Summary of lecture 5 - Critical points & classification

- \((a, b)\) is a critical point (stationary point) of \(f\) provided \(f_x(a, b) = 0\) and \(f_y(a, b) = 0\).

- The sign of
  \[
  \Delta(h, k) = f(a + h, b + k) - f(a, b)
  \]
  reveals whether \((a, b)\) is a local maximum or minimum.

- If \(\Delta(h, k) > 0\) for all \((h, k) \neq (0, 0)\) sufficiently close to \((0, 0)\), \((a, b)\) is a local minimum. If \(\Delta(h, k) > 0\) then \((a, b)\) is a local maximum,

- otherwise, \((a, b)\) is a saddle point.

- To classify a critical point we first use the second derivative test and if \(D = 0\) then we use first principals and look at \(\Delta(h, k)\).

2nd derivative test

Let \((a, b)\) be a critical point of \(f\) and let \(A = f_{xx}\) and \(D = f_{xx}f_{yy} - f_{xy}^2\), where all derivatives are evaluated at \((a, b)\). Then

1. If \(A > 0\) and \(D > 0\) then \((a, b)\) is a minimum point,
2. If \(A < 0\) and \(D > 0\) then \((a, b)\) is a maximum point,
3. If \(D < 0\) then \((a, b)\) is a saddle point,
4. If \(D = 0\) then no conclusion about the nature of \((a, b)\) is made.