Abstracts for Talks

Discrete Riccati equation, hypergeometric functions and circle patterns of Schramm type

S. Agafonov

Abstract

Square grid circle patterns with prescribed intersection angles, mimicking holomorphic maps z^γ and $\log(z)$ are studied. It is shown that the corresponding circle patterns are embedded and described by special separatrix solutions of discrete Painlevé and Riccati equations. General solution of this Riccati equation is expressed in terms of the hypergeometric function. Global properties of these solutions, as well as of the discrete z^γ and $\log(z)$, are established.

Notes

Bialecki

Integrable cellular automata

Mariusz Bialecki

Abstract

We consider the discrete KP equation over a finite field. Applying the algebro-geometric method of construction of solutions we obtain a two soliton solution (of breather type) out of hyperelliptic curve of genus two. We describe the Jacobian of the curve over a finite field, which form a finite abelian group. The "evolution" can be explicitly presented as linear flows in the Jacobian. We also discuss global properties (periodicity, selfsimilarity) of this solution.

Notes

Bobenko

Projective geometry of the Yang-Baxter equation

Alexander Bobenko

Abstract

In this joint work with V.Adler and Yu.Suris we classify rational Yang-Baxter maps and give their geometric interpretation in terms of pencils of quadrics.

Notes

Bullough

Quantum solitons

Robin Bullough

Abstract

A short paper under the title 'Quantum solitons for quantum information and quantum computing' has already appeared [1]. This present talk makes a substantial reference to the article [1] but on the one hand covers material that was not included in that article [1] (as mentioned there) together with new material concerning recent observations [2, 3] of quantum solitons in ultra-cold gases of ^7Li atoms at temperatures less than *micro*Kelvin. In first instance the material was stimulated by the popular article [4] actually called 'Quantum Solitons'.

First of all I try to pin down as a matter of definition a quantum soliton solution of the attractive quantum NLS model in one space dimension which in terms of second quantised Bose fields $\hat{\Psi}, \hat{\Psi}^\dagger$ satisfies (with $\hbar \equiv \frac{h}{2\pi}$, h Planck's constant)

$$\begin{aligned} +i\hbar\frac{\partial\hat{\Psi}}{\partial t} &= -\hbar^2\frac{\partial^2\hat{\Psi}}{\partial x^2} + 2c\hat{\Psi}^\dagger\hat{\Psi}^2 \\ -i\hbar\frac{\partial\hat{\Psi}^\dagger}{\partial t} &= -\hbar^2\frac{\partial^2\hat{\Psi}^\dagger}{\partial x^2} + 2c(\hat{\Psi}^\dagger)^2\hat{\Psi} \end{aligned} \quad (1)$$

in which the real-valued coupling constant c satisfies $c < 0$. I indicate why the natural definition of a quantum soliton is a certain *matrix element* of the form $\langle S|\hat{\Psi}|S'\rangle$ in which $|S\rangle$ is a 'one-soliton' state: $|S\rangle$ is a simultaneous eigenstate of the Hamiltonian operator $\hat{H}$ governing Eqn.(1), of a number operator $\hat{N}$ commuting with $\hat{H}$ ($[\hat{H}, \hat{N}] = 0$), and of a further countably infinite set of commuting operators - the quantum NLS models being 'quantum integrable' for any $c \in \mathbb{R}$. I then investigate the possible "squeezing" of the state $|S\rangle$: since $\hat{N}|S\rangle = n|S\rangle$ with eigennumber n this state cannot be squeezed contrary to the actual experiments reported in [4]! I next look at $|S\rangle$ as a qubit (of quantum information) and show how quantum sine-Gordon, the relativistic version of quantum attractive NLS, has a natural qubit state also.

In order to establish the *bona fides* of this talk at this meeting on 'Discrete systems and geometry' I make a brief reference to the quantum solitons of certain q-deformed quantum *lattices*. Thereafter I shall be concerned with the observations of quantum solitons in ultra-cold ^7Li atoms as described in [2, 3]. An obvious question is 'Do these quantum solitons show specific quantum features?' A phenomenon of 'collapse' is displayed but is this evidence of the quantum instability of quantum NLS in the attractive case?

References

[1] R.K. Bullough and Miki Wadati, 'Quantum solitons for quantum information and quantum computing' in 'Nonlinear Physics: Theory and Experiments. II' Eds. M.J. Ablowitz, M. Boiti, F. Pempinelli and B. Prinari (World Scientific Publishing Co. Pte. Ltd., Singapore, 2003)

[2] L. Khaykovich *et al.*, *Science*, **296**, 1290 (2002)

[3] K.E. Strecker *et al.*, *Nature*, **417**, 150 (2002)

[4] I. Abram, Quantum Solitons, *Physics World*, February (1999)

Notes

Quadrilateral Koenigs and Bianchi lattices

A. Doliwa

Abstract

The integrable discrete analog of Koenigs nets is introduced in a geometric way. It is shown how integrability of the Koenigs lattices is connected with the celebrated Pascal's six-point theorem. The appropriate reduction of the fundamental transformation is also constructed and its geometric properties are described.

Then the dual Koenigs lattices are introduced and their relation to a novel geometric interpretation of the discrete Bianchi system is presented. This new interpretation supports the opinion that geometry, especially the discrete geometry, helps to understand integrability of the underlying systems.

Notes

Estevez

A generalization of the Sine Gordon equation to $2+1$ dimensions

P. G. Estevez

Abstract

The singular manifold method is applied to an equation in $2+1$ dimensions that can be considered as a generalization of the Sine Gordon equation. This method allows us to relate the equation with the AKNS equation in $2+1$ dimensions via a Miura transformation.

Notes

Ferapontov

On integrability of (2+1)-dimensional quasilinear systems

E. V. Ferapontov and K. Khusnutdinova

Abstract

A (2+1)-dimensional quasilinear system is said to be 'integrable' if it can be decoupled in infinitely many ways into a pair of compatible n-component one-dimensional systems in Riemann invariants. Exact solutions described by these reductions, known as nonlinear interactions of planar simple waves, can be viewed as natural dispersionless analogs of n-gap solutions. It is demonstrated that the requirement of the existence of 'sufficiently many' n-component reductions provides the effective classification criterion. As an example of this approach we classify integrable (2+1)-dimensional systems of conservation laws possessing a convex quadratic entropy.

Notes

Affine Weyl group actions on isomonodromic systems

J. Harnad

Abstract

The action of affine Weyl groups on solutions to isomonodromic deformation equations is developed within the context of dual symplectic reductions. The parameter space is the root space and the points are identified with values of the momentum map generating a complexified torus action on the pre-reduced space. Critical points are shown to correspond to linearizations of the systems in terms of generalized hypergeometric functions.

Notes

Searching for lattice equations that are "consistent around a cube"

J. Hietarinta

Abstract

Only rather few integrable lattice equations are known. One class is defined by multi-linear expressions on the corners of a square; it is called integrable, if one can build a consistent cube from the squares. A partial classification of such lattice maps has been done by Adler, Bobenko and Suris under the assumptions of symmetry and "tetrahedron property". We report partial results of our search for maps without the tetrahedron property.

Notes

Hirota

Ultradiscretization of the Tzitzeica equation

Ryogo Hirota and Daisuke Takahashi

Abstract

The Tzitzeica equation was transformed into a trilinear form by Schief, who discretized the trilinear form. However the discrete trilinear Tzitzeica equation can not be ultradiscretized in its present form. We have found that the Tzitzeica equation is nothing but the Toda molecule equation with a special boundary condition. Using this fact we have succeeded in extending it to a higher equation and ultradiscretizing both equations.

Notes

Ultradiscrete Hamiltonian systems

Masataka Iwao

Abstract

The method of ultradiscrete limit can be applied for a series of discrete systems derived from such kind of Hamiltonian systems as parametrized with corresponding lattice polygons. For every ultradiscrete system, general solution can be obtained from the polar set of each lattice polygon.

Notes

Kakei

Weyl group action on a derivative nonlinear Schödinger equation

Saburo Kakei and Tetsuya Kikuchi

Abstract

A class of Bäcklund transformations of a derivative nonlinear Schödinger (∂NLS) equation,

$$\frac{\partial q}{\partial t} = \frac{1}{2}\frac{\partial^2 q}{\partial x^2} - 2q^2\frac{\partial r}{\partial x} - 4q^3r^2, \quad \frac{\partial r}{\partial t} = -\frac{1}{2}\frac{\partial^2 r}{\partial x^2} - 2r^2\frac{\partial q}{\partial x} + 4r^3q^2,$$

is investigated in terms of the Weyl group $W(A_1^{(1)})$. Under a similarity reduction, the ∂NLS is reduced to the Painlevé IV and the translation in the Weyl group generates two different equations of discrete Painlevé-type.

Notes

Kaptsov

Involutive distributions, invariant manifolds and determining equations

O. V. Kaptsov

Abstract

We introduce the notion of an invariant solution under an involutive distribution. We give sufficient conditions for existence of such a solution to a system of differential equations. In the case an evolution system of partial differential equations we describe how to construct auxiliary equations for functions determining differential constrains compatible with the original system. Using this theorem, we introduce linear and quasilinear determining equations which enable us to find some classes of involutive distributions, nonclassical symmetries, and differential constraints. We give examples of reductions and exact solutions to some nonlinear partial differential equations arising in applications.

References

[1] Kaptsov O.V. Siberian Mathematical Journal, Vol.43, No. 3, 2002.
[2] Andreev V K, Kaptsov O V, Pukhnachev V V and Rodionov A A 1998 *Applications of Group–Theoretic Methods in Hydrodynamics* (Kluwer Academic Publishers).

Notes

Konopelchenko (given by Schief)

A novel generalization of Clifford's classical point-circle configuration. Geometric interpretation of the quaternionic discrete Schwarzian KP equation

B. G. Konopelchenko and W. K. Schief

Abstract

There exists an extensive literature on generalizations of Clifford's planar point-circle configurations. Based on an observation made by Ziegenbein, we present a novel extension of Clifford's C4 configuration to four dimensions. These generalized Clifford configurations are then shown to encapsulate the quaternionic discrete Schwarzian KP equation.

Notes

Masuda

${}_{10}E_9$ solution to the elliptic Painlevé equation

Kenji Kajiwara, Tetsu Masuda, Masatoshi Noumi, Yasuhiro Ohta and Yasuhiko Yamada

Abstract

A τ function formalism for Sakai's elliptic Painlevé equation is presented. This establishes the equivalence between the two formulations by Sakai and by Ohta-Ramani-Grammaticos. We also construct a particular solution of the elliptic Painlevé equation expressed in terms of the elliptic hypergeometric function ${}_{10}E_9$.

Notes

Matsukidaira

Euler-Lagrange correspondence of generalised Burgers cellular automaton

Junta Matsukidaira

Abstract

Recently, we have proposed a Euler-Lagrange transformation for cellular automata (CA) by developing new explicit transformation formulas. Applying this method to the Burgers CA (BCA), we have succeeded in obtaining the Lagrange representation of the BCA. In this talk, we apply this method to generalised Burgers CA (GBCA) which include the Fukui-Ishibashi model and the quick-start model associated with traffic flow. As a result, we have succeeded in clarifying the Euler-Lagrange correspondence of these models. We have also obtained multi-value extension of these models, which are prototype of the conservative CA models in multi-lane.

Notes

Darboux type transformations for 5-point and 7-point self-adjoint schemes

Maciej Nieszporski

Abstract

We present a discretization of second order elliptic linear equations in two independent variables including a discretization of 2D Schrodinger equation. We show the discrete equations are covariant under Darboux type transformations. We point out some applications of the equations in discrete geometry.

Notes

Nijhoff

Integrable hierarchies of soliton equations: II. Associated lattice structures and reductions

F. W. Nijhoff

Abstract

This talk is to clarify a number of connections between the integrable continuous partial differential equations generating the hierarchies as mentioned in part I (see Tongas) and certain partial difference and differential-difference equations which can be identified as discrete versions of the Boussinesq family of equations. Some reductions on the level of the nonlinear lattices will be discussed as well.

Notes

Movable discrete breathers

V.Novokshenov

Abstract

Nonlinear classical Hamiltonian lattices exhibit generic solutions in the form of discrete breathers. These solutions are time-periodic and exponentially localized in space. Two models, the discrete sine-Gordon and Lioville lattices, are discussed in view to get moving breathers, i.e. true time-periodic (multi) solitons with free velocity and amplitude.

Notes

Some aspects of Toda molecule

Yasuhiro Ohta

Abstract

The Toda molecule equation appears in many places in soliton theory since its solution is nothing but the general determinant. Some properties of the solutions of Toda molecule equation and their relations with the rational solutions of Painlevé III equation are discussed.

Notes

Orlov

Hypergeometric tau functions and discrete equations

Alexandre Orlov

Abstract

We show that hypergeometric type series solves integrable discrete equations.

Notes

Schiebold

Negatons of the Toda lattice

Cornelia Schiebold

Abstract

In the talk we explain how to use some tools from operator theory (the concept of quasi-Banach ideals) to derive an explicit solution formula for the Toda lattice. The essential advantage of our formula is that it contains an operator-valued parameter, which means that we can plug in almost arbitrary operators. This yields a systematic approach to the construction of solutions. For example it can be shown that all solutions covered by the Inverse Scattering Method (and more) can be realized in this frame.

As application, we focus on (free and bound) nonlinear superposition. First we discuss countable superpositions of solitons. In our result we are able to simplify and generalize work of Gesztesy and Renger (Comm.Math.Phys.184, 27-54, 1997).

Second we give a detailed study of negatons, a solution class with the following properties: They consist of solitons which are organized in groups, and solitons in the same group are weakly coupled. In contrast, groups as a whole show a particle-like behaviour. Our main result is a complete characterization of negatons in terms of their asymptotic behaviour. It confirms expectations of Matveev (J.Math.Phys.35, 2955-2970, 1994) in the case of negatons.

Notes

Schief

On the integrability of infinitesimal and finite deformations of polyhedral surfaces

W. K. Schief

Abstract

In 1892, Bianchi determined under what circumstances a conjugate net on a surface is preserved by finite isometric deformations. His investigation led to a class of surfaces which has come to be known as Bianchi surfaces. These surfaces are integrable and, as shown by Bianchi, the deformation parameter may be identified with the 'spectral' parameter. In this talk, we adopt Sauer's kinematic approach to isolate similar hidden integrable structure in the canonical discrete analogue of Bianchi's problem.

Notes

Symmetry approach for non-evolution equations

V. Sokolov

Abstract

The main concepts of the symmetry approach are generalized to the case of Boussinesq type models. All integrable equations with dispersion law $\omega = k^{3/2}$ are found.

Notes

Szafraniec

Continuous vs. discrete orthogonal polynomials. Two examples

F. H. Szafraniec

Abstract

The topics is presented by exibiting some pecular properties of two pairs of orthogonal polynomials: Hermite and Charlier, Laguerre and Meixner.

References

1. FH Szafraniec, Duality in the quantum harmonic oscillator, J. Phys. A: Math. Gen. 34(2001), 10487-10492.
2. FH Szafraniec, Laguerre and Meixner polynomials in duality, submitted.
3. FH Szafraniec, Laguerre and Meixner polynomials in one, work in progress.

Notes

Asymptotic behavior of the fundamental Cycle of Periodic Box-Ball Systems

Tetsuji Tokihiro

Abstract

We investigate asymptotic behavior of the fundamental cycle of periodic box-ball systems (PBBSs) when the system size N goes to infinity. According to the integrable nature of the PBBS, the trajectory is confined to qualitatively smaller number of states than that of the total states. We prove that, although the maximum fundamental cycle is of order of $\exp[\sqrt{N}]$, almost all fundamental cycle is less than $\exp[(\log N)^2]$.

Notes

Tongas

Integrable hierarchies of soliton equations: I. Generalized Ernst equations for colliding plane spacetimes.

Anastasios Tongas

Abstract

We present a method of constructing integrable nonlinear partial differential equations which encode the complete hierarchy of soliton equations. The first nontrivial case shows that there is a very close relation between the hierarchy of the KdV equation and the Ernst-Weyl equation describing the interaction of gravitational and neutrino waves in Einstein's theory of General Relativity. The next case leads to a new family of nonlinear partial differential equations of hyperbolic type which represents a generalization of the Ernst-Maxwell-Weyl equations describing the interaction of the fields in the former case with the electromagnetic field. The new system under consideration is classically exactly integrable and in fact it can be related to the Boussinesq hierarchy of soliton equations.

Notes

Separation of variables for cofactor pair systems and cofactor-elliptic coordinates

Claes Waksjö

Abstract

A Newton system of cofactor type (a cofactor system) is a non-conservative system $\ddot{q} = M(q)$ with a force that can be written $M(q) = -A(q)^{-1}\nabla W(q)$, where $A(q) = \operatorname{cof} G(q)$ is the cofactor matrix of the second degree matrix polynomial $G(q) = \alpha q q^{\mathrm{T}} + \beta q^{\mathrm{T}} + q\beta^{\mathrm{T}} + \gamma$, and $W(q)$ is a function called quasi-potential. These systems do not have any direct Hamiltonian formulation as in the special case of conservative systems $G = I$, $W = V$, making it impossible to speak of separation of variables in the usual Hamilton–Jacobi sense. However, if the cofactor system admits an extra quadratic first integral $\frac{1}{2}\dot{q}^{\mathrm{T}}\tilde{A}\dot{q} + \tilde{W}$, also of cofactor type, in addition to the energy $\frac{1}{2}\dot{q}^{\mathrm{T}}A\dot{q} + W$, then it is possible to define a non-canonical transformation that brings the vector field $(\det \tilde{G})(p, M)$ into canonical Hamiltonian form. This vector field is simply a rescaled version of the vector field (p, M) corresponding to the cofactor system, so they share trajectories and are simultaneously integrable. The Hamiltonian for the rescaled vector field has Stäckel form, which means that the Hamilton–Jacobi equation is solvable by separation of variables. The separation coordinates involved are related to the usual elliptic and parabolic coordinates that arise in classical separability theory.

It is in this sense that we will discuss separability for cofactor systems. It naturally generalizes the classical separability concept for conservative systems. Incidentally, it also sheds light on the meaning of separation of variables for non-canonical Hamiltonian systems.

Notes

Willox

A generalized Tzitzeica equation and its discretization

Ralph Willox

Abstract

Extending the so called coupled KP hierarchy (a hierarchy of 2+1 dimensional integrable equations related to the $D(\infty)$ algebra, that possess solutions expressed in terms of Pfaffian tau functions) to negative time flows, one obtains coupled Toda-type equations on a two-dimensional lattice. It can be shown that there exist reductions of these integrable systems to 1+1 dimensional integrable systems that "live" on a finite part of this lattice. The topic of this talk will be a system of coupled Hirota bilinear equations, obtained from such a reduction, defined on only 5 points of the lattice. These equations will be shown to correspond to a coupling of a Tzitzeica equation to two linear equations. The Lax representation of this system will be presented and if time permits, a fully discrete version of this system will also be discussed.

Notes