HECKE ALGEBRAS ACTING ON ELLIPTIC COHOMOLOGY

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Introduction.

In our earlier papers [2,3,4,5,6], we investigated stable operations and cooperations in elliptic cohomology and its variants, relating these to known operations on rings of modular forms. The purpose of this article is to give an introduction to these stable operation algebras, in particular explaining the connections with Hecke algebras and Morava stabilizer algebras; further details will appear in [7]. In [9] we make use of some of the Hecke operations described here to calculate parts of the Adams E_2-term based on elliptic homology. In work currently in progress we describe the operation algebra for the supersingular elliptic cohomology of §4 in terms of isogenies of supersingular elliptic curves over finite fields and use this to study the v_2-periodic part of elliptic cohomology Adams E_2-term which was determined in [8].

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§1 Recollections on elliptic cohomology.

We refer to [17,18,19] for detailed expositions of the basic notions of elliptic cohomology; for discussions of reductions modulo prime ideals see [3,4]. For actions of Hecke algebras see [2,3].

In the present exposition we will concentrate on elliptic cohomology of level 1, having coefficient ring $E\ell\ell_* = \mathbb{Z}[1/6][E_4, E_6, \Delta^{-1}]$, the ring of modular forms for the full modular group $\mathrm{SL}_2(\mathbb{Z})$ which are meromorphic at infinity and have q-expansion coefficients in the ring $\mathbb{Z}[1/6]$. Here, as usual, the topological grading is twice the weight. By definition,

$$E\ell\ell^*(X) = E\ell\ell_* \underset{MU_*}{\otimes} MU^*(X)$$

for any finite CW-complex X. The functor $E\ell\ell^*(\ )$ is a multiplicative complex oriented cohomology theory and for our purposes is the 'universal example' of an elliptic cohomology theory (at least for those which are localised away from the primes 2 and 3).

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In [2], we extended the natural action of the Hecke operator T_N on the coefficient ring $E\ell\ell_*$ to a stable operation

$$\mathrm{T}_N \colon E\ell\ell^*(\ ) \longrightarrow E\ell\ell[1/N]^*(\ ),$$

where the target is elliptic cohomology with N inverted. These operations satisfy analogues of the usual relations amongst the Hecke operators and this characterises our extensions. These operations are $\mathbb{Z}[1/6N]$-linear but *not* multiplicative, although by construction they are symmetrisations of multiplicative stable operations into higher level versions of elliptic cohomology.

§2 Recollections on Hecke algebras and a twisted analogue.

For basic definitions of *Hecke algebras* and their structure we refer the reader to [16]. However, we will need a twisted version which so far as we can determine does not explicitly appear in the literature.

Let $\Bbbk$ be a commutative unital ring, L be a commutative $\Bbbk$-algebra, and G be a subgroup of the $\Bbbk$-algebra automorphism group of L. Let $H \leqslant G$ be a subgroup satisfying the *Hecke condition*, i.e., for each $g \in G$, the indices

$$[H, gHg^{-1} \cap H] \quad \text{and} \quad [gHg^{-1}, gHg^{-1} \cap H]$$

are finite. Then we can define the Hecke algebra $\Bbbk[H\backslash G/H]$ as in [16].

Now let L^H denote the subalgebra of L fixed by H. If we interpret G as acting trivially on L^H, we can also form the Hecke algebra $L^H[H\backslash G/H]$. However, we wish to form a *twisted Hecke algebra* $L^H\{H\backslash G/H\}$ which in the case where $H = N$ is normal in G, agrees with the usual *twisted group ring* $L^N\{G/N\}$, well-known to group theorists.

We define the underlying $\Bbbk$-module of $L^H\{H\backslash G/H\}$ to be

$$\left\{ \sum_{g\colon G/H} \ell_g gH : \forall h \in H,\ \forall g,\ {}^h\ell_g = \ell_{hg} \right\},$$

where ℓ_g depends only on the coset gH and the sum has finitely nonvanishing terms. We also use the notation of [16] in which $g\colon G/H$ means that g ranges over a complete set of representatives of cosets of H in G; similarly $g\colon H\backslash G/H$ will mean that gH ranges over a complete set of (left) representatives of the double coset HgH, etc. $L^H\{H\backslash G/H\}$ can also be identified with the set of left invariant elements in the free left L-module $L[G/H]$ under the left $\Bbbk$-linear action of H given by

$$h \cdot \sum_{g\colon G/H} \ell_g gH = \sum_{g\colon G/H} {}^h\ell_g hgH.$$

Clearly $L^H\{H\backslash G/H\}$ is a left L^H-module. In order to define a product on $L^H\{H\backslash G/H\}$, it suffices to define the product of generators of the form

$$\lambda_g = \sum_{k\colon H/gHg^{-1}\cap H} \ell_{kg} kgH$$

and

$$\lambda_{g'} = \sum_{k' :\ H/g'Hg'^{-1}\cap H} \ell_{k'g'} k'g'H.$$

Therefore we set

$$\lambda_g \cdot \lambda_{g'} = \sum_{\substack{k:\ H/gHg^{-1}\cap H \\ k':\ H/g'Hg'^{-1}\cap H}}\sum \ell_{kg}{}^{kg}\ell_{k'g'} kgk'g'H.$$

It can be verified that this product does indeed make $L^H\{H\backslash G/H\}$ into a $\mathbb{k}$-algebra, but *not* always an L^H-algebra. In fact, there is far more structure in this object. The most transparent way to understand this is to pass to a dual object,

$$\mathrm{Map}^{\mathrm{f}}_H(G/H, L) = \left\{f\colon G/H \longrightarrow L : \forall g\colon G/H, \forall h \in H,\ f(hgH) = {}^h f(gH)\right\},$$

where $\mathrm{Map}^{\mathrm{f}}$ denotes the set of finitely supported maps $G/H \longrightarrow L$.

It is then possible to define a Hopf algebroid structure upon $\mathrm{Map}^{\mathrm{f}}_H(G/H, L)$, as explained in [7]. In examples of interest to topologists, this gives twisted Hecke algebras closely related to cooperation algebras in (co)homology theories. For example, in the case of elliptic cohomology we have the twisted Hecke algebra

$$E\ell\ell_* \otimes \mathbb{Q}\{\mathrm{SL}_2(\mathbb{Z})\backslash \mathrm{GL}_2(\mathbb{Q})/\mathrm{SL}_2(\mathbb{Z})\} = \\ (\widetilde{E\ell\ell_*} \otimes \mathbb{Q})^{\mathrm{SL}_2(\mathbb{Z})}\{\mathrm{SL}_2(\mathbb{Z})\backslash \mathrm{GL}_2(\mathbb{Q})/\mathrm{SL}_2(\mathbb{Z})\},$$

where $\widetilde{E\ell\ell_*}$ is the union of a tower of finite Galois extensions of $E\ell\ell_*$. This is closely related to the structure of the cooperation Hopf algebroid $E\ell\ell_*E\ell\ell$, the details appearing in [6,7].

§3 Ordinary reduction.

For any prime p greater than 3, there is a reduction of elliptic cohomology modulo p, whose coeeficient ring is $E\ell\ell_*/(p)$. This coefficient ring is a graded PID, since the ring $E\ell\ell_*$ has dimension 2. We can map this ring into the (ungraded) ring $\mathbb{F}_p((q))$ by sending the residue class $F \pmod p \in E\ell\ell_{2n}/(p)$ to its reduced q-expansion,

$$\tilde{F}(q) \pmod p.$$

However, this is not a monomorphism, since for example we have

$$\tilde{E}_{p-1}(q) \equiv 1 \pmod p. \tag{3.1}$$

Indeed, for any pair of modular forms

$$F \in E\ell\ell_{2n}, \quad G \in E\ell\ell_{2n+p-1}$$

satisfying

$$\tilde{F}(q) \equiv \tilde{G}(q) \pmod p,$$

the element $FE_{p-1} - G$ is in the kernel of this q-expansion modulo p map. We use this to define the *ordinary reduction at* p of elliptic cohomology. Note that the element E_{p-1} will be invertible as a consequence of Equation (3.1). At the other extreme, in §4 we will consider the *supersingular reduction at* p, in which E_{p-1} will be contained in a prime ideal.

We follow closely the ideas described by Serre in [25,24]. However, it should be noted that we are working with the ring of modular forms which are meromorphic at infinity, whereas he works in the ring of holomorphic modular forms. This difference is insignificant because of the following easily verified fact.

Lemma 3.1. *Let p be a prime greater than 3. Then in the ring of holomorphic modular forms with q-expansion coefficients in $\mathbb{Z}_{(p)}$, the element E_{p-1} is not divisible by Δ; this continues to hold after reduction modulo p.*

We can view the image of $E\ell\ell_*/(p)$ in $\mathbb{F}_p((q))$ as a graded field, with grading group some quotient of $\mathbb{Z}/(p-1)$, since E_{p-1} is of weight $p-1$. The following result is discussed in [25] and was proved originally by Swinnerton-Dyer and Serre.

Theorem 3.2. *For each prime $p > 3$, the image of $E\ell\ell_*/(p)$ in $\mathbb{F}_p((q))$ is the $\mathbb{Z}/2(p-1)$-graded ring*

$$E\ell\ell_*/(p, E_{p-1} - 1).$$

Moreover, this is a graded integral domain; equivalently, $E_{p-1} - 1$ is a prime element in $E\ell\ell_*/(p)$ viewed as an ungraded ring.

As a consequence, we can form the ordinary reduction of elliptic cohomology at p as the functor

$$E\ell\ell[1]^*(\ ) = E\ell\ell[1]_* \underset{E\ell\ell_*/(p)}{\otimes} (E\ell\ell/(p))^*(\ ),$$

where we set

$$E\ell\ell[1]_* = E\ell\ell/J_1,$$

with the ideal J_1 being generated by p together with all the homogeneous elements in the kernel of the map into $\mathbb{F}_p((q))$.

Theorem 3.3. *The functor $E\ell\ell[1]^*(\ )$ is a multiplicative complex oriented cohomology theory on finite CW-complexes.*

The proof of this uses the Landweber Exact Functor Theorem in its modulo p version, together with the fact that working modulo p we can identify E_{p-1} with the Hazewinkel generator v_1 (i.e., the $[p]$-series of the canonical formal group law over $E\ell\ell_*/(p)$ begins with the term $E_{p-1}X^p$ up to a unit in $\mathbb{F}_p^\times$). It seems more natural to replace this $\mathbb{Z}$-graded theory with the $\mathbb{Z}/2(p-1)$-graded theory

$$E\ell\ell[1]^\bullet(\ ) = E\ell\ell[1]_\bullet \underset{E\ell\ell_*/(p)}{\otimes} (E\ell\ell/(p))^*(\ )$$

where

$$E\ell\ell[1]_\bullet = E\ell\ell_*/(p, E_{p-1} - 1).$$

The even part of this grading is best viewed as coming from the subgroup of $(p-1)$st roots of unity in $\mathbb{Z}_p^\times$, the units of p-adic integers $\mathbb{Z}_p$. Indeed there is an integral version of this cohomology theory whose coefficient ring can be identified with the ring of *meromorphic p-adic modular forms* obtained from the holomorphic version of Serre [24] (with integral q-expansion coefficients); this is naturally graded on $\mathbb{Z}_p^\times$.

The Hecke algebra

$$H_{(p)} = \bigotimes_{\substack{\ell \text{ prime} \\ \ell \neq p}} \mathbb{Z}_{(p)}[\mathrm{T}_\ell, \mathrm{T}_{\ell,\ell}, \mathrm{T}_{\ell,\ell}^{-1}]$$

acts upon the cohomology theory $E\ell\ell[1]^*(\ )$. However, there are also two other operations $\mathrm{U}_p, \mathrm{V}_p$ acting, as described in [3]. Moreover, these do not commute, so there is a non-commutative stable operation algebra acting. To understand this, we need to recall a result of Serre [24].

Theorem 3.4. *Let F be a modular form for the congruence subgroup $\Gamma_0(p)$, whose q-expansion at each cusp has coefficients in $\mathbb{Z}_{(p)}$. Then F is a p-adic modular form for $\mathrm{SL}_2(\mathbb{Z})$. In particular, for each natural number $k > 0$, there exists a modular form G for $\mathrm{SL}_2(\mathbb{Z})$ with q-expansion coefficients in $\mathbb{Z}_{(p)}$ and an integer $t \geqslant 0$ such that*

$$\tilde{F}(q) \equiv \tilde{G}(q) E_{p-1}^{tp^{k-1}} \pmod{p^k}.$$

Thus when working modulo p or p-adically, we are in effect working with modular forms for $\Gamma_0(p)$ in a suitable sense, since any modular form for $\mathrm{SL}_2(\mathbb{Z})$ is automatically one for $\Gamma_0(p)$.

Now modular forms for $\Gamma_0(p)$ can be viewed as functions on the space of pairs (L, P), where L is a lattice and $P \leqslant \mathbb{C}/L$ is a subgroup of order p. Such a function has two q-expansions, corresponding to the two cusps $i\infty$ and 0, and these are interchanged by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Following the account in [16], for each $N \geqslant 1$, we introduce the set of matrices

$$M_2^0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}) : c \equiv 0 \pmod{p},\ p \nmid a,\ \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = N \right\}$$

for $0 < N \in \mathbb{Z}$. Then the Hecke algebra $\mathbb{Z}[\Gamma_0(p) \backslash \bigcup_{p \nmid N} M_2^0(N)/\Gamma_0(p)]$ acts upon $E\ell\ell[1]_*$ and the cohomology theory $E\ell\ell[1]^*(\ )$.

§4 Supersingular reduction.

Let $p > 3$ be a prime. We will consider *supersingular elliptic cohomology at p*. This is described in detail in [4] and here we confine ourselves to a brief account.

In general, the graded ideal $J_2 = (p, E_{p-1}) \triangleleft E\ell\ell_*$ is not prime. However, an argument due to Serre and related to the proof of Theorem 3.2 gives the following.

Proposition 4.1. *In $E\ell\ell_*/(p)$, the prime factors $\pi_1, \pi_2, \ldots, \pi_d$ of E_{p-1} all have multiplicity 1. Hence, the quotient ring is a product of graded fields*

$$E\ell\ell_*/(p, E_{p-1}) \cong \prod_{1 \leqslant k \leqslant d} E\ell\ell_*/(p, \pi_k).$$

In [4] we showed there is a ring homomorphism $h_2 \colon K(2)_* \longrightarrow E\ell\ell_*/(p, E_{p-1})$ classifying the natural formal group law over the target (or rather its canonical p-typification), and so we can form the functor

$$(E\ell\ell/(p, E_{p-1}))^*(\ ) = E\ell\ell_*/(p, E_{p-1}) \underset{K(2)_*}{\otimes} K(2)^*(\ ),$$

on finite CW-complexes. Similarly, we can form theories

$$(E\ell\ell/(p, \pi_k))^*(\ ) = E\ell\ell_*/(p, \pi_k) \underset{K(2)_*}{\otimes} K(2)^*(\ ) \quad (1 \leqslant k \leqslant d).$$

Theorem 4.2. *The functor* $(E\ell\ell/(p, E_{p-1}))^*(\ )$ *is a multiplicative complex oriented cohomology theory on finite CW-complexes. Moreover, there is a natural splitting of algebra theories over* $K(2)^*(\ )$*,*

$$(E\ell\ell/(p, E_{p-1}))^*(\ ) \cong \prod_{1 \leqslant k \leqslant d} (E\ell\ell/(p, \pi_k))^*(\ ).$$

We refer to all of these theories *supersingular reductions of elliptic cohomology*.

The p-local Hecke algebra $H_{(p)}$ acts on the theory $(E\ell\ell/(p, E_{p-1}))^*(\ )$ since the ideal (p, E_{p-1}) is invariant under the actions of all the Hecke operators. This can be seen using the fact that the q-expansion of E_{p-1} is congruent to 1 (mod p); indeed the supersingular quotient ring is investigated in [22]. It also follows from our construction of the Hecke operators since in $E\ell\ell_*/(p)$, E_{p-1} is the image of v_1 under the map $MU_* \longrightarrow E\ell\ell_*$ classifying the canonical formal group law. However, in general the prime ideals (p, π_k) are not preserved under the Hecke operators, as was shown explicitly in the MSc thesis of R. Field [11]. Thus we cannot expect that the Hecke algebra will preserve the decomposition of Theorem 4.2.

Let us analyse in detail one of the quotient graded fields of Theorem 4.1. We set $\pi = \pi_k$, for $1 \leqslant k \leqslant d$. Using ideas of [4], we define the element $W = W_\pi \in E\ell\ell_{2w}/(p, \pi)$ by

$$W = \begin{cases} E_6 & \text{if } \pi = E_4, \\ E_4 & \text{if } \pi = E_6, \\ E_6 E_4^{-1} & \text{otherwise.} \end{cases}$$

Here we also set

$$w = \begin{cases} 6 & \text{if } \pi = E_4, \\ 4 & \text{if } \pi = E_6, \\ 2 & \text{otherwise.} \end{cases}$$

Using results from [4] we obtain the following.

Theorem 4.3. *There is an isomorphism of graded fields*

$$E\ell\ell_*/(p, \pi) \cong \mathbb{F}_{p^e}[W, W^{-1}],$$

where $e = 1, 2$ *depending upon* p *and* π*. Moreover, if* $e = 2$*, there are exactly two such isomorphisms.*

Proof. Only the last sentence needs comment. Let $\mathbb{F}$ be the degree 0 part of $E\ell\ell_*/(p, \pi)$. Then there is an $\alpha \in \mathbb{F}$ such that

$$E_4 = \alpha W^2, \quad \text{and} \quad E_6 = \alpha W^3.$$

Now we can identify $\mathbb{F}$ with $\mathbb{F}_{p^2}$ by identifying α with one of the roots of the minimal polynomial of α in $\mathbb{F}_{p^2}$, but of course there are exactly two ways to do this.

The details of this proof suggest that we consider the *Frobenius operation*

$$\mathrm{Fr}\colon E\ell\ell_*/(p, \pi) \longrightarrow E\ell\ell_*/(p, \pi)$$

defined by

$$\mathrm{Fr}(F) = F^{(p)} = F^p W^{(p-1)\,\mathrm{wt}\,F/2},$$

which agrees with the usual Frobenius operation Fr on $\mathbb{F}$.

We also recall a formula that holds in connection with the homomorphism

$$h_2 \colon K(2)_* = \mathbb{F}_p[v_2, v_2^{-1}] \longrightarrow E\ell\ell_*/(p, E_{p-1}),$$

and in which $\left(\dfrac{-1}{p}\right)$ is a Legendre symbol.

Theorem 4.4. *In $E\ell\ell_*/(p, E_{p-1})$ we have the relation*

$$h_2(v_2) = \left(\frac{-1}{p}\right) \Delta^{(p^2-1)/12}.$$

Now we can tensor the field $E\ell\ell_*/(p,\pi)$ with the Galois field $\mathbb{F}_{p^2}$, and obtain a homomorphism

$$\mathbb{F}_{p^2} \otimes K(2)_* \xrightarrow{1\otimes h_2} \mathbb{F}_{p^2} \otimes E\ell\ell_*/(p,\pi).$$

Notice that the target is either

$$\mathbb{F}_{p^2}[W, W^{-1}] \quad \text{if } e = 1,$$

or

$$\mathbb{F}_{p^2}[W, W^{-1}] \times \mathbb{F}_{p^2}[W, W^{-1}] \quad \text{if } e = 2,$$

and the latter has an action of the Galois group

$$G_2 = \mathrm{Gal}(\mathbb{F}_{p^2}/\mathbb{F}_p) \cong \mathbb{Z}/2,$$

generated by the Frobenius map Fr which interchanges the factors. But by Theorem 4.4, the composite

$$h_2' \colon K(2)_* \xrightarrow{\mathrm{incl}} \mathbb{F}_{p^2} \otimes K(2)_* \xrightarrow{1\otimes h_2} \mathbb{F}_{p^2} \otimes E\ell\ell_*/(p,\pi)$$

is invariant under this action of G_2.

We can consider the natural transformation of cohomology theories

$$(E\ell\ell/(p,\pi))^*(\ ) \longrightarrow \mathbb{F}_{p^2} \otimes (E\ell\ell/(p,\pi))^*(\ )$$

and observe that the natural orientation (coming form $K(2)^*(\ )$) is invariant under the action of the group G_2 acting via the coefficients. The (strict) automorphism group of the corresponding formal group law over the ring $(E\ell\ell/(p,\pi))_*$ now acts as a group of multiplicative stable operations which are Galois invariant in the sense that

$$\mathrm{Fr}(\overline{\theta}(t)) = (\overline{{}^{\mathrm{Fr}}\theta})(\mathrm{Fr}(t)).$$

Here we recall that the group of strict automorphisms of $F^{K(2)}$ is the group of units $\theta \in \mathbb{D}_2$ (a certain central division algebra over $\mathbb{Q}_p$ of dimension 4) satisfying

$$|\theta|_p \leqslant 1$$

and

$$|1-\theta|_p < 1,$$

with ${}^{\mathrm{Fr}}\theta = S\theta S^{-1}$ where $S \in \mathbb{D}_2$ satisfies $S^2 = p$. This is all explained in Ravenel's book [21]. We denote this group of strict automorphisms by $\mathbb{S}_2$. Using the subgroup $\widetilde{\mathbb{S}}_2 \leqslant \mathbb{D}_2^\times$ generated by $\mathbb{S}_2$ together with the central element $S^2 = p$, we can form the double coset space $\langle p\rangle \backslash\widetilde{\mathbb{S}}_2/\langle p\rangle$. Then the associated Hecke algebra $\mathbb{Z}_p[\langle p\rangle \backslash\widetilde{\mathbb{S}}_2/\langle p\rangle]$ acts continuously upon

$${}^{\mathrm{Fr}}(\mathbb{F}_{p^2} \otimes E\ell\ell/(p,\pi))^*(\ ) \cong (E\ell\ell/(p,\pi))^*(\ ).$$

Hence we can form the twisted Hecke algebra

$$(\mathbb{F}_{p^2} \otimes E\ell\ell/(p,\pi)_*)^{\langle p\rangle}\left\{\langle p\rangle\backslash\widetilde{\mathbb{S}}_2/\langle p\rangle\right\}$$

and this acts continuously upon $(E\ell\ell/(p,\pi))^*(\ )$.

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