Dr A. J. Baker: Level 5 projects for 2009-10

As well as the following I am willing to discuss possible projects on **Algebra** or **Number Theory**.

Title: Homological algebra.
Length: ⩽50 pages
Subject area: Algebra/Geometry/Topology/
Supervisor: Dr A. J. Baker

Description: Homological algebra plays a major rôle in subjects such as differential/algebraic geometry, algebraic topology, ring theory similar to that played by calculus in analysis and applied mathematics.

A basic strategy in homological algebra involves replacing modules or objects in some abelian category $\mathcal{A}$ by 'cofibrant' or 'fibrant' objects in some larger category $\mathrm{Ch}\mathcal{A}$ then defining functors on the latter and forming their 'derived functors', which specialise to (co)homology functors on the original category viewed as functors on the 'derived category' of $\mathrm{Ch}\mathcal{A}$.

Examples of this include (co)homology of spaces, de Rham cohomology of manifolds, (co)homology of groups, Lie algebras, commutative rings and many others.

This project would involve understanding the basic classical theory at least for modules over a ring, then perhaps studying more specialised examples in geometry, algebraic topology or algebra. Some of the most interesting examples begin with a non-abelian category such as the category of commutative rings and give rise to André-Quillen (co)homology theories.

Prerequisites: Strong background in abstract algebra, an interest in category theory, differential/algebraic geometry or algebraic topology would also be useful.

Title: Lie theory and its applications
Length: ⩽50 pages
Subject area: Algebra/Geometry/Topology/Analysis
Supervisor: Dr A. J. Baker

Description: Lie groups are groups on which notions of analysis make sense. They appear throughout Mathematics and its applications, especially wherever smoothly varying symmetries occur. Lie groups are amongst the key ingredients in modern descriptions of fundamental particles and quantum field theory.

In this project aspects of Lie theory could be explored in depth (e.g., the classification of simply connected simple Lie groups) or specific contexts in which Lie groups play a part could be studied (e.g., geometry and symmetry, theoretical physics especially quantum mechanics).

Prerequisites: Strong background in abstract algebra and differential geometry, algebraic topology would also be useful.