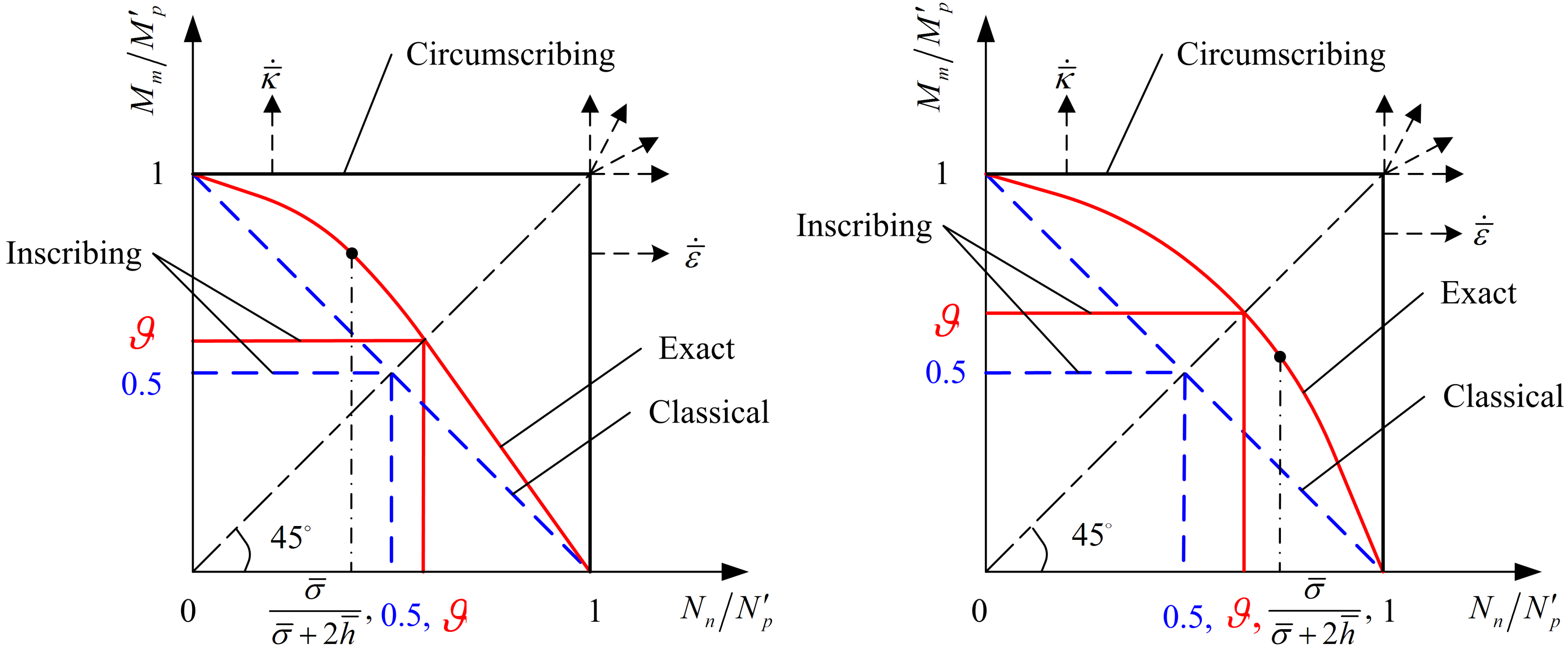
$$\begin{cases} |\bar{m}| + \dfrac{(\bar{\sigma}+2\bar{h})^2}{4\bar{\sigma}\bar{h}\left[1+\bar{h}/(1-\varepsilon_c)\right]+\bar{\sigma}^2}\bar{n}^2 = 1, & 0 \le |\bar{n}| \le \dfrac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}} \\ |\bar{m}| + \dfrac{(\bar{\sigma}+2\bar{h})\left(|\bar{n}|-1\right)\times\left[|\bar{n}|(\bar{\sigma}+2\bar{h})+2\bar{h}-\bar{\sigma}+2(1-\varepsilon_c)\right]}{4\bar{h}(1-\varepsilon_c+\bar{h})+\bar{\sigma}(1-\varepsilon_c)} = 0, & \dfrac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}} \le |\bar{n}| \le 1 \end{cases}$$



$\bar{\sigma} = \sigma_c/\sigma_f = 0.1$ and $\bar{h} = h/c = 0.1$

and $\bar{h} = h/c = 0.1$

Approximate yield criterion



$$8\bar{h}^2\left[1+\bar{h}/(1-\varepsilon_c)\right]-\bar{\sigma}^2 \geq 0$$

$$8\bar{h}^2\left[1+\bar{h}/(1-\varepsilon_c)\right]-\bar{\sigma}^2 \leq 0$$

Circumscribing

$$|N_n| = N_p', \qquad |M_m| = M_p'$$

Inscribing

$$|N_n| = \vartheta' \cdot N_p', \qquad |M_m| = \vartheta' \cdot M_p'$$

$$\vartheta' = \begin{cases} \dfrac{\sqrt{1+4k_0'}-1}{2k_0}, & 8\bar{h}^2\left(1+\dfrac{\bar{h}}{1-\varepsilon_c}\right)-\bar{\sigma}^2 \le 0 \\ \dfrac{\sqrt{k_1'^2+4k_2'}-k_1'}{2}, & 8\bar{h}^2\left(1+\dfrac{\bar{h}}{1-\varepsilon_c}\right)-\bar{\sigma}^2 > 0 \end{cases}$$

where

$$k_0' = \frac{(\bar{\sigma}+2\bar{h})^2}{4\bar{\sigma}\bar{h}\left(1+\dfrac{\bar{h}}{1-\varepsilon_c}\right)+\bar{\sigma}^2}, \quad k_1' = 1+\frac{(1-\varepsilon_c-\bar{\sigma})(3\bar{\sigma}+8\bar{h})}{(\bar{\sigma}+2\bar{h})^2}, \quad k_2' = 1+\frac{2(1-\varepsilon_c-\bar{\sigma})}{\bar{\sigma}+2\bar{h}}$$

$\downarrow$ $\varepsilon_c = 0$

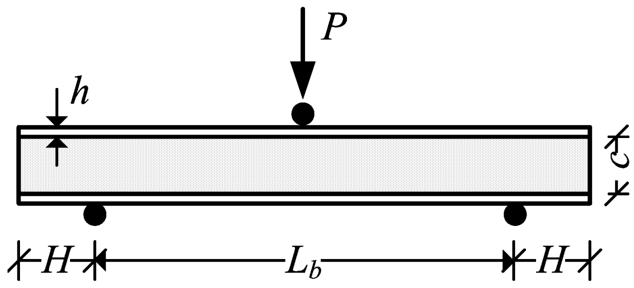
Perfect sandwich cross-section ϑ

$\downarrow$ $\bar{\sigma} = \sigma_c / \sigma_f = 1$

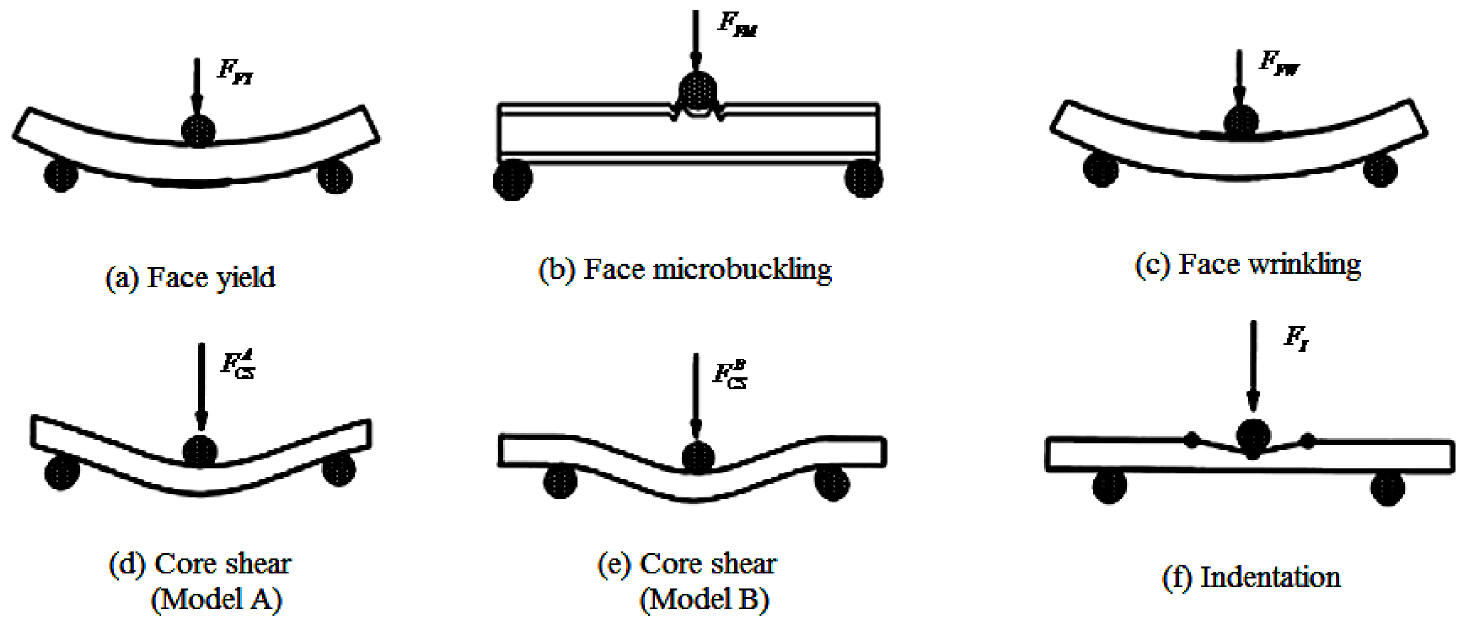
Monolithic solid cross-section $\vartheta = 0.618$ $\Leftarrow$ **Hodge, 1963**

Analytic solution for the large deflection of metallic foam core sandwich beam

Initial collapse modes of metallic sandwich beam

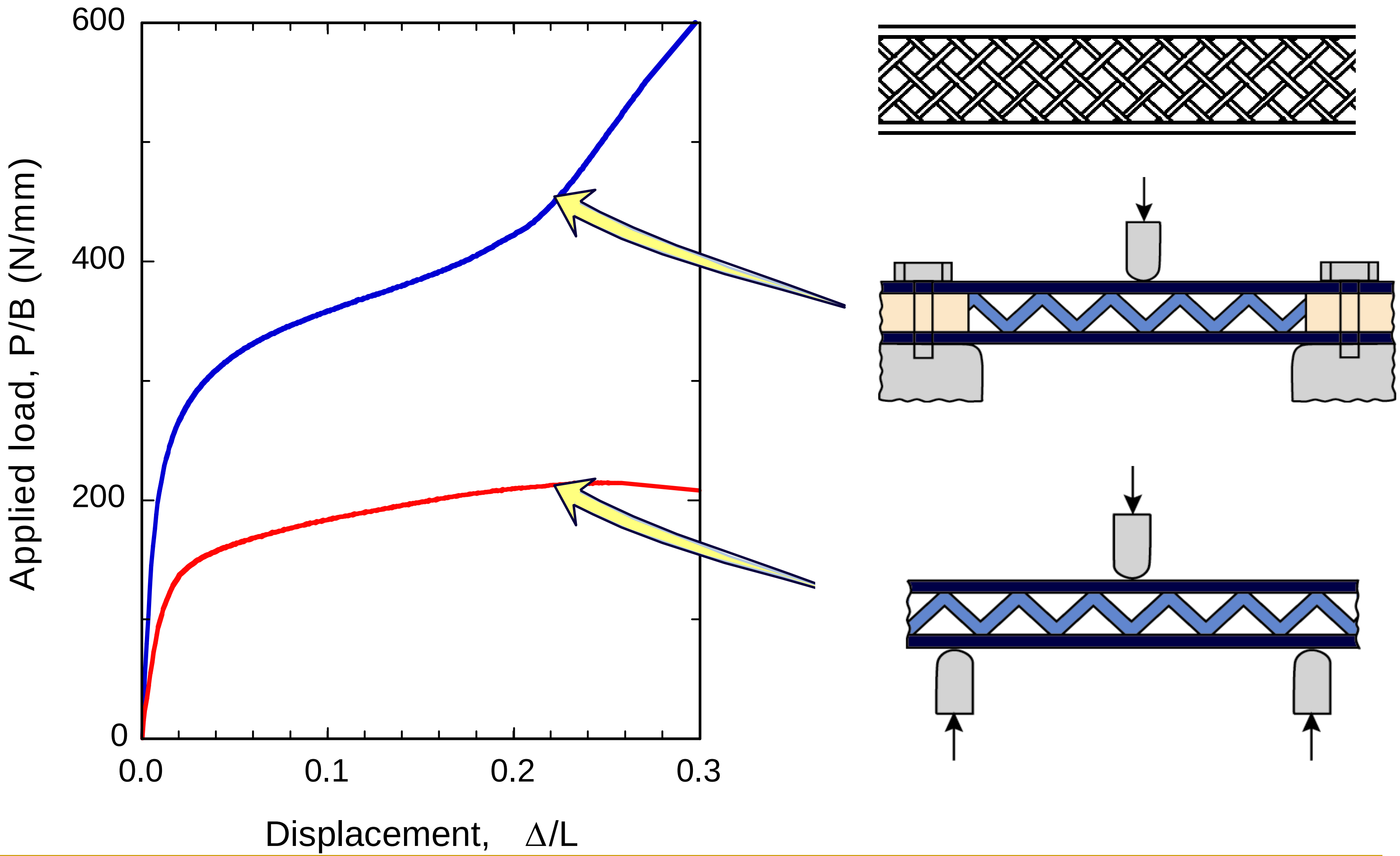


Three-point bending of sandwich beam

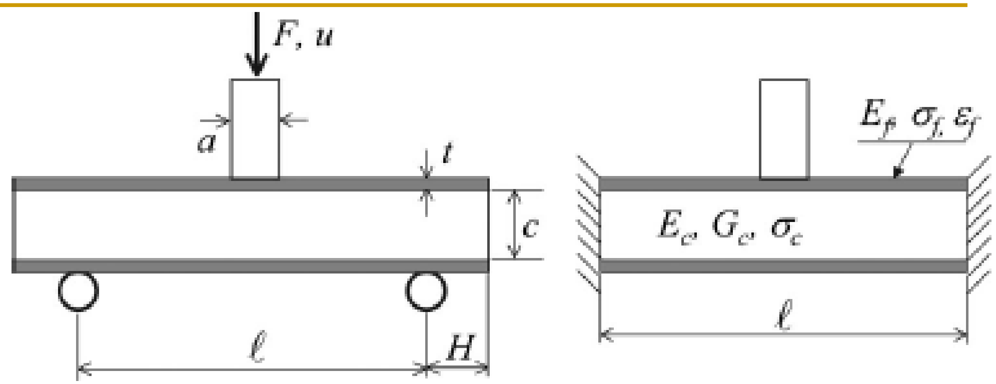


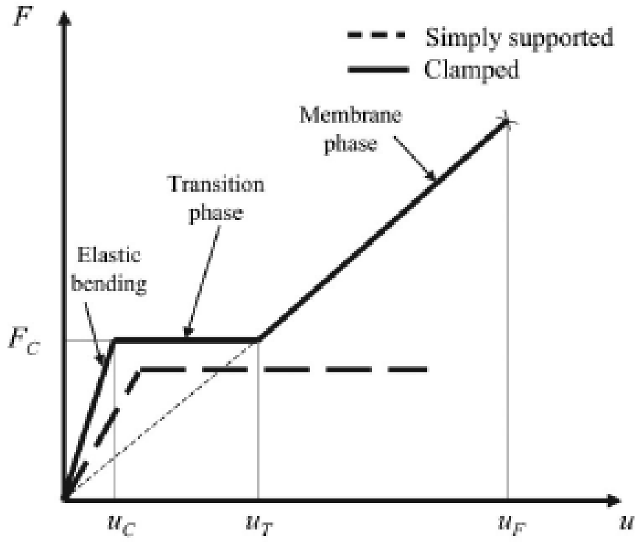
Failure modes

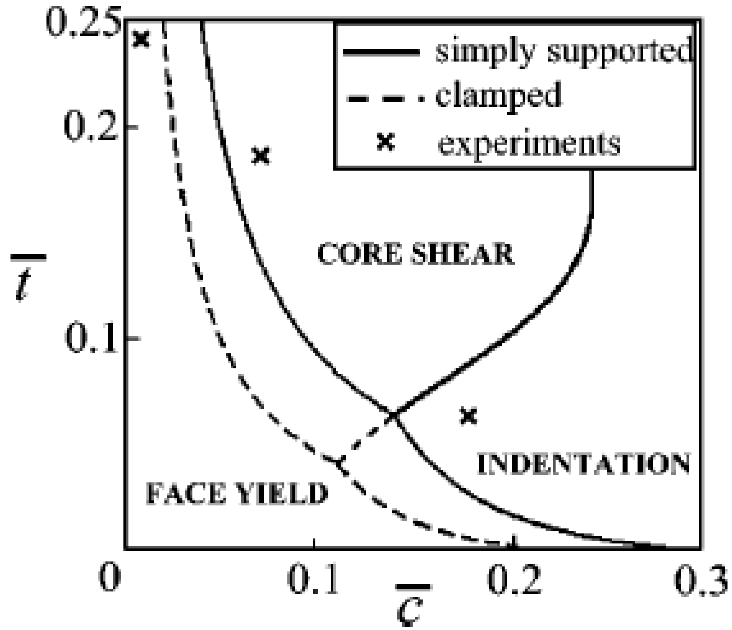
Influence of finite deflections



Simply supported and fully clamped sandwich beams

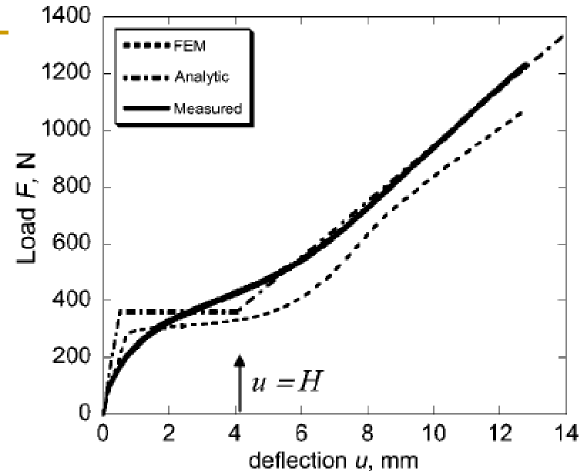




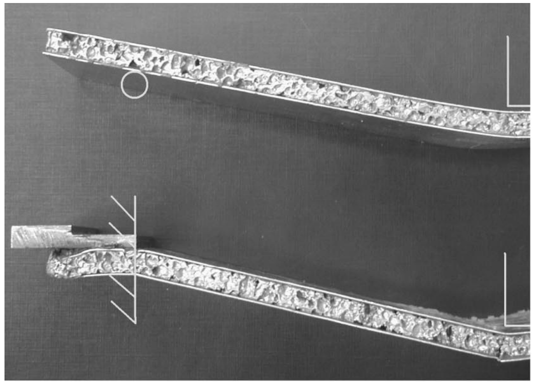


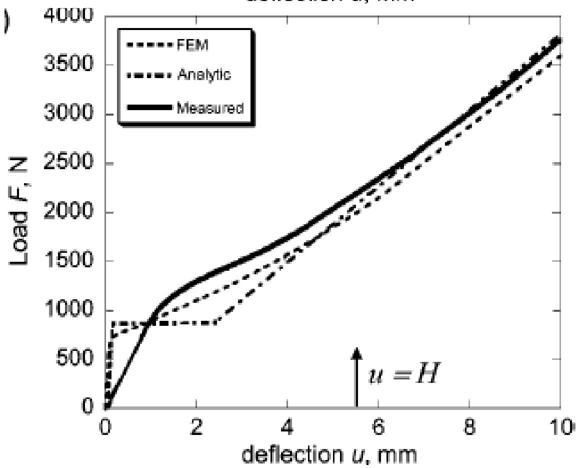
Analytical model **Initial collapse mechanism map**

Tagarielli and Fleck, ASME, J. Appl. Mech., 2005. 72: 408-417.

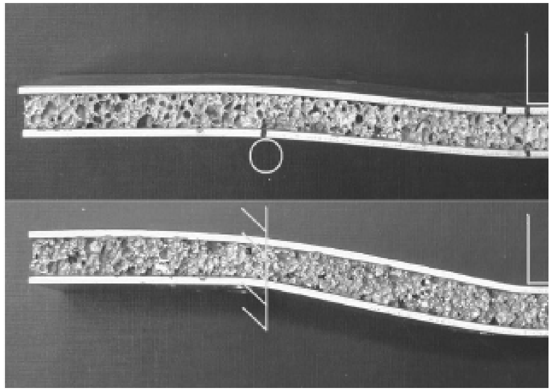


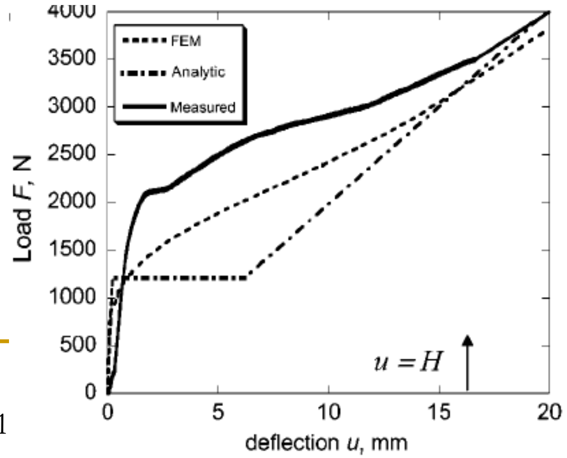
Face yield



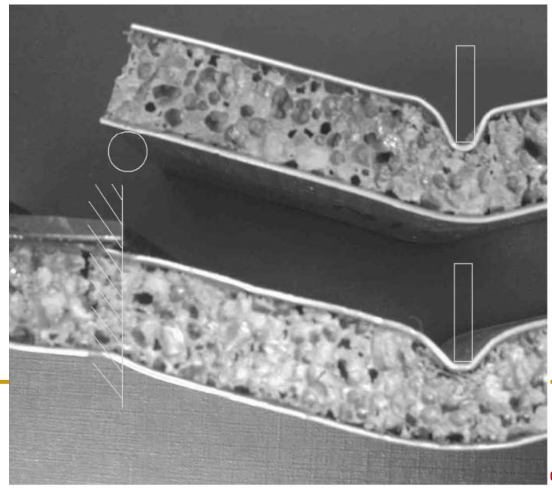


Core shear

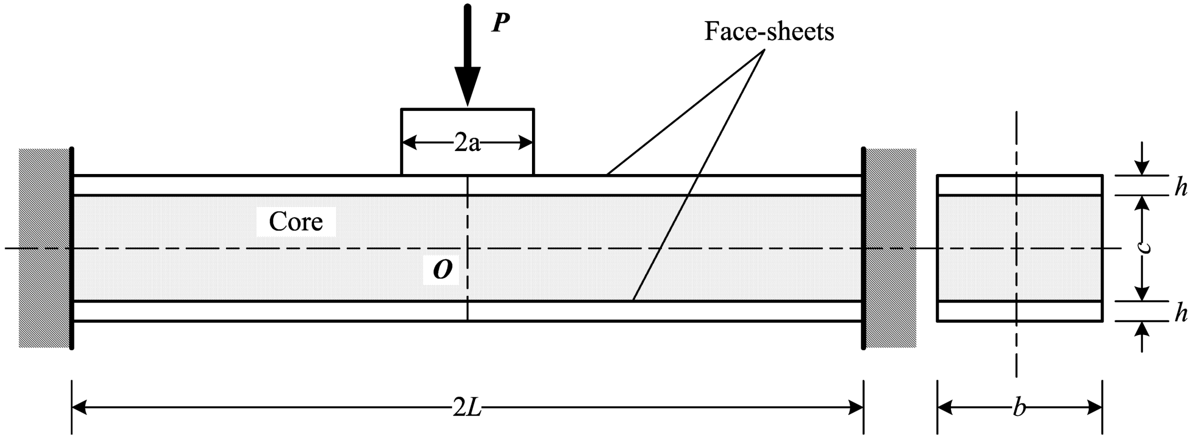




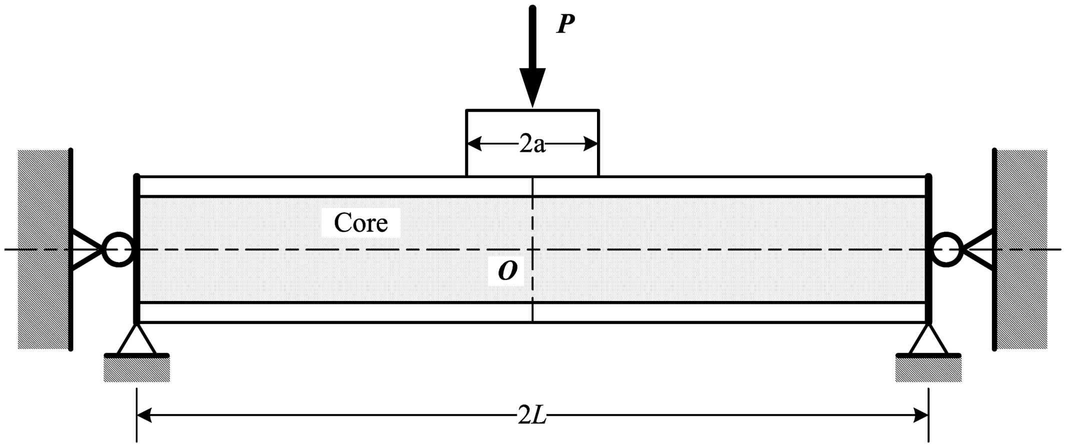
Indentation



Sandwich beams with axial restraints

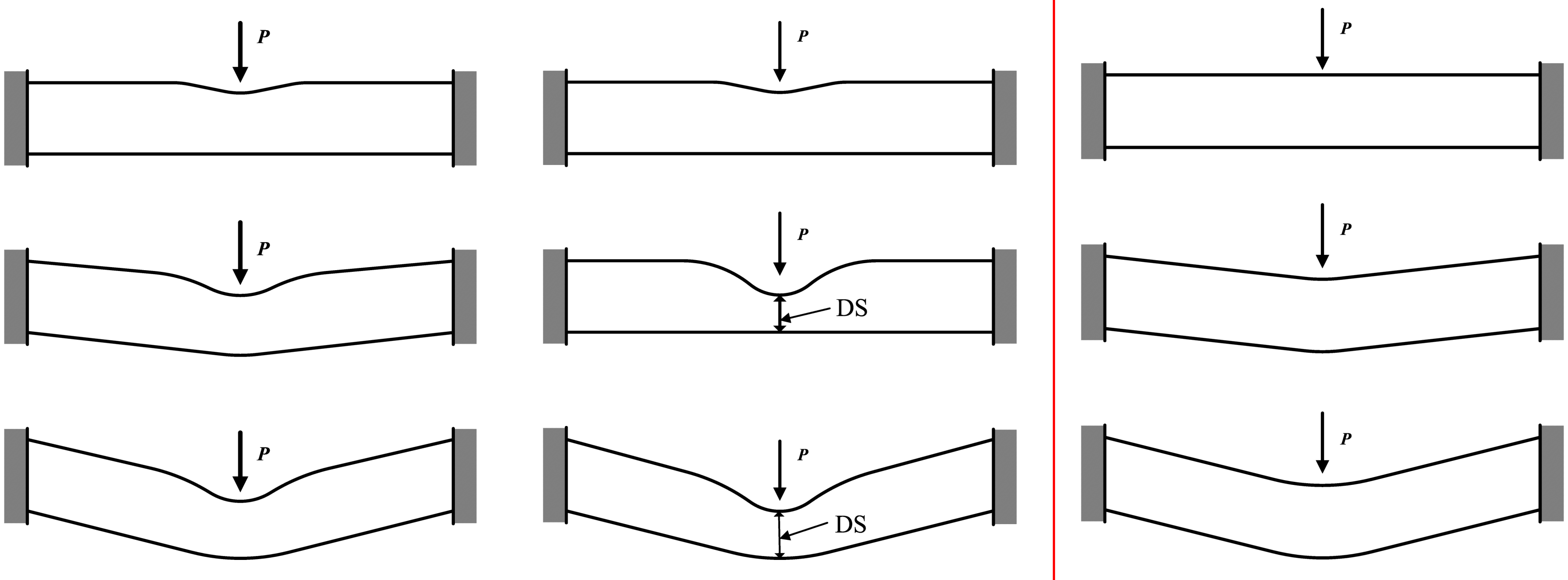


(a)

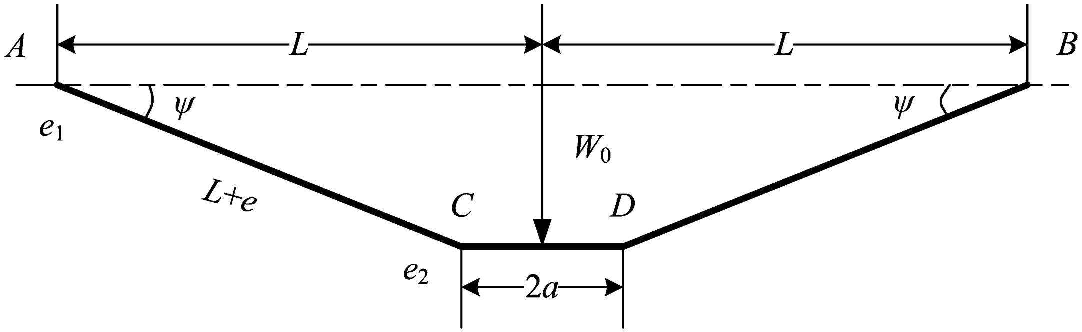


(b)

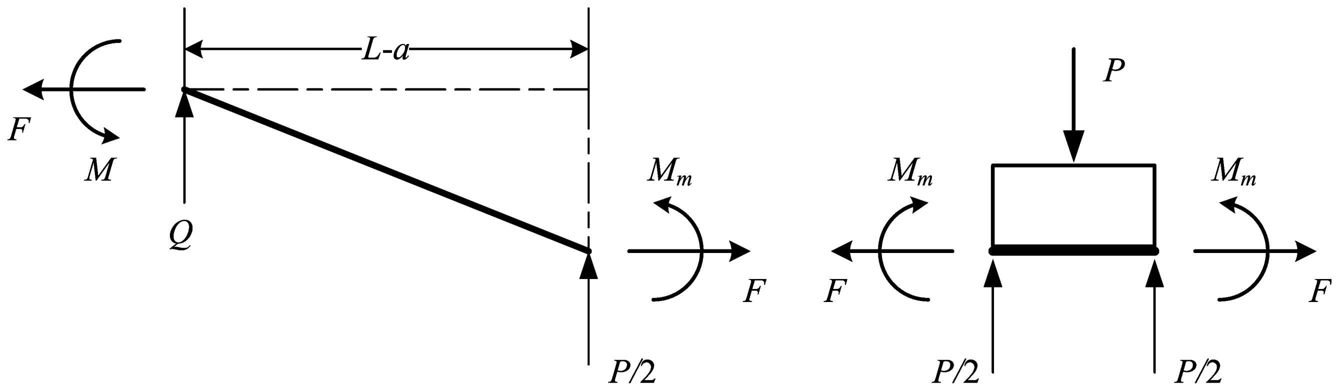
Deformation modes in post-yield regime



Mode I **Mode II** **Mode III**



(a)



(b)

Overall deformation of fully clamped sandwich beam

The total extension

$$e = e_1 + e_2 = \frac{1}{2}\frac{W_0^2}{L}$$

The angular rotation

$$\psi \cong \frac{W_0}{L}$$

The moment equilibrium equation

$$4M - PL + 2FW_0 = 0$$

According to the associated flow rule

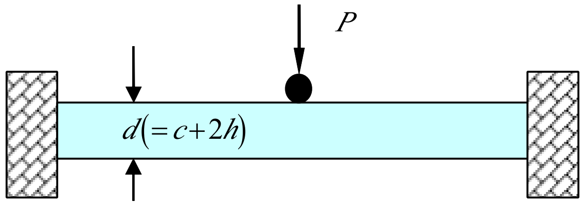
$$\frac{\dot{e}_1}{\dot{\psi}} = \frac{\dot{e}_2}{\dot{\psi}} = \begin{cases} \left(\frac{1}{2} + \frac{\bar{h}}{\bar{\sigma}}\right)c|n|, & 0 \le |n| \le \frac{\bar{\sigma}}{\bar{\sigma} + 2\bar{h}} \\ \frac{c}{2}\left[|n|\left(\bar{\sigma} + 2\bar{h}\right) - \bar{\sigma} + 1\right], & \frac{\bar{\sigma}}{\bar{\sigma} + 2\bar{h}} \le |n| \le 1 \end{cases}$$

Analytical solution for large deflection

$$
\begin{cases}
P^* = \dfrac{1}{1-\bar{a}}\left[1+\dfrac{\bar{\sigma}\left(1+2\bar{h}\right)^2}{4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}}W_0^{*2}\right], & 0 \le W_0^* \le \dfrac{1}{\left(1+2\bar{h}\right)} \\[2ex]
P^* = \dfrac{(1+2\bar{h})\left[(1+2\bar{h})(W_0^{*2}+1)+2(\bar{\sigma}-1)W_0^*\right]}{\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}, & \dfrac{1}{1+2\bar{h}} \le W_0^* \le 1 \\[2ex]
P^* = \dfrac{2\left(\bar{\sigma}+2\bar{h}\right)\left(1+2\bar{h}\right)}{\left[4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}\right]\left(1-\bar{a}\right)}W_0^*, & W_0^* \ge 1
\end{cases}
$$

$\bar{\sigma} = 1$ $\quad$ $\bar{a} \to 0$

$$
P^* = \begin{cases} W_0^{*2}+1, & 0 \le W_0^* \le 1 \\ 2W_0^*, & W_0^* \ge 1 \end{cases}
$$



Monolithic solid beam

Jones, Structural Impact, 1989.

The energy absorption

$$U = \int_0^{W_0} P(W_0)\, dW_0$$

Analytical solution for the energy absorption

$$\begin{cases} P^* = \dfrac{1}{1-\bar{a}}\left[1 + \dfrac{\bar{\sigma}\left(1+2\bar{h}\right)^2}{4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}} W_0^{*2}\right], & 0 \le W_0^* \le \dfrac{1}{\left(1+2\bar{h}\right)} \\ P^* = \dfrac{(1+2\bar{h})\left[(1+2\bar{h})(W_0^{*2}+1)+2(\bar{\sigma}-1)W_0^*\right]}{\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}, & \dfrac{1}{1+2\bar{h}} \le W_0^* \le 1 \\ P^* = \dfrac{2\left(\bar{\sigma}+2\bar{h}\right)\left(1+2\bar{h}\right)}{\left[4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}\right](1-\bar{a})} W_0^*, & W_0^* \ge 1 \end{cases}$$

$\bar{\sigma} = 1$ $\bar{a} = 0$

where $U^* = \dfrac{U}{P_c(c+2h)}.$

$$U^* = \begin{cases} \dfrac{1}{3} W_0^{*3} + W_0^*, & 0 \le W_0^* \le 1 \\ W_0^{*2} + \dfrac{1}{3}, & W_0^* \ge 1 \end{cases}$$

monolithic solid beam

Jones, Structural Impact, 1989.

Large deflection of pin-supported sandwich beam

Analytical solution for large deflection

$$
\begin{cases}
P^* = \dfrac{1}{1-\bar{a}}\left[\dfrac{2\bar{\sigma}(1+2\bar{h})^2}{4\bar{h}(1+\bar{h})+\bar{\sigma}}W_0^{*2}+\dfrac{1}{2}\right], & 0 \le W_0^* \le \dfrac{1}{2\left(1+2\bar{h}\right)} \\
P^* = \dfrac{(1+2\bar{h})\left[(1+2\bar{h})(4W_0^{*2}+1)+4(\bar{\sigma}-1)W_0^*\right]}{2\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}, & \dfrac{1}{2\left(1+2\bar{h}\right)} \le W_0^* \le \dfrac{1}{2} \\
P^* = \dfrac{2(\bar{\sigma}+2\bar{h})(1+2\bar{h})}{\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}W_0^*, & W_0^* \ge \dfrac{1}{2}
\end{cases}
$$

$\bar{\sigma}=1$ $\bar{a}=0$

$$
P^* = \begin{cases}
2W_0^{*2}+\dfrac{1}{2}, & 0 \le W_0^* \le \dfrac{1}{2} \\
2W_0^*, & W_0^* \ge \dfrac{1}{2}
\end{cases}
$$

monolithic solid beam

Jones, Structural Impact, 1989.

Analytical solution for the energy absorption

$$
\begin{cases}
U^* = \dfrac{1}{1-\bar{a}}\left[\dfrac{2\bar{\sigma}(1+2\bar{h})^2}{3\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right]}W_0^{*3}+\dfrac{1}{2}W_0^*\right], \quad 0 \le W_0^* \le \dfrac{1}{2\left(1+2\bar{h}\right)} \\
U^* = \dfrac{(1+2\bar{h})^2\left[4\left(W_0^{*3}-W_{01}^{*3}/8\right)/3+W_0^*-W_{01}^*/2\right]+2(\bar{\sigma}-1)(1+2\bar{h})\left(W_0^{*2}-W_{01}^{*2}/4\right)}{2\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}+\dfrac{3\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}}{3\left(1+2\bar{h}\right)\left[4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}\right](1-\bar{a})}, \\
\qquad \dfrac{1}{2\left(1+2\bar{h}\right)} \le W_0^* \le \dfrac{1}{2} \\
U^* = \dfrac{(\bar{\sigma}+2\bar{h})(1+2\bar{h})}{\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right](1-\bar{a})}\left(W_0^{*2}-\dfrac{1}{4}\right)+\dfrac{2\bar{h}(1+2\bar{h})^2+\bar{h}(1+\bar{h})(3\bar{\sigma}+1)+\bar{\sigma}}{3(1+2\bar{h})\left[4\bar{h}(1+\bar{h})+\bar{\sigma}\right]\left(1-\bar{a}\right)}, \quad W_0^* \ge \dfrac{1}{2}
\end{cases}
$$

$\bar{\sigma} = 1$ $\bar{a} = 0$

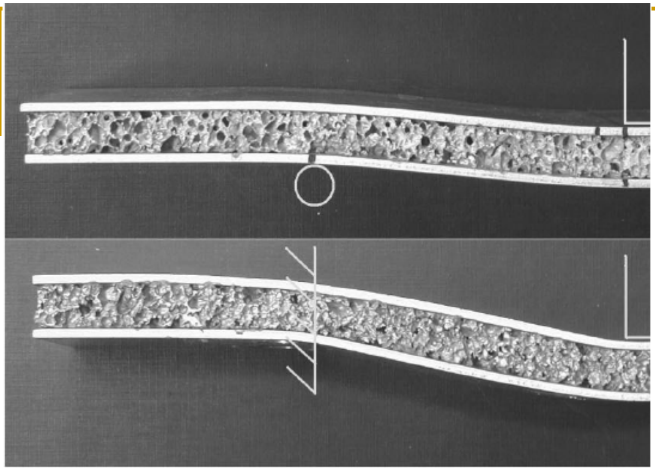
$$
U^* = \begin{cases}
\dfrac{2}{3}W_0^{*3}+\dfrac{1}{2}W_0^*, & 0 \le W_0^* \le \dfrac{1}{2} \\
W_0^{*2}+\dfrac{1}{12}, & W_0^* \ge \dfrac{1}{2}
\end{cases}
$$

monolithic solid beam

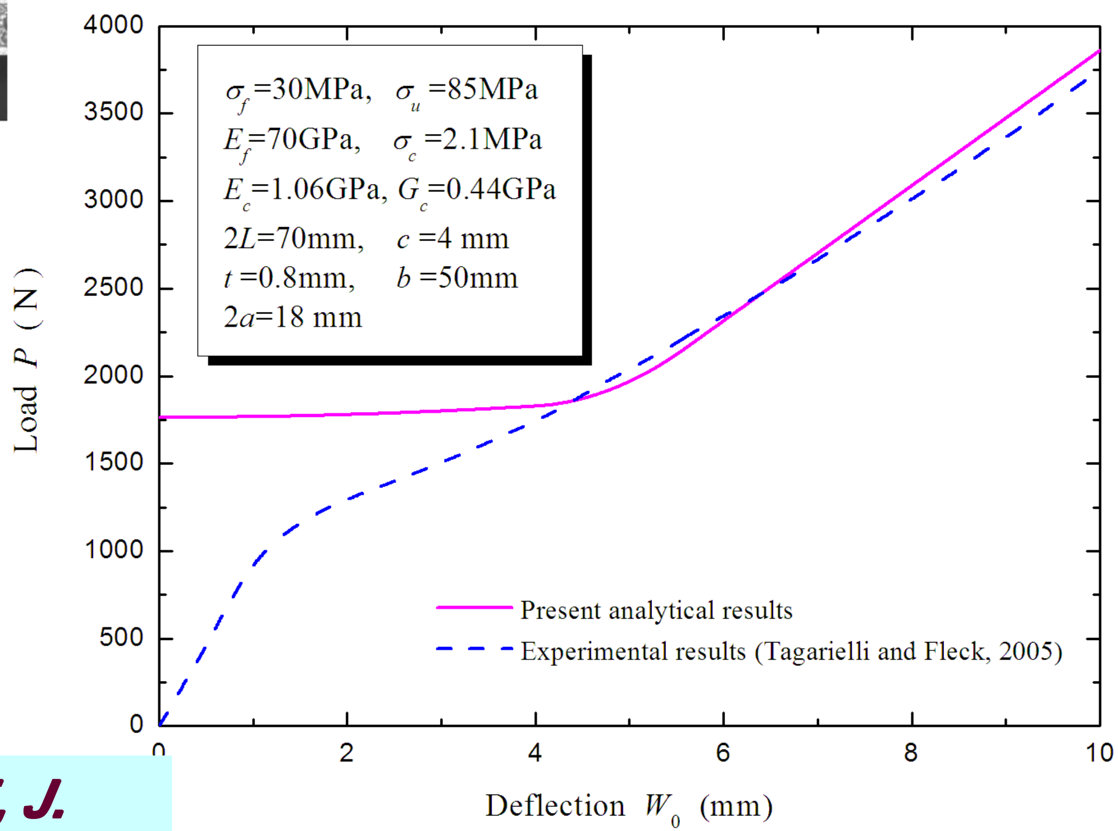
Jones, Structural Impact, 1989.

Results and Discussion

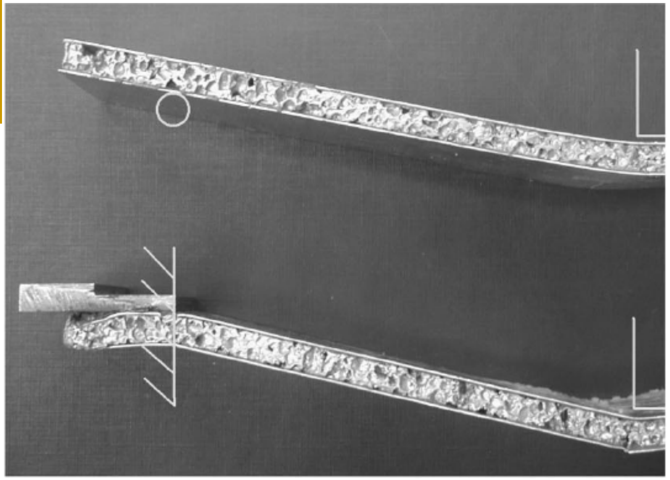
Comparison of the present analytical solution with experimental results (*Tagarielli and Fleck, 2005*)



(Tagarielli and Fleck, 2005)

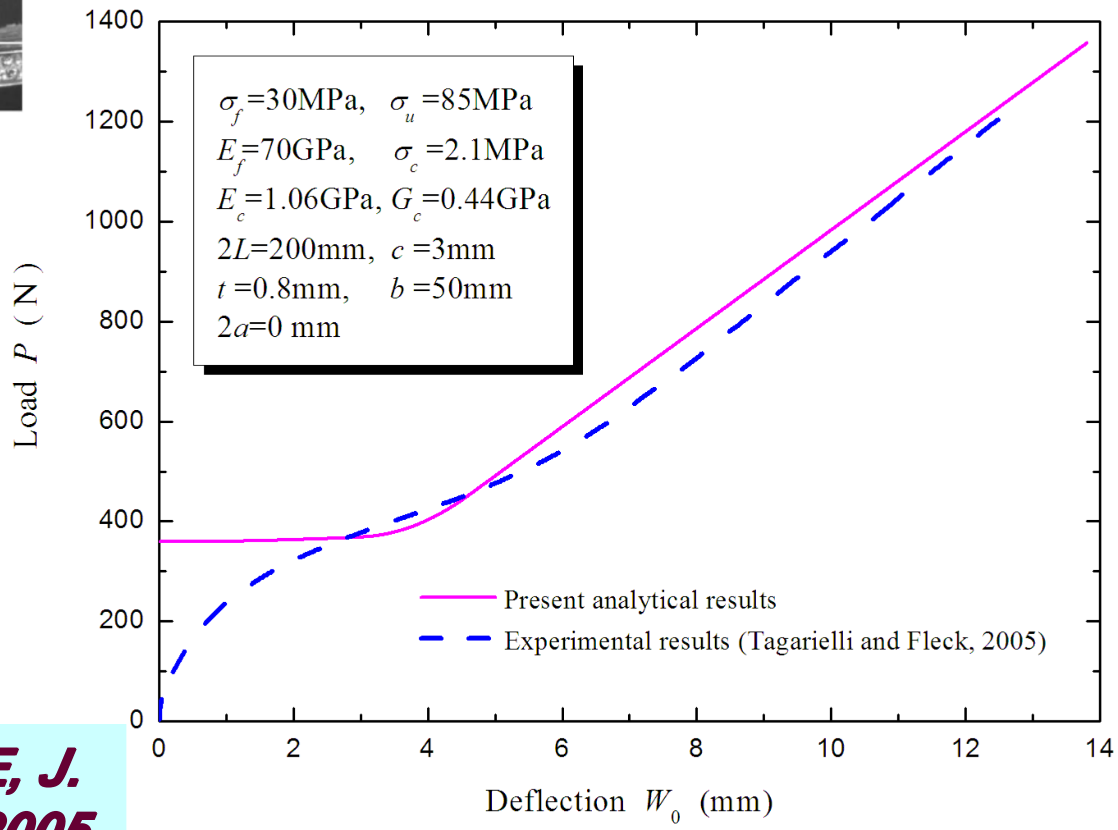


Tagarielli and Fleck, ASME, J. Appl. Mech., 72, 408-417, 2005.

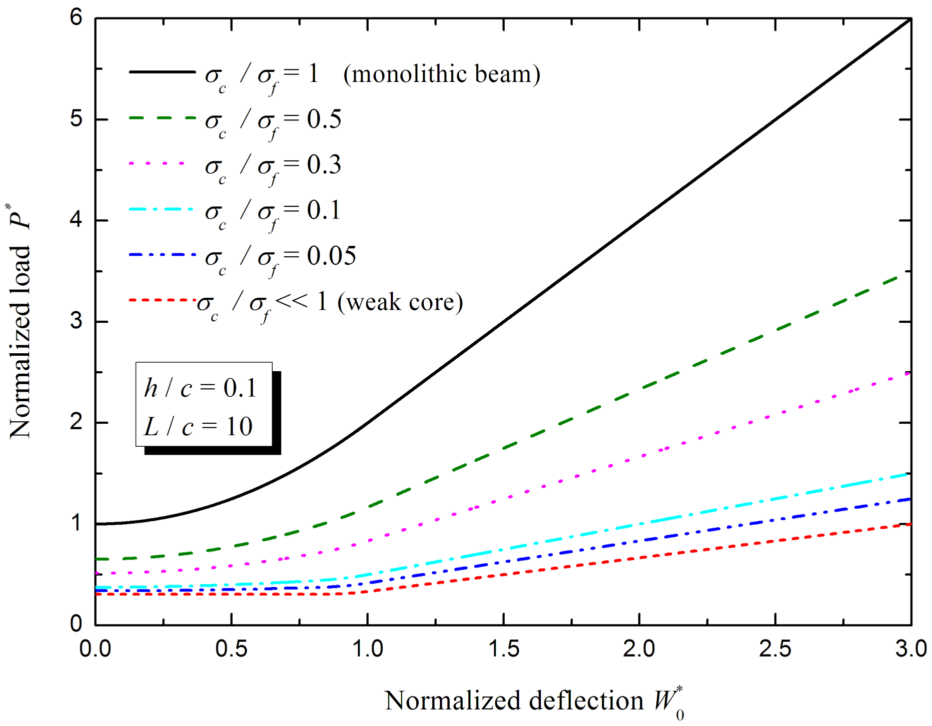


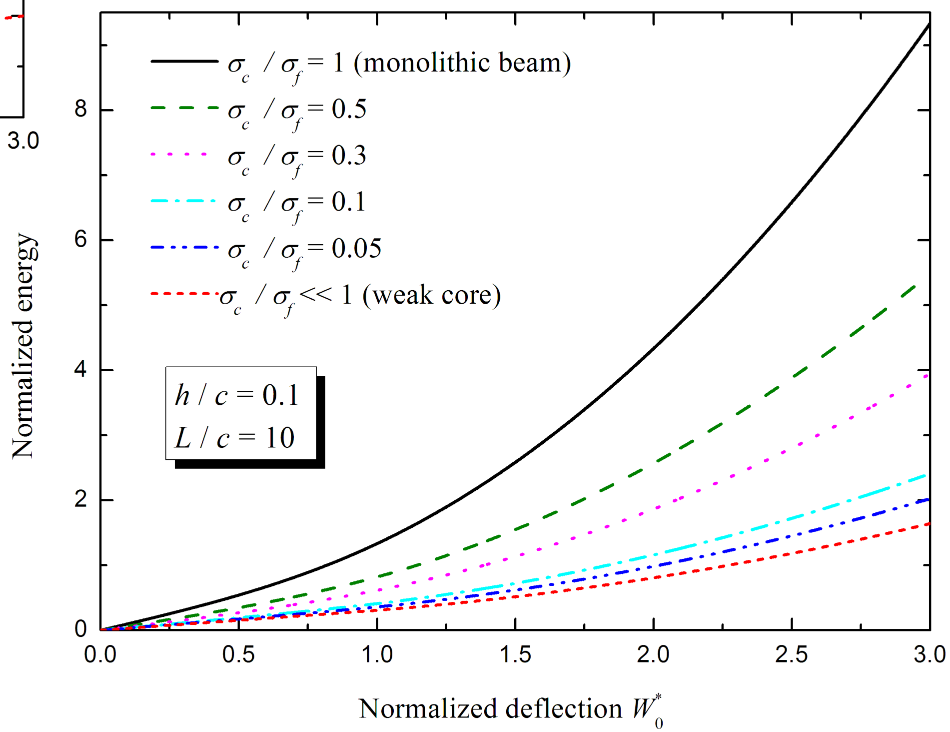
(Tagarielli and Fleck, 2005)

Comparison of the present analytical solution with experimental results (*Tagarielli and Fleck, 2005*)



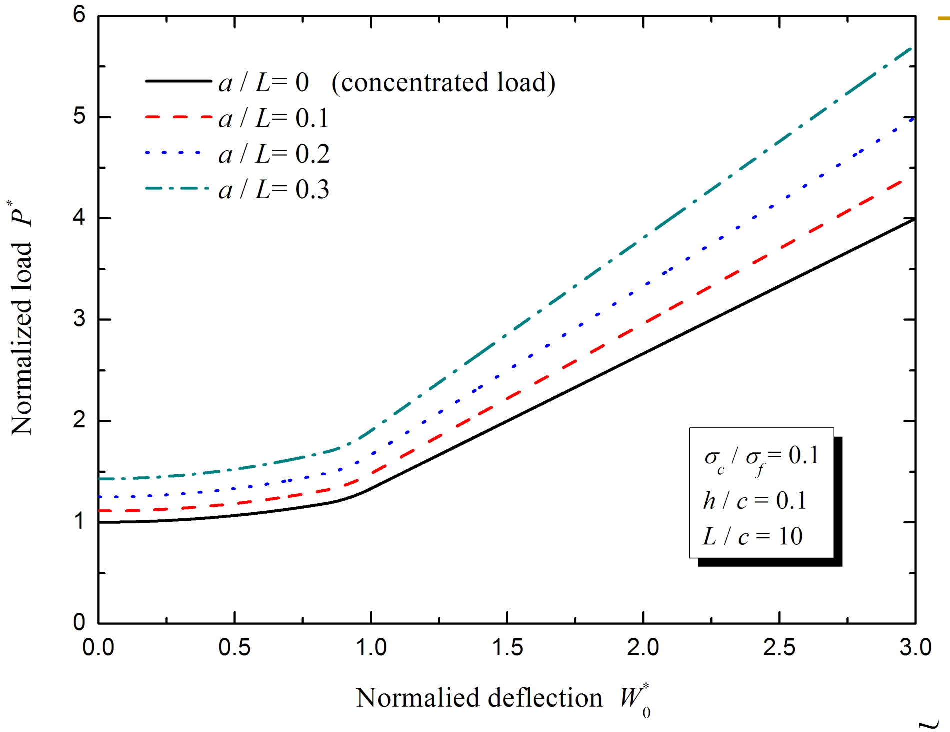
Tagarielli and Fleck, ASME, J. Appl. Mech., 72, 408-417, 2005.

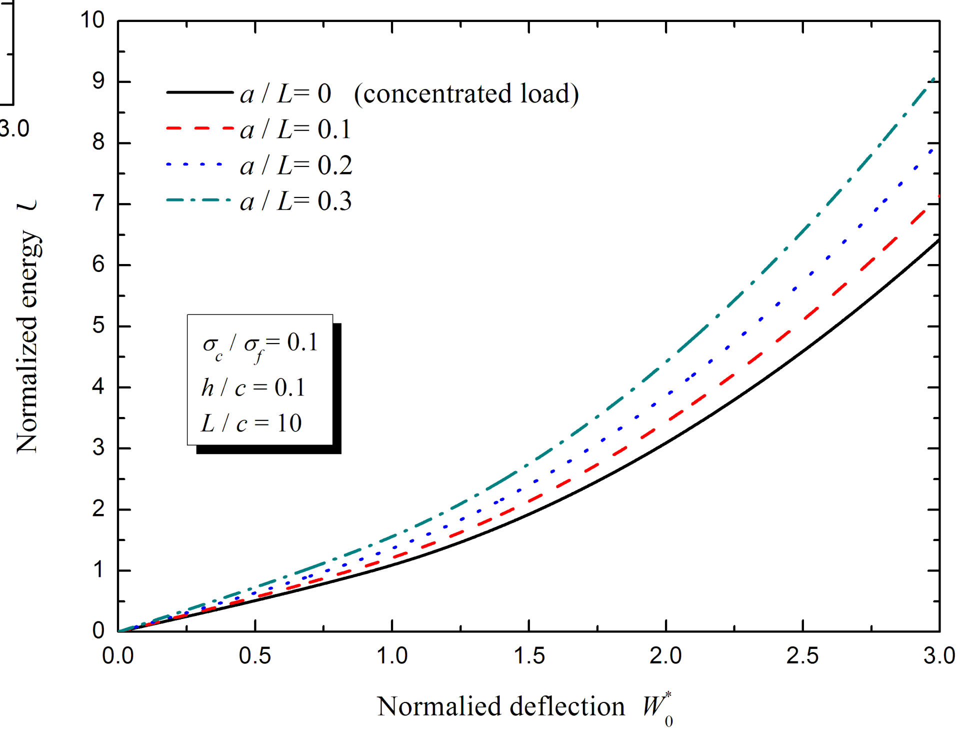




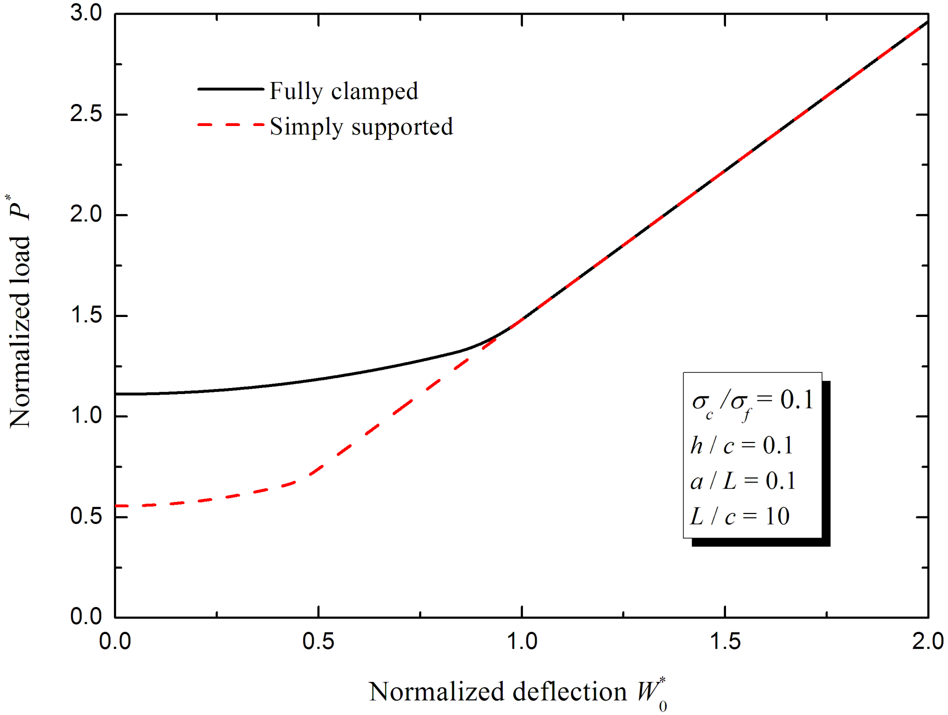
Effect of core strength

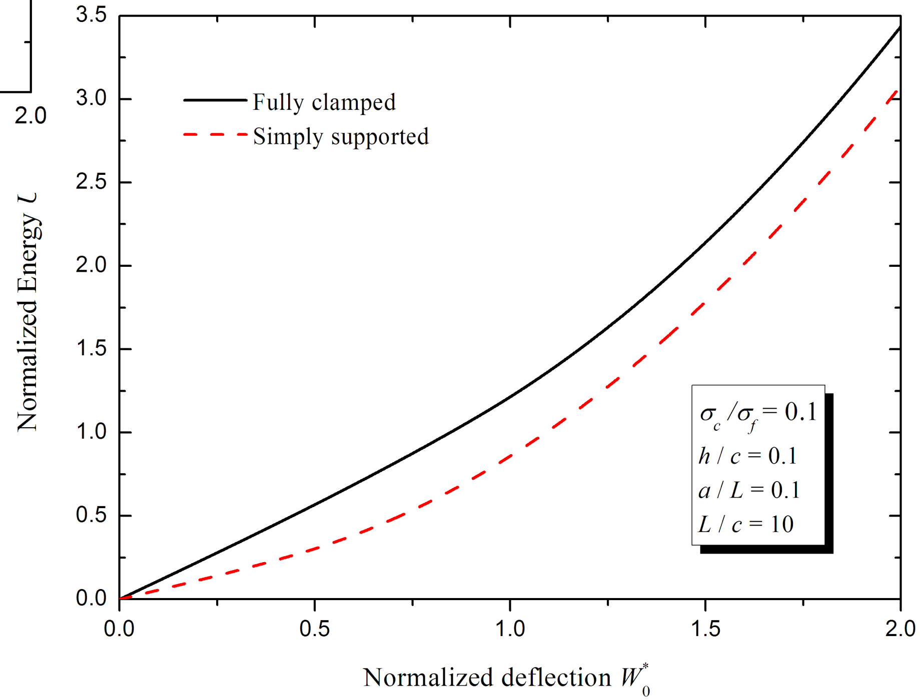
$$\bar{a} \to 0$$





Effect of geometry of punch





Effect of boundary condition

Summary

- **Yield criterion for sandwich structures incorporating the effect of core strength**

  which is valid for the sandwich structures with various core strength and geometries,

  can reduce to the well-known one for solid monolithic structures and the classical one for the sandwich structures with weak core, respectively.

- **Analytical solutions for the large deflections of fully clamped and simply supported metallic foam core sandwich beams under concentrated load.**
- **Large-deflection- induced axial force increases the capacity of load-carrying and energy absorption of foam core sandwich beam in post-yield regime.**
- **Core strength should be considered in analyzing large deflection of sandwich beam with strong core inside.**

Thank you very much for your attention