Differentiation Structured Worksheet 1

1. Differentiate $x \sin \frac{1}{x}$ with respect to x, and hence evaluate the derivative of this function at $x = \frac{6}{\pi}$ and $x = -2$.
(You should be able to give exact expressions and do so without the use of a calculator!)

Solution We begin by using the $\boxed{}$ Rule. We can write $x \sin \frac{1}{x}$ as $f(x)g(x)$ where $f(x) = \boxed{}$ and $\boxed{} = \sin \frac{1}{x}$. By the $\boxed{}$ Rule,

$$\frac{d}{dx}\big(f(x)g(x)\big) = \boxed{}.$$

Now $f'(x) = \boxed{}$, while to find $g'(x)$ we need the $\boxed{}$ Rule. We write

$$g(x) = \sin\big(u(x)\big) \text{ where } u(x) = \boxed{}.$$

So

$$\begin{aligned} g'(x) &= \frac{d}{dx}\big(\sin u(x)\big) \\ &= \frac{d}{du}(\sin u)\frac{du}{dx} \\ &= \cos u \times \boxed{} \\ &= \boxed{} \cos \frac{1}{x}. \end{aligned}$$

Hence

$$\frac{d}{dx}\left(x \sin \frac{1}{x}\right) = \boxed{}.$$

By substituting in, the derivative at $x = \frac{6}{\pi}$ is $\boxed{}$, while the derivative at $x = -2$ is $\boxed{}$.

2. Differentiate the function

$$\frac{(2x+1)^3}{(3x-2)^5}$$

with respect to x, simplifying your answer as far as possible. Hence find the derivative of this function at $x = 0$. (Once again it should be possible to give an exact answer without using a calculator.)

Solution We begin by using the $\boxed{}$ Rule. We write the function as $\dfrac{f(x)}{g(x)}$, where $f(x) = \boxed{}$ and $g(x) = \boxed{}$. By the $\boxed{}$ Rule,

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \boxed{}.$$

To find $f'(x)$ and $g'(x)$ we need the $\boxed{}$ Rule. We have that $f(x) = \big(u(x)\big)^3$, where $u(x) = \boxed{}$, and $g(x) = \big(v(x)\big)^5$, where $v(x) = \boxed{}$. So

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\big(u(x)\big)^3\right) \\ &= \frac{d}{du}(u^3)\frac{du}{dx} \\ &= 3u^2 \times \square \\ &= \square(2x+1)^{\square} \end{aligned} \qquad \text{and} \qquad \begin{aligned} g'(x) &= \frac{d}{dx}\left(\big(v(x)\big)^5\right) \\ &= \frac{d}{dv}(v^5)\frac{dv}{dx} \\ &= \square \times 3 \\ &= \square(3x+2)^{\square}. \end{aligned}$$

Thus

$$\begin{aligned} \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \boxed{} \\ &= \frac{3(2x+1)^{\square}(3x-2)^{\square}\boxed{}}{(3x-2)^{\square}} \\ &= \square\frac{3(2x+1)^{\square}\boxed{}}{(3x-2)^{\square}}. \end{aligned}$$

Hence the derivative at $x = 0$ is $\boxed{}$.