

# Stable Homotopy Theory: Structured Ring Spectra and Their Invariants

3rd–5th September 2014

School of Mathematics, University of Manchester

Organisers: Andrew Baker, Nigel Ray & Birgit Richter

Schedule [05/10/2014]			
	Wednesday	Thursday	Friday
9.45	<b>Welcome</b>		
10.00	Dundas	Schwede	Roitzheim
11.00	<b>Coffee/Tea</b>	<b>Coffee/Tea</b>	<b>Coffee/Tea</b>
11.30	Barnes	Mazel-Gee	Horel
12.00	Sagave	Stojanoska	Greenlees
13.00	<b>Lunch</b>	<b>Lunch</b>	<b>Lunch</b>
14.00	Whitehouse	<b>Adams Afternoon</b>	Ching
14.30	Whitehouse	Tillmann	Lindenstrauss
15.00	Harper	Tillmann	Lindenstrauss
15.30	<b>Coffee/Tea</b>	<b>Coffee/Tea</b>	<b>Coffee/Tea</b>
16.00	Beardsley	Heller	<b>Discussion &amp; Problem Session</b>
16.30	<b>Poster Session</b>	Heller	
18.45		<b>Conference Dinner</b>	

**Note:** Unfortunately, Prof. Gunnar Carlsson is unable to attend the conference.

**David Barnes:** *Rational Orthogonal Calculus.*

Orthogonal calculus is a calculus of functors, inspired by Goodwillie calculus. It takes as input a functor from finite dimensional inner product spaces to topological spaces and as output gives a tower of approximations by well-behaved functors. The output captures a lot of important homotopical information and is an important tool for calculations. This talk will describe current work on setting up model structures for a rational version of Weiss orthogonal calculus. Slides: <http://www.maths.gla.ac.uk/~ajb/Conference2014/Barnes.pdf>

**Jon Beardsley:** *Homotopical Descent Theory and Twisted Forms.*

Given a descent morphism of rings  $R \rightarrow S$ , and an  $R$ -module  $M$ , one can ask what other  $R$ -modules are equivalent to  $M$  after tensoring with  $S$ . Such modules are called twisted forms of  $M$ . If the above morphism is Galois or Hopf-Galois, such forms can be computed using variants of Galois cohomology. We describe the analogous machinery in ring spectra in terms of infinity categories, as well as draw the relevant connections between the existing work of Kathryn Hess and Jacob Lurie. If there is time, we will also describe several interesting examples from chromatic homotopy theory, and ongoing work on constructing a Galois correspondence for the Hopf-Galois extension of the sphere spectrum by  $MU$ .

Slides: <http://www.maths.gla.ac.uk/~ajb/Conference2014/Beardsley.pdf>

**Michael Ching:** *Calculus for the algebraic K-theory of  $E_n$ -ring spectra.*

Functors between categories of algebras over operads (of spectra) can be classified by certain structures on their Goodwillie derivatives. In this talk, I will describe this classification and outline joint work with Andrew Blumberg to calculate the relevant description for algebraic  $K$ -theory as a functor from  $E_n$ -ring spectra to spectra. From this calculation we should be able to recover the fact that the  $K$ -theory of an  $E_n$ -ring spectrum is an  $E_{n-1}$ -ring spectrum.

**Bjørn Dundas:** *Continuity of topological Hochschild homology.*

(This is joint work with Matthew Morrow.)

Let  $A$  be a ring which is complete with respect to an ideal  $I$ . Roughly, a functor  $F$  is *continuous* at  $A$  if  $F(A)$  is equivalent to the homotopy limit of the  $F(A/I^s)$ 's. We give criteria on  $A$  for topological Hochschild and cyclic homology to be continuous. For instance, completing at a prime  $p$ , these functors are continuous for Noetherian commutative  $\mathbb{Z}_{(p)}$ -algebras  $A$  with the property that  $A/pA$  is finitely generated over its subring of  $p$ -th powers. On our way we have to resolve finiteness questions that are interesting in their own right.

**John Greenlees:** *Gorenstein duality for topological Hochschild homology.*

Calculations of Bökstedt, Ausoni, Rognes and many others have shown that THH has striking duality property. The talk will show how this duality can be established without making complete calculations. Indeed, it will be shown that the appropriate ring spectrum has Gorenstein duality in the sense of Dwyer-Greenlees-Iyengar, and this applies in many cases where complete calculations are not accessible. The key new ingredients are a Gorenstein ascent theorem and a very useful lemma of Dundas.

**John Harper:** *On a structured ring spectra analog of Quillen-Sullivan theory.*

An important theme in current work in homotopy theory is the investigation and exploitation of enriched algebraic structures on spectra that naturally arise, for instance, in algebraic topology, algebraic  $K$ -theory, and derived algebraic geometry. Such structured ring spectra or 'geometric rings' are most simply viewed as algebraic-topological generalizations of the notion of ring from algebra and algebraic geometry. This talk will describe recent progress, in joint work with Michael Ching, on an analog of Quillen-Sullivan theory for structured ring spectra. Along the way, I will explain how Kathryn Hess' homotopic descent ideas have played a critical guiding role in establishing these new results.

Slides: <http://www.maths.gla.ac.uk/~ajb/Conference2014/Harper.pdf>

**Jeremiah Heller: *Endomorphisms of the equivariant motivic sphere.***

(This is joint work in progress with David Gepner.)

I will explain how to compute the zeroth homotopy group of the equivariant motivic sphere spectrum, for a finite group. The result is a combination of the zeroth homotopy group of the equivariant topological sphere spectrum (which equals the Burnside ring by a result of Segal) and that of the motivic sphere spectrum (which equals the Grothendieck-Witt ring of quadratic forms by a result of Morel). Our computation is a corollary to a tom Dieck style splitting result for certain equivariant motivic homotopy groups.

**Geoffroy Horel: *Cobordisms and operations on higher Hochschild cohomology.***

Using the methods of factorization homology, I will construct operations on higher Hochschild cohomology or ring spectra based on cobordisms. These operations imply in particular the higher dimensional version of Deligne’s conjecture but they are actually a lot richer than that. For example they also allow to construct an action of the Grothendieck Teichmüller group on Hochschild cohomology of an algebra over the little disk operad.

**Ayelet Lindenstrauss: *Calculations of Higher Topological Hochschild Homology.***

(Joint with I. Bobkova, K. Poirier, B. Richter, I. Zakharevich, M. Basterra, M. Mandell, and B. Dundas, in different combinations.)

T. Pirashvili defined the higher Hochschild homology groups of a commutative ring  $R$  with coefficients in an  $R$ -bimodule  $M$  as the Loday construction  $\mathcal{L}(R; M)$  evaluated on  $S^n$ . I would like to discuss calculations of higher Hochschild homology of some basic rings, leading to calculations of higher topological Hochschild homology of some commutative ring spectra, namely:  $\mathrm{THH}^{[n]}(H\mathbb{F}_p)$  for all  $n$  (extending work of M. Bökstedt, J. Rognes, and T. Veen), and  $\mathrm{THH}^{[n]}(HA; H(A/p))$  for some number rings  $A$ , including the integers and those number rings whose completions at  $p$  are either wildly ramified or unramified over  $\widehat{\mathbb{Z}}_p$  at all maximal ideals which contain  $(p)$  (extending work of M. Bökstedt, I. Madsen, and myself).

**Aaron Mazel-Gee: *Goerss-Hopkins obstruction theory for  $\infty$ -categories.***

I’ll begin with a brief overview of Goerss-Hopkins obstruction theory, a tool for obtaining moduli spaces of structured ring spectra from purely algebraic invariants. Then, I’ll give a black-box description of a version that works in any presentable  $\infty$ -category. If time remains, I’ll also say a few words about its construction, which involves ‘model  $\infty$ -categories’ – that is, of model structures on  $\infty$ -categories.

Slides: <http://math.berkeley.edu/~aaron/writing/ytm-cghost-beamer.pdf>

**Constanze Roitzheim: *Homological Localisation and Postnikov Towers.***

Bousfield localisation with respect to generalised homology theories of either spaces or spectra has proved to be a powerful tool in topology. We present a feasible version of homological localisation for general model categories and show some examples and applications including Postnikov towers.

**Steffen Sagave: *Logarithmic THH of topological K-theory spectra.***

(This is joint work in progress with John Rognes and Christian Schlichtkrull.)

In this talk I will review the definition of logarithmic ring spectra and their logarithmic topological Hochschild homology. Based on these notions, I will explain how the inclusion of the Adams summand can be viewed as a formally log étale map. Together with a localization sequence for logarithmic  $\mathrm{THH}$ , this leads to a computation of the  $V(1)$ -homotopy of the logarithmic topological Hochschild homology of the  $p$ -local complex connective topological  $K$ -theory spectrum  $ku$  and its Adams summand. As an application of these results about logarithmic  $\mathrm{THH}$ , I will outline how one can recover Ausoni’s computation of the  $V(1)$ -homotopy of the ordinary  $\mathrm{THH}$  of  $ku$ .

**Stefan Schwede:** *Global deloopings of ultra-commutative monoids.*

An ultra-commutative monoid can be thought of as a compatible collection of genuine equivariant  $E_\infty$  spaces, one for each compact Lie group. A point-set level model for these highly structured objects is given by orthogonal spaces under global equivalences, endowed with a strictly commutative multiplication with respect to the convolution product (aka ‘box product’). In this talk I will describe the basic homotopy theory of ultra-commutative monoids, give examples and explain how an ultra-commutative monoid can be ‘globally delooped’ to an orthogonal spectrum consisting of genuine equivariant infinite loop spaces.

**Vesna Stojanowska:** *Picard groups of topological modular forms.*

(This is joint work with Akhil Mathew.)

I will present computations of the Picard groups of several spectra of topological modular forms, with and without level structures, periodic and non-periodic. The toolbox for these computations will consist of descent theory and a technical lemma allowing us to compare stable and unstable information in cosimplicial infinite loop spaces.

**Ulrike Tillmann:** *Stabilising diffeomorphism groups.*

The topology of the diffeomorphism group of a manifold is in general very difficult to understand. Historically one has stabilised the manifolds by thickening them and then study the limit via  $K$ -theory. More recently it has proved fruitful to stabilise by taking repeated connected sums with a fixed manifold of the same dimension. Homology stability is at the heart of this second approach. We will discuss this in particular with reference to configuration spaces.

**Sarah Whitehouse:** *Derived  $A$ -infinity algebras from the point of view of operads.*

The notion of a derived  $A$ -infinity algebra arose in the work of Sagave as a natural generalisation of the classical  $A$ -infinity algebra, providing a framework for a minimal model theorem for differential graded algebras over a general commutative ground ring. I will discuss joint work with Livernet and Roitzheim giving an operadic interpretation of these structures, as well as further developments of this work with Aponte Román, Livernet, Robertson and Ziegenhagen, including the study of representations of derived  $A$ -infinity algebras.