A NEW DESCRIPTION OF EQUIVARIANT COHOMOLOGY FOR TOTALLY DISCONNECTED GROUPS

CHRISTIAN VOIGT

Abstract. We consider smooth actions of totally disconnected groups on simplicial complexes and compare different equivariant cohomology groups associated to such actions. Our main result is that the bivariant equivariant cohomology theory introduced by Baum and Schneider can be described using equivariant periodic cyclic homology. This provides a new approach to the construction of Baum and Schneider as well as a computation of equivariant periodic cyclic homology for a natural class of examples. In addition we discuss the relation between cosheaf homology and equivariant Bredon homology. Since the theory of Baum and Schneider generalizes cosheaf homology we finally see that all these approaches to equivariant cohomology for totally disconnected groups are closely related.

1. Introduction

In this paper we compare different cohomology theories defined for smooth proper actions of totally disconnected groups on appropriate topological spaces. More precisely, we consider bivariant equivariant cohomology in the sense of Baum and Schneider [4], equivariant periodic cyclic homology [19], cosheaf homology [3] and equivariant Bredon homology [7], [13]. Our main result explains the relation between the first two theories. Moreover, by the work of Baum and Schneider, cosheaf homology can be viewed as a special case of bivariant equivariant cohomology. We complete the picture by showing that equivariant Bredon homology is naturally isomorphic to cosheaf homology for proper actions on simplicial complexes.

The approach of Baum and Schneider is based on sheaf theory and unifies several constructions which appeared previously in the literature. As already mentioned, it contains as a special case the cosheaf homology groups considered by Baum, Connes and Higson in connection with the Baum-Connes conjecture [3]. Moreover it covers a construction of Baum and Connes for discrete groups [2]. Equivariant Bredon homology is an equivariant generalization of cellular homology. It has been used by Lück to describe the rationalized left hand side of the assembly maps in K- and L-theory for discrete groups [13]. Finally, equivariant cyclic homology can be viewed as a noncommutative generalization of the equivariant de Rham cohomology of manifolds.

The latter theory is different in nature to the previous ones since it is defined not only for spaces but also for possibly noncommutative algebras equipped with a group action. Its construction is based on an extension of the Cuntz-Quillen approach to cyclic homology. However, computations tend to be more difficult than in ordinary cyclic homology due to the fact that the basic ingredient in the theory is not a complex in the usual sense of homological algebra. In fact, our original motivation was to calculate equivariant periodic cyclic homology for some basic examples.

2000 *Mathematics Subject Classification.* 19D55, 55N91.

This research was supported by the EU-Network *Quantum Spaces and Noncommutative Geometry* (Contract HPRN-CT-2002-00280) and the *Deutsche Forschungsgemeinschaft* (SFB 478).

Let us now describe in more detail the contents of this paper. In section 2 we review some facts about totally disconnected groups and smooth representations. We recall the concept of a covariant module which plays a central role in equivariant cyclic homology. In particular we discuss the decomposition of a covariant module into an elliptic part and a hyperbolic part in the case of totally disconnected groups. In section 3 we review the general construction of equivariant cyclic homology. Bivariant equivariant periodic cyclic homology $HP_*^G(A,B)$ is defined for pairs of G-algebras A and B. We are interested in the case of certain algebras of smooth functions on simplicial complexes acted upon by a totally disconnected group G. The appropriate notion of a smooth function on a simplicial complex X is introduced in section 4 where we also study some properties of the resulting algebra $C_c^\infty(X)$ of smooth functions with compact support. Let us point out that simplicial complexes are a convenient choice of spaces for two reasons. On the one hand they are special enough to have a nice de Rham-theoretic description of their cohomology. On the other hand they are general enough to cover natural examples, most notably in connection with the Baum-Connes conjecture. In section 5 we review the definition of bivariant equivariant cohomology in the sense of Baum and Schneider. We study in particular the case of group actions on simplicial complexes and introduce the notion of a G-simplicial complex. Section 6 contains an equivariant version of the Hochschild-Kostant-Rosenberg theorem. This theorem is an important ingredient in our main result which is formulated and proved in section 7. We show that there exists a natural isomorphism

$$HP_*^G(C_c^\infty(X), C_c^\infty(Y)) \cong \bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y)$$

if X and Y are finite dimensional locally finite G-simplicial complexes and X is proper. Here H_G^* denotes the theory of Baum and Schneider. A small variant of this result in the case of discrete groups acting on manifolds yields a description of the theory of Baum and Connes [2] in terms of equivariant cyclic homology. Finally, in section 8 we review the definitions of equivariant Bredon homology and cosheaf homology and show that these theories are naturally isomorphic on the category of G-simplicial complexes. This fact is certainly well-known, but we could not find a reference.

In a subsequent paper the results obtained here will be used to construct an equivariant Chern character for totally disconnected groups [20].

This paper is based on the last chapter of my thesis [18]. I would like to thank J. Cuntz for his constant support and P. Schneider and H. Glöckner for some helpful comments.

2. Totally disconnected groups

Let G be locally compact and totally disconnected group. We call an element $t \in G$ elliptic if it is contained in a compact subgroup. The set of all elliptic elements of G is denoted by G_{ell}. It is known that G_{ell} is a closed subset of G [22]. In contrast we shall say that an element $t \in G$ is hyperbolic if it is not elliptic. Let G_{hyp} be the set of all hyperbolic elements of G. Hence, according to these definitions, we obtain a disjoint union decomposition

$$G = G_{\mathrm{ell}} \cup G_{\mathrm{hyp}}$$

of the space G. Moreover it follows from the structure theory developped in [22] that G_{ell} is the union of all compact open subgroups of G. Consequently the set G_{hyp} is again open and closed.

In connection with equivariant cyclic homology we work with smooth representations of totally disconnected groups on bornological vector spaces. Roughly speaking, a bornological vector space is a vector space V together with a collection of subsets of V satisfying some axioms modelled on the properties of bounded subsets in a locally convex vector space. In particular, every locally convex vector space can be viewed as a bornological vector space with the bounded bornology consisting of its bounded subsets. In fact, even in the context of locally convex vector spaces, the bornological approach has several advantages which make it very convenient to work with. For more information we refer to [14], [15], [19].

Let us recall some basic facts concerning smooth representations and fix our notation. All bornological vector spaces in this paper are assumed to be convex and complete. A representation of a totally disconnected group G on a bornological vector space V is called smooth if the the stabilizers of small subsets in V are open subgroups of G. To avoid confusion we point out that $t \in G$ is in the stabilizer of a subset $S \subset V$ iff $t \cdot v = v$ for all $v \in S$. For instance, the trivial representation of G on any bornological vector space is a smooth representation. We will frequently also speak of G-modules instead of smooth representations. A bounded linear map $f : M \to N$ between G-modules is called equivariant if it commutes with the action of G.

If G is a totally disconnected group we denote by $\mathcal{D}(G)$ the space of locally constant functions on G with compact support. This space is equipped with the fine bornology, that is, the smallest possible bornology. Elements of $\mathcal{D}(G)$ are also referred to as smooth functions on G with compact support. The left regular representation of G on $\mathcal{D}(G)$ given by

$$(s \cdot f)(t) = f(s^{-1}t)$$

is a basic example of a smooth representation.

Lemma 2.1. *Let G be a totally disconnected group and let $t \in G$ be hyperbolic. Then there are no fixed points in the left regular representation $\mathcal{D}(G)$ for the action of t.*

Proof. Assume $f \in \mathcal{D}(G)$ is a nonzero element satisfying $t \cdot f = f$. It suffices to consider the case that f is the characteristic function of some nonempty compact open subset K of G. In this case $t \cdot f = f$ just means $t \cdot K = K$. After possibly translating K from the right we may assume that the identity element e is contained in K. It follows that the closed subgroup generated by t is contained in K and hence compact. This contradicts the assumption that t is hyperbolic. □

A bornological algebra is a bornological vector space A with an associative multiplication given as a bounded linear map $A\hat{\otimes}A \to A$ where $\hat{\otimes}$ denotes the completed bornological tensor product. Remark that we do not require the existence of a unit in a bornological algebra. A basic example of a bornological algebra is the Hecke algebra of a totally disconnected group G. It is obtained by equipping the space $\mathcal{D}(G)$ with the convolution product

$$(f * g)(t) = \int f(s)g(s^{-1}t)ds$$

where ds is a fixed left Haar measure on G. This algebra is unital iff the group G is discrete. We will denote the Hecke algebra of G again by $\mathcal{D}(G)$.

A module M over a bornological algebra A is called nondegenerate if the module action $A\hat{\otimes}M \to M$ is a bornological quotient map. We remark that the category of smooth representations of G is isomorphic to the category of nondegenerate $\mathcal{D}(G)$-modules.

A G-algebra is a bornological algebra A which is at the same time a G-module such that the multiplication $A\hat{\otimes}A \to A$ is equivariant. Here the tensor product $A\hat{\otimes}A$ is

equipped with the diagonal action as usual. A particular example of a G-algebra is the algebra $\mathcal{K}_G$ which is defined as follows. As a bornological vector space we have $\mathcal{K}_G = \mathcal{D}(G)\hat{\otimes}\mathcal{D}(G) = \mathcal{D}(G \times G)$. The multiplication in $\mathcal{K}_G$ is given by

$$(k \cdot l)(s,t) = \int_G k(s,r)l(r,t)dr$$

and the G-action is defined by

$$(r \cdot k)(s,t) = k(r^{-1}s, r^{-1}t).$$

This algebra can be viewed as a dense subalgebra of the algebra of compact operators $\mathbb{K}(L^2(G))$ on the Hilbert space $L^2(G)$.
Next we recall the definition of a covariant module. For more details we refer to [19]. Let $\mathcal{O}_G$ be the space $\mathcal{D}(G)$ equipped with pointwise multiplication and the action of G by conjugation. A covariant module M is a smooth representation of G which is at the same time a nondegenerate $\mathcal{O}_G$-module. The G-module structure and the $\mathcal{O}_G$-module structure are required to be compatible in the sense that

$$s \cdot (f \cdot m) = (s \cdot f) \cdot (f \cdot m)$$

for all $s \in G, f \in \mathcal{O}_G$ and $m \in M$. A bounded linear map $f : M \to N$ between covariant modules is called covariant if it is $\mathcal{O}_G$-linear and equivariant. The category of covariant modules is denoted by G-$\mathfrak{Mod}$.
Covariant modules are closely related to equivariant sheaves over the space G-space G with the adjoint action.

Proposition 2.2. *Let G be a totally disconnected group. The functor of taking global sections with compact support induces an equivalence between the category $\mathrm{Sh}_G(G)$ of equivariant sheaves over the G-space G with the adjoint action and the category of G-covariant modules whose underlying bornology is fine.*

In other words, covariant modules can be viewed as equivariant sheaves except that we include a bornology as extra information. The proof of of proposition 2.2 is sketched in [4].
The category of covariant modules is isomorphic to the category of nondegenerate $\mathfrak{Cov}(G)$-modules where $\mathfrak{Cov}(G) = \mathcal{O}_G \rtimes G$ is the smooth crossed product of $\mathcal{O}_G$ with respect to the adjoint action. In particular $\mathfrak{Cov}(G)$ itself is a covariant module in a natural way.
Every covariant module M is equipped with a natural automorphism $T : M \to M$. For $M = \mathfrak{Cov}(G)$ this automorphism is defined by

$$T(f)(s,t) = f(s,st)$$

where we view an element of $\mathfrak{Cov}(G)$ as a smooth function with compact support on $G \times G$, the first variable corresponding to $\mathcal{O}_G$. To define the operator T for an arbitrary covariant module M one uses the canonical isomorphism $\mathfrak{Cov}(G)\hat{\otimes}_{\mathfrak{Cov}(G)}M \cong M$ and applies the map $T : \mathfrak{Cov}(G) \to \mathfrak{Cov}(G)$ from above to the first tensor factor in $\mathfrak{Cov}(G)\hat{\otimes}_{\mathfrak{Cov}(G)}M$.
In some situations we will have to look at the stalks of the sheaf underlying a covariant module. Let $t \in G$ be a point and consider the ideal I_t in $\mathcal{O}_G$ of functions vanishing at t. Clearly I_t is a prime ideal and hence we may consider the localisation M_t of a covariant module M at I_t. Since G is totally disconnected the space M_t can be identified with M/I_tM in a natural way. The localisation M_t is no longer a covariant module in general, in addition to the natural $\mathcal{O}_G$-module structure we only have an action of the centralizer G_t of the element t on M_t. A basic observation is that a sequence $0 \to K \to E \to Q \to 0$ of (fine) covariant modules is exact iff the localized sequences $0 \to K_t \to E_t \to Q_t \to 0$ are exact for all $t \in G$.

Recall from [15] that the multiplier algebra $M(A)$ of a bornological algebra A consists of all two-sided multipliers of A. That is, an element (L, R) of $M(A)$ consists of a bounded right A-module map $L : A \to A$ and a bounded left A-module map $R : A \to A$ such that $a(Lb) = (aR)b$ for all $a, b \in A$.
Since the group G is a disjoint union of the open sets of elliptic and hyperbolic elements we can define two elements P_{ell} and P_{hyp} in the multiplier algebra $M(\mathcal{O}_G)$ of $\mathcal{O}_G$ as follows. The multiplier P_{ell} is the characteristic function of the set G_{ell} whereas $P_{\text{hyp}} = 1 - P_{\text{ell}}$ is the characteristic function of G_{hyp}. For every covariant module M we obtain a natural direct sum decomposition of covariant modules $M = M_{\text{ell}} \oplus M_{\text{hyp}}$ where $M_{\text{ell}} = P_{\text{ell}} \cdot M$ and $M_{\text{hyp}} = P_{\text{hyp}} \cdot M$.
We conclude this section with the construction of a canonical projection on the elliptic part M_{ell} of an arbitrary covariant module M. Consider the operator $T : M_{\text{ell}} \to M_{\text{ell}}$ and let $m \in M_{\text{ell}}$. By the definition of G_{ell} and since M_{ell} is a smooth representation there exists a natural number n such that $T^n(m) = m$. We can thus write M_{ell} as direct limit of the spaces $M(n)$ consisting of all elements m such that $T^n(m) = m$. Here the direct limit is taken over the natural numbers where by definition $k \leq l$ iff k divides l. We define a covariant map $E : M(n) \to M$ by

$$E(m) = \frac{1}{n} \sum_{j=0}^{n-1} T^j(m).$$

This definition is compatible with the structure maps in the inductive limit and yields a covariant map $E : M_{\text{ell}} \to M_{\text{ell}}$. Moreover the relation $E^2 = E$ holds by construction. The map E is the natural projection onto the T-invariant elements in M_{ell}.

3. Equivariant periodic cyclic homology

In this section we recall the definition of bivariant equivariant periodic cyclic homology $HP^G_*(A, B)$ given in [19]. An important property of the construction is that the fundamental object in the theory, the equivariant Hodge tower $\theta\Omega_G(A)$ of a G-algebra A is not a complex in the usual sense. In fact, the differential ∂ in $\theta\Omega_G(A)$ fails to satisfy $\partial^2 = 0$.
First we have to discuss noncommutative equivariant differential forms. Let A be a G-algebra. The equivariant n-forms of A are defined by $\Omega^n_G(A) = \mathcal{O}_G\hat{\otimes}\Omega^n(A)$ where $\Omega^n(A) = A^+\hat{\otimes}A^{\hat{\otimes}n}$ and A^+ denotes the unitarization of A. The group G acts diagonally on $\Omega^n_G(A)$ and we have an obvious $\mathcal{O}_G$-module structure. In this way $\Omega^n_G(A)$ becomes a covariant module.
The equivariant Hochschild boundary $b : \Omega^n_G(A) \to \Omega^{n-1}_G(A)$ is defined by

$$\begin{aligned} b(f(t)\otimes x_0 dx_1 \cdots dx_n) &= f(t) \otimes x_0 x_1 dx_2 \cdots dx_n \\ &+ \sum_{j=1}^{n-1} (-1)^j f(t) \otimes x_0 dx_1 \cdots d(x_j x_{j+1}) \cdots dx_n \\ &+ (-1)^n f(t) \otimes (t^{-1} \cdot x_n) x_0 dx_1 \cdots dx_{n-1}. \end{aligned}$$

Moreover we have the equivariant Connes operator $B : \Omega^n_G(A) \to \Omega^{n+1}_G(A)$ which is given by

$$B(f(t) \otimes x_0 dx_1 \cdots dx_n) = \sum_{i=0}^{n} (-1)^{ni} f(t) \otimes t^{-1} \cdot (dx_{n+1-i} \cdots dx_n) dx_0 \cdots dx_{n-i}.$$

It is straightforward to check that b and B are covariant maps. The natural symmetry operator T for covariant modules is of the form

$$T(f(t) \otimes \omega) = f(t) \otimes t^{-1} \cdot \omega$$

on $\Omega^n_G(A)$. One easily obtains the relations $b^2 = 0, B^2 = 0$ and $Bb+bB = \mathrm{id} - T$ for these operators. This shows that $\Omega_G(A)$ is a paramixed complex in the following sense.

Definition 3.1. *A paramixed complex M of covariant modules is a sequence of covariant modules M_n together with differentials b of degree -1 and B of degree $+1$ satisfying $b^2 = 0$, $B^2 = 0$ and*

$$[b, B] = bB + Bb = \mathrm{id} - T.$$

The most important examples of paramixed complexes are bounded below in the sense that $M_n = 0$ if $n < N$ for some fixed $N \in \mathbb{Z}$. In particular, the equivariant differential forms $\Omega_G(A)$ of a G-algebra A satisfy this condition for $N = 0$.
The Hodge filtration of a paramixed complex M of covariant modules is defined by

$$F^n M = bM_{n+1} \oplus \bigoplus_{j>n} M_j.$$

Clearly $F^n M$ is closed under the operators b and B. We write

$$L^n M = F^{n-1} M / F^n M$$

for the nth layer of the Hodge filtration. If M is bounded below such that $M_n = 0$ for $n < 0$ we define the nth level $\theta^n M$ of the Hodge tower of M by

$$\theta^n M = \bigoplus_{j=0}^{n-1} M_j \oplus M_n / bM_{n+1}.$$

By definition, the Hodge tower of M is the projective system $\theta M = (\theta^n M)_{n\in\mathbb{N}}$.
We remark that the image of the Hochschild operator b is not closed in general. In this case the spaces $F^n M$ are possibly incomplete and $L^n M$ and $\theta^n M$ may fail to be separated. However, in the examples that we will study the image of b is always closed and hence these problems do not show up.
The spaces $\theta^n M$ are equipped with the grading into even and odd forms and the differential $\partial = B + b$. In this way the Hodge tower becomes a projective system of paracomplexes in the following sense.

Definition 3.2. *A paracomplex of covariant modules is a $\mathbb{Z}_2$-graded covariant module C with a boundary operator $\partial : C \to C$ of degree one such that $\partial^2 = \mathrm{id} - T$.*

Chain maps of paracomplexes and homotopy equivalences are defined by the usual formulas.
We will work with the following definition of equivariant periodic cyclic homology which is equivalent to the one given in [19] using X-complexes.

Definition 3.3. *Let G be a totally disconnected group and let A and B be G-algebras. The bivariant equivariant periodic cyclic homology of A and B is*

$$HP^G_*(A, B) = H_*(\mathfrak{Hom}_G(\theta\Omega_G(A\hat{\otimes}\mathcal{K}_G), \theta\Omega_G(B\hat{\otimes}\mathcal{K}_G))).$$

To explain this definition we first remark that the G-algebra $\mathcal{K}_G$ was defined in section 2. Secondly, the definition involves covariant maps between projective systems of covariant modules. Maps between projective systems are always understood in the sense

$$\mathrm{Hom}((M_i)_{i\in I}, (N_j)_{j\in J}) = \varprojlim_{j\in J} \varinjlim_{i\in I} \mathrm{Hom}(M_i, N_j)$$

of pro-categories. Finally, we consider the usual differential for a Hom-complex given by

$$\partial(\phi) = \phi\partial_A - (-1)^{|\phi|}\partial_B\phi$$

for a homogeneous element ϕ in order to define homology. This makes sense since the failure of the individual differentials in $\theta\Omega_G(A\hat{\otimes}\mathcal{K}_G)$ and $\theta\Omega_G(B\hat{\otimes}\mathcal{K}_G)$ to satisfy $\partial^2 = 0$ is cancelled out by naturality of the operator T.
According to the decomposition of a covariant module into an elliptic and a hyperbolic part discussed in section 2 we obtain a decomposition

$$HP_*^G(A,B) = HP_*^G(A,B)_{\mathrm{ell}} \oplus HP_*^G(A,B)_{\mathrm{hyp}}$$

of equivariant periodic cyclic homology where by definition

$$HP_*^G(A,B)_{\mathrm{ell}} = H_*(\mathfrak{Hom}_G(\theta\Omega_G(A\hat{\otimes}\mathcal{K}_G)_{\mathrm{ell}}, \theta\Omega_G(B\hat{\otimes}\mathcal{K}_G)_{\mathrm{ell}}))$$

and accordingly for $HP_*^G(A,B)_{\mathrm{hyp}}$. As we shall see, and as is familiar from the computation of the cyclic homology of group algebras, the elliptic and hyperbolic parts of equivariant cyclic homology behave differently.
Let us have a closer look at the elliptic part and recall the definition of the projection E from section 2. Since the map $E : \theta\Omega_G(A)_{\mathrm{ell}} \to \theta\Omega_G(A)_{\mathrm{ell}}$ commutes with the boundary operators b and B we get a direct sum decomposition

$$\theta\Omega_G(A)_{\mathrm{ell}} = E\theta\Omega_G(A)_{\mathrm{ell}} \oplus (1-E)\theta\Omega_G(A)_{\mathrm{ell}}$$

of paracomplexes. Actually, $E\theta\Omega_G(A)_{\mathrm{ell}}$ is an ordinary complex since $T = \mathrm{id}$ on this space.

Proposition 3.4. *Let A be a G-algebra. The paracomplex $(1-E)\theta\Omega_G(A)_{\mathrm{ell}}$ is covariantly contractible. Hence the canonical projection $\theta\Omega_G(A)_{\mathrm{ell}} \to E\theta\Omega_G(A)_{\mathrm{ell}}$ is a covariant homotopy equivalence.*

Proof. The covariant map $\mathrm{id} - T : \theta\Omega_G(A)_{\mathrm{ell}} \to \theta\Omega_G(A)_{\mathrm{ell}}$ is diagonal with respect to the direct sum decomposition $E\theta\Omega_G(A)_{\mathrm{ell}} \oplus (1-E)\theta\Omega_G(A)_{\mathrm{ell}}$. It follows from the definitions that $\mathrm{id} - T$ is injective on $(1-E)\theta\Omega_G(A)_{\mathrm{ell}}$ and $\mathrm{id} - T = 0$ on $E\theta\Omega_G(A)_{\mathrm{ell}}$. Now consider an element $(1-E)m \in (1-E)\theta^n\Omega_G(A)_{\mathrm{ell}}$ with $m \in \theta^n\Omega_G(A)_{\mathrm{ell}}$. We find $k \in \mathbb{N}$ such that $T^k(m) = \mathrm{id}$ and hence

$$(1-E)(m) = (\mathrm{id} - T)\sum_{j=0}^{k-1} \frac{k-j}{k}\, T^j(m)$$

is in the image of $\mathrm{id} - T$. It follows that the map $\mathrm{id} - T : (1-E)\theta\Omega_G(A)_{\mathrm{ell}} \to (1-E)\theta\Omega_G(A)_{\mathrm{ell}}$ is an isomorphism. Since $\mathrm{id} - T$ is homotopic to zero we conclude that $(1-E)\theta\Omega_G(A)_{\mathrm{ell}}$ is covariantly contractible. □

4. Smooth functions on simplicial complexes

In this section we study smooth functions and smooth differential forms on simplicial complexes.
First we have to fix some notation. We denote by Δ^k the k-dimensional standard simplex

$$\Delta^k = \{(x_0, \ldots, x_k) \in \mathbb{R}^{k+1} \mid 0 \le x_j \le 1, \sum_{j=0}^{k} x_j = 1\}$$

in $\mathbb{R}^{k+1}$. By construction Δ^k is contained in a unique k-dimensional affine subspace of $\mathbb{R}^{k+1}$ which will be denoted by A^k. A function $f : \Delta^k \to \mathbb{C}$ is called smooth if it is the restriction of a smooth function on the affine space A^k.
To obtain an appropriate class of functions for our purposes we have to require conditions on the behaviour of such smooth functions near the boundary $\partial\Delta^k$ of the simplex Δ^k. Roughly speaking, we shall consider only those functions which are constant in the direction orthogonal to the boundary in a neighborhood of $\partial\Delta^k$.
Let us explain this precisely. We denote by $\partial^i\Delta^k$ the i-th face of the standard simplex consisting of all points $(x_0, \cdots, x_k) \in \Delta^k$ satisfying $x_i = 0$. Then $\partial^i\Delta^k$

defines a hyperplane $A_i^k \subset A^k$ in a natural way. To this hyperplane we associate the vector space V_i which contains all vectors in $\mathbb{R}^k$ that are orthogonal to A_i^k. For $v \in \mathbb{R}^k$ denote by $\partial_v(f)$ the partial derivative of a smooth function f on A^k in direction v. We say that a smooth function $f : \Delta^k \to \mathbb{C}$ is i-regular if there exists a neighborhood U_i of $\partial^i\Delta^k$ such that $\partial_v(f)(x) = 0$ for all $x \in U_i$ and all $v \in V_i$. If we want to emphasize the particular neighborhood U_i we also say that f is i-regular on U_i. The function f is called regular if it is i-regular for all $i = 0, \ldots, k$. We denote by $C^\infty(\Delta^k)$ the algebra of regular smooth functions on Δ^k.

The idea behind these definitions is as follows. Let us denote by $C^\infty(\Delta^k, \partial\Delta^k) \subset C^\infty(\Delta^k)$ the subalgebra consisting of those functions that vanish on the boundary $\partial\Delta^k$ of Δ^k. It is not hard to check that $C^\infty(\Delta^k, \partial\Delta^k)$ can be identified with the algebra $C_c^\infty(\Delta^k \setminus \partial\Delta^k)$ of smooth functions with compact support on the open set $\Delta^k \setminus \partial\Delta^k$. Moreover the inclusion $\partial^i : \partial^i\Delta^k \to \Delta^k$ of a face induces a homomorphism $C^\infty(\Delta^k) \to C^\infty(\partial^i\Delta^k)$. Our definition yields a natural class of smooth functions satisfying these properties.

If we identify Δ^1 with the unit interval $[0, 1]$ the algebra $C^\infty(\Delta^1)$ corresponds to the algebra of smooth functions on $[0, 1]$ which are constant around the endpoints. Moreover $C^\infty(\Delta^1, \partial\Delta^1)$ can be identified with the algebra $C_c^\infty(0, 1)$ of smooth functions with compact support on the open interval $(0, 1)$.

We want to extend the definition of regular smooth functions to arbitrary simplicial complexes. A regular smooth function on a simplicial complex X is given by a family $(f_\sigma)_{\sigma\subset X}$ of regular smooth functions on the simplices of X which is compatible with restriction to faces in the obvious way. The function f is said to have compact support if only finitely many f_σ in the corresponding family are different from zero. We denote by $C_c^\infty(X)$ the algebra of regular smooth functions with compact support on X. If the simplicial complex X is finite we simply write $C^\infty(X)$ instead of $C_c^\infty(X)$.

Let us now describe the natural locally convex topology on the algebra $C_c^\infty(X)$ of regular smooth functions on the simplicial complex X. First we consider again the case $X = \Delta^k$. Let $\mathcal{U} = (U_0, \cdots, U_k)$ be a family of open subsets of Δ^k where each U_i is a neighborhood of $\partial^i\Delta^k$. The collection of all such families is partially ordered where $\mathcal{U} \prec \mathcal{V}$ iff $V_j \subset U_j$ for all j in the corresponding families. For a family $\mathcal{U} = (U_0, \cdots, U_k)$ we let $C^\infty(\Delta^k, \mathcal{U}) \subset C^\infty(\Delta^k)$ be the subalgebra of smooth functions which are i-regular on U_i for all i. We equip $C^\infty(\Delta^k, \mathcal{U})$ with the natural Fréchet topology of uniform convergence of all derivatives on Δ^k. For $\mathcal{U} \prec \mathcal{V}$ we have an obvious inclusion $C^\infty(\Delta^k, \mathcal{U}) \subset C^\infty(\Delta^k, \mathcal{V})$ which is compatible with the topologies. Moreover $C^\infty(\Delta^k)$ is obtained as the union of the algebras $C^\infty(\Delta^k, \mathcal{U})$. We equip $C^\infty(\Delta^k)$ with the resulting inductive limit topology. In this way the algebra $C^\infty(\Delta^k)$ becomes a nuclear LF-algebra and the natural restriction homomorphism $C^\infty(\Delta^k) \to C^\infty(\partial^i\Delta^k)$ associated to the inclusion of a face is continuous.

In order to introduce a topology on $C_c^\infty(X)$ for arbitrary X let $K \subset X$ be a finite subcomplex. A function $f = (f_\sigma)_{\sigma\subset X} \in C_c^\infty(X)$ is said to have support in K if $f_\sigma = 0$ for all simplices $\sigma \subset X$ not contained in K. We let $C_K^\infty(X) \subset C_c^\infty(X)$ be the algebra of regular smooth functions with support in K. The algebra $C_K^\infty(X)$ is equipped with the subspace topology from the finite direct sum of algebras $C^\infty(\sigma)$ for $\sigma \subset K$. If $K \subset L$ are finite subcomplexes the obvious inclusion $C_K^\infty(X) \to C_L^\infty(X)$ is compatible with the topologies. Moreover $C_c^\infty(X)$ is the union over all finite subcomplexes K of the algebras $C_K^\infty(X)$. Hence we obtain a natural inductive limit topology on $C_c^\infty(X)$. We equip the algebra $C_c^\infty(X)$ with the associated precompact bornology (which is equal to the bounded bornology). In this way $C_c^\infty(X)$ becomes a complete bornological algebra.

Let us have a closer look at the natural restriction homomorphism $C^\infty(\Delta^k) \to$

$C^\infty(\partial\Delta^k)$. As above we denote by $C^\infty(\Delta^k, \partial\Delta^k) \subset C^\infty(\Delta^k)$ the kernel of this homomorphism which consists of all regular smooth functions on Δ^k that have compact support in the interior $\Delta^k \setminus \partial\Delta^k$.

Proposition 4.1. *For all k the restriction homomorphism $C^\infty(\Delta^k) \to C^\infty(\partial\Delta^k)$ has a continuous linear splitting. Hence we obtain a linearly split extension*

$$C^\infty(\Delta^k, \partial\Delta^k) \rightarrowtail C^\infty(\Delta^k) \twoheadrightarrow C^\infty(\partial\Delta^k)$$

of complete bornological algebras.

Proof. By definition we have $C^\infty(\partial\Delta^0) = 0$ and hence the case $k = 0$ is trivial.
For $k = 1$ we identify Δ^1 with the unit interval $[0, 1]$. Choose a smooth function $h : [0, 1] \to [0, 1]$ such that $h = 1$ on $[0, 1/3]$ and $h = 0$ on $[2/3, 1]$. We set $e_0 = h, e_1 = 1 - h$ and define $\sigma_1 : C^\infty(\partial\Delta^1) = \mathbb{C} \oplus \mathbb{C} \to C^\infty(\Delta^1)$ by $\sigma_1(f_0, f_1) = f_0e_0 + f_1e_1$. It is clear that σ_1 is a continuous linear splitting for the restriction map.
In order to treat the case $k = 2$ we first consider the corresponding lifting problem for a corner of Δ^2 which can be formulated as follows. Let us write $\mathbb{R}^+$ for the set of nonnegative real numbers. A corner of Δ^2 can be viewed as a neighborhood of the point $(0, 0)$ in $\mathbb{R}^+ \times \mathbb{R}^+$. Given smooth functions $f_1, f_2 : \mathbb{R}^+ \to \mathbb{C}$ that are both constant in a neighborhood of 0 and satisfy $f_1(0) = f_2(0)$ we want to construct a smooth function $f : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{C}$ such that

a) $f(x_1, 0) = f_1(x_1)$ and $f(0, x_2) = f_2(x_2)$ for all $x_1, x_2 \in \mathbb{R}^+$,
b) the function f is constant in the transversal direction in a neighborhood of the boundary $(\mathbb{R}^+ \times \{0\}) \cup (\{0\} \times \mathbb{R}^+)$,
c) f depends linearly and continuously on f_1 and f_2.

In order to construct such a function we first extend f_1 and f_2 to smooth functions F_1 and F_2 on $\mathbb{R}^+ \times \mathbb{R}^+$ by setting

$$F_1(x_1, x_2) = f_1(x_1), \qquad F_2(x_1, x_2) = f_2(x_2).$$

Then we use polar coordinates (r, θ) in $(\mathbb{R}^+ \times \mathbb{R}^+) \setminus \{(0, 0)\}$ to define a smooth function g_1 by

$$g_1(r, \theta) = h\left(\frac{2\theta}{\pi}\right)$$

where h is the function from above. We extend g_1 to $\mathbb{R}^+ \times \mathbb{R}^+$ by setting $g_1(0, 0) = 0$. Moreover we define g_2 by $g_2 = 1 - g_1$ on $(\mathbb{R}^+ \times \mathbb{R}^+) \setminus \{(0, 0)\}$ and $g_2(0, 0) = 0$. Remark that g_1 and g_2 are not continuous in $(0, 0)$. We can now define the desired function $f : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{C}$ by

$$f = f_1(0)\delta + F_1g_1 + F_2g_2 = f_2(0)\delta + F_1g_1 + F_2g_2$$

where δ is the characteristic function of the point $(0, 0)$. The function f is smooth in $(0, 0)$ since the assumptions on f_1 and f_2 imply that f is constant in a neighborhood of this point. Moreover it is easy to verify that f satisfies the conditions a), b) and c) above. Hence this construction solves the lifting problem for a corner of Δ^2.
Now we want to show that the restriction map $C^\infty(\Delta^2) \to C^\infty(\partial\Delta^2)$ has a continuous linear splitting. One can combine the functions g_0 and g_1 constructed above for the corners of Δ^2 to obtain functions e_j on Δ^2 for $j = 0, 1, 2$ such that

a) each e_j is a regular smooth function on Δ^2 except in the vertices where e_j is zero,
b) $e_j = 1$ in the interior of $\partial^j\Delta^2$ and $e_j = 0$ in the interior of $\partial^i\Delta^2$ for $i \neq j$,

c) for each j there exists a neighborhood U_j of the j-th vertex v_j of Δ^2 such that

$$\sum_{i \neq j} e_i = 1$$

holds on U_j except in v_j.

Now assume that a regular smooth function $f = (f_0, f_1, f_2)$ on $\partial\Delta^2$ is given where f_i is defined on the face $\partial^i\Delta^2$. The functions f_i can be extended to i-regular smooth functions F_i on Δ^2 by

$$F_0(x_0, x_1, x_2) = f_0\bigg(x_1 + \frac{x_0}{2}, x_2 + \frac{x_0}{2}\bigg)$$
$$F_1(x_0, x_1, x_2) = f_1\bigg(x_0 + \frac{x_1}{2}, x_2 + \frac{x_1}{2}\bigg)$$
$$F_2(x_0, x_1, x_2) = f_2\bigg(x_0 + \frac{x_2}{2}, x_1 + \frac{x_2}{2}\bigg).$$

Moreover let $\chi : \Delta^2 \to \mathbb{C}$ be the characteristic function of the set $\{v_0, v_1, v_2\}$ consisting of the three vertices of Δ^2. Using these functions we define $\sigma_2(f) : \Delta^2 \to \mathbb{C}$ by

$$\sigma_2(f) = f\chi + F_0 e_0 + F_1 e_1 + F_2 e_2.$$

To avoid confusion we point out that $f\chi$ is the function which is equal to f in the vertices of Δ^2 and extended by zero to the whole simplex. It is easy to see that the restriction of $\sigma_2(f)$ to the boundary of Δ^2 is equal to f. Using the fact that F_i is i-regular one checks that $\sigma_2(f)$ is a regular smooth function on Δ^2. Our construction yields a continuous linear map $\sigma_2 : C^\infty(\partial\Delta^2) \to C^\infty(\Delta^2)$ which splits the natural restriction homomorphism.

To prove the assertion for $k > 2$ one proceeds in a similar way as in the case $k = 2$. Essentially one has to combine the functions constructed above in an appropriate way. First we consider again the lifting problem for a corner of Δ^k. Such a corner can be viewed as a neighborhood of $(0, \ldots, 0)$ in $(\mathbb{R}^+)^k$. We are given smooth functions $f_1, \ldots, f_k : (\mathbb{R}^+)^{k-1}$ which are transversally constant in a neighborhood of the boundary and satisfy certain compatibility conditions. The function f_j is extended to a smooth function $F_j : (\mathbb{R}^+)^k \to \mathbb{C}$ by setting

$$F_j(x_1, \ldots, x_k) = f_j(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_k).$$

For $1 \leq i < j \leq k$ we define

$$g_{ij}(x_1, \ldots, x_k) = g_1(x_i, x_j), \qquad g_{ji}(x_1, \ldots, x_k) = g_2(x_i, x_j)$$

where g_1 and g_2 are the functions from above. Each function g_{ij} is smooth except in some $k-2$-dimensional subspace inside the boundary and transversally constant in a neighborhood of the boundary. If we expand the product

$$\prod_{0 \leq i < j \leq k} (g_{ij} + g_{ji})$$

we obtain a sum of functions which are smooth except in the boundary and transversally constant in a neighborhood of the boundary. Moreover these functions vanish in the interior of all faces except possibly one. Using the functions F_j constructed before we can proceed as in the case $k = 2$ to solve the lifting problem for the k-dimensional corner. Since this is a straightforward but lengthy verification we omit the details.

To treat the simplex Δ^k we construct functions e_j for $j = 0, \ldots, k$ such that

a) e_j is regular smooth except in the $(k-2)$-skeleton of Δ^k where $e_j = 0$,
b) $e_j = 1$ in the interior of $\partial^j\Delta^k$ and $e_j = 0$ in the interior of $\partial^i\Delta^k$ for $i \neq j$,

c) for each point $x \in \partial\Delta^k$ there exists a neighborhood U_x of x such that

$$\sum_{j \in S(x)} e_j = 1$$

in U_x except the $(k-2)$-skeleton of Δ^k where $S(x)$ is the collection of all i such that $x \in \partial^i \Delta^k$.

Assume that a regular smooth function $f = (f_0, \ldots, f_k)$ on $\partial\Delta^k$ is given where f_i is defined on the face $\partial^i\Delta^k$. The function f_i can be extended to an i-regular smooth function F_i on Δ^k by

$$F_i(x_0, \ldots, x_k) = f_i\bigg(x_0 + \frac{x_i}{k}, \ldots, x_{i-1} + \frac{x_i}{k}, x_{i+1} + \frac{x_i}{k}, \ldots, x_k + \frac{x_i}{k}\bigg).$$

Moreover let $\chi : \Delta^k \to \mathbb{C}$ be the characteristic function of the $k-2$-skeleton of Δ^k. We define $\sigma_k(f) : \Delta^k \to \mathbb{C}$ by

$$\sigma_k(f) = f\chi + \sum_{j=0}^{k} F_j\, e_j.$$

Using the properties of the functions e_j and the fact that F_j is j-regular one checks that $\sigma_k(f)$ is a regular smooth function. The restriction of $\sigma_k(f)$ to $\partial\Delta^k$ is equal to f. In this way we obtain a continuous linear splitting $\sigma_k : C^\infty(\partial\Delta^k) \to C^\infty(\Delta^k)$ for the natural restriction homomorphism. □

For a simplicial complex X let X^k denote its k-skeleton. Consider the natural continuous restriction homomorphism $C_c^\infty(X^k) \to C_c^\infty(X^{k-1})$. This homomorphism is surjective provided X is locally finite and its kernel will be denoted by $C_c^\infty(X^k, X^{k-1})$.

Proposition 4.2. *Let X be a locally finite simplicial complex. For all k the restriction homomorphism $C_c^\infty(X^k) \to C_c^\infty(X^{k-1})$ has a continuous linear splitting. Hence we obtain a linearly split extension*

$$C^\infty(X^k, X^{k-1}) \rightarrowtail C_c^\infty(X^k) \twoheadrightarrow C_c^\infty(X^{k-1})$$

of complete bornological algebras.

Proof. We construct a retraction $\rho : C_c^\infty(X^k) \to C_c^\infty(X^k, X^{k-1})$ for the natural inclusion. The algebra $C_c^\infty(X^k, X^{k-1})$ can be identified with a direct sum $\bigoplus_{i\in I} C^\infty(\Delta^k, \partial\Delta^k)$. Recall that the elements $f \in C_c^\infty(X^k)$ are families $(f_\sigma)_{\sigma \subset X^k}$. For each k-simplex $\eta \in X^k$ we define a map

$$\rho_\eta : C_c^\infty(X^k) \to C^\infty(\Delta^k, \partial\Delta^k), \qquad \rho_\eta((f_\sigma)) = \rho_k(f_\eta)$$

where $\rho_k : C^\infty(\Delta^k) \to C^\infty(\Delta^k, \partial\Delta^k)$ is the retraction obtained in proposition 4.1. It is easy to check that ρ_η is continuous. The maps ρ_σ assemble to yield a map

$$\rho : C_c^\infty(X^k) \to \bigoplus_{i\in I} C^\infty(\Delta^k, \partial\Delta^k) = C^\infty(X^k, X^{k-1})$$

which is again continuous. Moreover by construction ρ is a retraction for the inclusion $C^\infty(X^k, X^{k-1}) \to C^\infty(X^k)$. □

We say that a complete bornological algebra K has local units if for every small subset $S \subset K$ there exists an element $e \in K$ such that $ex = xe = x$ for all $x \in S$. Clearly every unital complete bornological algebra has local units. In the bornological context the existence of local units has similar consequences as H-unitality [23], [24] in the algebraic setting. Clearly a complete bornological algebra which has local units is in particular H-unital in the purely algebraic sense. The proof of

excision in (algebraic) Hochschild homology for H-unital algebras can immediately be adapted to show that every extension

$$K \rightarrowtail E \twoheadrightarrow Q$$

of complete bornological algebras with bounded linear splitting induces a long exact sequence in (bornological) Hochschild homology provided K has local units. A similar assertion holds for the homology with respect to the equivariant Hochschild boundary in the equivariant context.

Proposition 4.3. *Let X be a locally finite simplicial complex. For every finite subcomplex $K \subset X$ there exists a positive function $e \in C_c^\infty(X)$ such that $e = 1$ on K. In particular $C_c^\infty(X)$ has local units.*

Proof. First recall that a simplicial complex X is called locally finite if every vertex of X is contained in only finitely many simplices of X. A simplicial complex is locally finite iff it is a locally compact space in the weak topology.
The desired function e will be constructed inductively. On X^0 we define $e(x) = 1$ if $x \in K^0$ and $e(x) = 0$ otherwise. Assuming that e is constructed on X^{k-1} we essentially have to extend functions which are defined on the boundary of k-dimensional simplices to the whole simplices. If e is constant on the boundary we extend it to the whole simplex as a constant function. In general the extension can be done using the liftings for the restriction map $C^\infty(\Delta^k) \to C^\infty(\partial\Delta^k)$ constructed in proposition 4.1. It is clear that the resulting regular smooth function e is equal to 1 on K. The fact that X is locally finite guarantees that e has compact support. Since every small subset of $C_c^\infty(X)$ is contained in $C_K^\infty(X)$ for some finite subcomplex $K \subset X$ the previous discussion shows that $C_c^\infty(X)$ has local units. □
Apart from smooth functions we also have to consider differential forms on simplicial complexes. A smooth differential form on the standard simplex Δ^k is defined as the restriction of a smooth differential form on the k-dimensional affine space A^k to Δ^k. Again we have to impose some conditions on the behaviour near the boundary. Let us consider forms of a fixed degree p. For $v \in \mathbb{R}^k$ we denote by $\mathcal{L}_v$ the Lie derivative in direction v and by ι_v the interior product with the vector field associated to v. Using the notation established in the beginning of this section we say that a smooth p-form ω on Δ^k is i-regular if there exist a neighborhood U_i of $\partial^i\Delta^k$ such that $\mathcal{L}_v(\omega)(x) = 0$ and $\iota_v(\omega)(x) = 0$ for all $x \in U_i$ and all $v \in V_i$. The form ω is called regular if it is i-regular for all $i = 0, \ldots, k$.
Given a simplicial complex X a regular smooth p-form ω on X is a family $(\omega_\sigma)_{\sigma \subset X}$ of regular smooth p-forms on the simplices of X which is compatible with the natural restriction maps. A form $\omega = (\omega_\sigma)_{\sigma \subset X}$ is said to have compact support if only finitely many ω_σ in the corresponding family are nonzero. We denote by $\mathcal{A}_c^p(X)$ the space of regular smooth p-forms on X with compact support. The exterior differential d can be defined on $\mathcal{A}_c(X)$ in the obvious way and turns it into a complex. Also the exterior product of differential forms extends naturally. Note that $\mathcal{A}_c^0(X)$ can be identified with the algebra $C_c^\infty(X)$ of regular smooth functions defined above.
As in the case of functions there is a natural topology on the space $\mathcal{A}_c^p(X)$ of regular smooth p-forms. Let us start with $X = \Delta^k$ and consider a family $\mathcal{U} = (U_0, \cdots, U_k)$ of open subsets of Δ^k where each U_i is a neighborhood of $\partial^i\Delta^k$. We let $\mathcal{A}^p(\Delta^k, \mathcal{U}) \subset C^\infty(\Delta^k)$ be the space of smooth p-forms which are i-regular on U_i for all i and equip this space with the natural Fréchet topology. We obtain a corresponding inductive limit topology on $\mathcal{A}^p(\Delta^k)$. Since one proceeds for an arbitrary simplicial complex X as in the case of functions we shall not work out the details. Most of the time we will not have to take into account the resulting bornology on $\mathcal{A}_c^p(X)$ in our considerations anyway.
We will have to consider differential forms not only as globally defined objects but

also from the point of view of sheaf theory. The regularity conditions for smooth differential forms on a simplicial complex X obviously make sense also for an open subset U of X. Hence we obtain in a natural way sheaves $\mathcal{A}^p_X$ on X by letting $\Gamma(U, \mathcal{A}^p_X)$ be the space of regular smooth p-forms on the open set $U \subset X$. We also write C^∞_X for the sheaf $\mathcal{A}^0_X$. The sheaf C^∞_X is a sheaf of rings and the sheaves $\mathcal{A}^p_X$ are sheaves of modules for C^∞_X. Clearly the space $\Gamma_c(X, \mathcal{A}^p_X)$ of global sections with compact support of $\mathcal{A}^p_X$ can be identified with $\mathcal{A}^p_c(X)$.

Proposition 4.4. *Let X be a locally finite simplicial complex. The sheaves $\mathcal{A}^p_X$ are c-soft for all p and*

$$\mathbb{C}_X \longrightarrow \mathcal{A}^0_X \xrightarrow{d} \mathcal{A}^1_X \xrightarrow{d} \mathcal{A}^2_X \xrightarrow{d} \cdots$$

is a resolution of the constant sheaf $\mathbb{C}_X$ on X.

Proof. In this proof we will tacitly use some results from sheaf theory which can be found in [8]. Let us first show that the sheaves $\mathcal{A}^p_X$ are c-soft. Since the sheaves $\mathcal{A}^p_X$ are sheaves of modules for the sheaf of rings C^∞_X it suffices to show that C^∞_X is c-soft. Using the fact that a sheaf $\mathcal{F}$ on X is c-soft iff the restrictions $\mathcal{F}_{|K}$ are soft for all compact subsets $K \subset X$ we may assume that X is a finite complex. We have to show that the restriction map $\Gamma(X, C^\infty_X) \to \Gamma(K, C^\infty_X)$ is surjective for all closed subsets $K \subset X$. Given a regular smooth function f on K we shall construct a regular smooth function $F : X \to \mathbb{C}$ which extends f. For $x \in X^0$ we put $F(x) = f(x)$ if $x \in K$ and $F(x) = 0$ otherwise. Now assume that F has been constructed on X^{k-1}. In order to extend F to X^k we can consider each k-simplex of X separately. If σ is a k-simplex then F is already given on $\partial\sigma$ by induction hypothesis and on the closed subset $\sigma \cap K$ by assumption. The resulting function can be extended to a smooth regular function in a small neigborhood U of $\partial\sigma \cup (\sigma \cap K)$. We find a regular smooth function h on σ such that the support of h is contained in U and $h = 1$ on $\partial\sigma \cup (\sigma \cap K)$. Using the function h we can extend F to the whole simplex σ.
To show that the complex of sheaves $\mathcal{A}^\bullet_X$ is a resolution of the constant sheaf on X we have to prove that the stalks $(\mathcal{A}^\bullet_X)_x$ of this complex are resolutions of $\mathbb{C}$ for all $x \in X$. Each point $x \in X$ is contained in $X^k \setminus X^{k-1}$ for some k and we find a k-dimensional simplex σ in X such that x is an element in the interior $\sigma \setminus \partial\sigma$ of σ. >From the definition of regular smooth differential forms we see that the stalks $(\mathcal{A}^\bullet_X)_x$ depend only on the coordinates of σ. Hence we can identify these stalks in a natural way with stalks of the sheaves $\mathcal{A}^\bullet_{\mathbb{R}^k}$ of smooth differential forms on k-dimensional Euclidean space. The Poincaré lemma yields the assertion. □
For technical reasons we need a slightly more general class of spaces in the sequel. Namely, we will consider closed subspaces of spaces of the form $T \times K$ where T is locally compact and totally disconnected and K is a locally finite simplicial complex. By definition, a regular smooth function f on the space $T \times K$ is a function which can be written locally around any point as $f(t, x) = F(x)$ for some regular smooth function F on K. This condition clearly makes sense also for open subsets of $T \times K$. Now let $X \subset T \times K$ be a closed subspace. By definition, a regular smooth function on an open subset $U \subset X$ is a function which can be extended to a regular smooth function in an open neighborhood of U in $T \times K$. This defines a sheaf C^∞_X of regular smooth functions on X in a natural way. Similarly one can consider differential forms and one obtains corresponding sheaves $\mathcal{A}^p_X$.

Proposition 4.5. *Let T be a locally compact totally disconnected space and let K be a locally finite simplicial complex. For any closed subspace $X \subset T \times K$ the*

sheaves $\mathcal{A}_X^p$ are c-soft and

$$\mathbb{C}_X \longrightarrow \mathcal{A}_X^0 \xrightarrow{d} \mathcal{A}_X^1 \xrightarrow{d} \mathcal{A}_X^2 \xrightarrow{d} \cdots$$

is a resolution of the constant sheaf $\mathbb{C}_X$ on X.

Proof. The assertion concerning exactness can be proved as in proposition 4.4. It suffices to show that $C^\infty_{T\times K}$ is c-soft. Let $A \subset T \times K$ be a compact subset and let f be a regular smooth function on A. We have to construct a regular smooth function F on $T \times K$ which extends f. First choose open subsets $U_1, \ldots, U_n$ covering A with $U_i = V_i \times W_i$ where $V_i \subset T$ is compact open and $W_i \subset K$ is open such that f can be extended to a regular smooth function on $U = \bigcup_{i=1}^n U_i$ which depends only on the K-variable on each U_j. In order to extend f to $T \times K$ we proceed as follows. If we set $V = \bigcup_{i=1}^n V_i$ we find compact open subsets $T_i \subset V$ such that the restriction of the extended function f to $T_i \times K \cap U$ does not depend on the T-variable. Since the set $T_j \times K \cap A$ is compact the same is true for its projection $K_j = \pi_K(T_j \times K \cap A)$ to K. By hypothesis $g_j(x) = f(t, x)$ for $(t, x) \in T_j \times K \cap A$ yields a well-defined regular smooth function on K_j. We use proposition 4.4 to extend g_j to a regular smooth function G_j on K. Setting $F(t, x) = G_j(x)$ for $(t, x) \in T_j \times K$ we obtain a regular smooth function F on $T_j \times K$ which restricts to f on $T_j \times K \cap A$. Since $T \setminus V \subset T$ is an open and closed subset we may set in addition $F = 0$ on $(T \setminus V) \times K$ to obtain the desired extension of f to a regular smooth function on $T \times K$. □

5. Bivariant equivariant cohomology

In this section we review the definition of bivariant equivariant cohomology given by Baum and Schneider [4].

Let G be a totally disconnected group. A locally compact G-space is a locally compact space X with a continuous action of G. To every locally compact G-space X we associate the Brylinski space

$$\hat{X} = \{(t, x) \in G \times X |\, t \text{ is elliptic and } t \cdot x = x\} \subset G \times X$$

and the extended Brylinski space

$$\bar{X} = \{(t, x) \in G \times X |\, t \cdot x = x\} \subset G \times X.$$

Note that we have $\hat{X} = \bar{X}$ if the action of G on X is proper. If G is discrete we may view $\hat{X}$ as the disjoint union of the fixed point sets $X^t = \{x \in X |\, t \cdot x = x\}$ of elements $t \in G$ of finite order. Similarly, $\bar{X}$ is the disjoint union of the fixed point sets X^t of arbitrary elements $t \in G$ in this case.

Lemma 5.1. *Let G be a totally disconnected group and let X be a locally compact G-space. Then $\hat{X}$ and $\bar{X}$ are closed subspaces of $G \times X$.*

Proof. Let $\mu : G \times X \to X \times X$ be the map defined by $\mu(t, x) = (t \cdot x, x)$. Then $\bar{X} = \mu^{-1}(\Delta)$ is the preimage of the diagonal $\Delta \subset X \times X$ and hence closed. Since G_{ell} is a closed subspace of G and $\hat{X} = \bar{X} \cap (G_{\mathrm{ell}} \times X)$ it follows that $\hat{X}$ is closed. □

There is a G-action on the (extended) Brylinski space of a locally compact G-space X given by the formula

$$s \cdot (t, x) = (sts^{-1}, s \cdot x).$$

In this way $\hat{X}$ and $\bar{X}$ become locally compact G-spaces.

The space $\bar{X}$ will appear in the equivariant Hochschild-Kostant-Rosenberg theorem in section 6. For the remaining part of this section we will work only with the ordinary Brylinksi space $\hat{X}$.

Since the category $\mathrm{Sh}_G(\hat{X})$ has enough injectives we can choose an injective resolution

$$\mathbb{C}_{\hat{X}} \longrightarrow I^0 \longrightarrow I^1 \longrightarrow I^2 \longrightarrow \cdots$$

of the constant sheaf $\mathbb{C}_{\hat{X}}$ in the category of equivariant sheaves on $\hat{X}$. Consider the complex $C_c^\bullet(\hat{X})$ obtained by taking global sections with compact support in $I^\bullet$. Since the sheaves I^j are equivariant there is a natural G-action on $C_c^j(\hat{X})$ for all j. Moreover we have an $\mathcal{O}_G$-module structure on $C_c^j(\hat{X})$ given by

$$(f\sigma)(s,x) = f(s)\sigma(s,x)$$

for $f \in \mathcal{O}_G$ and $\sigma \in C_c^j(\hat{X})$. Observe that only the elliptic part of $\mathcal{O}_G$ acts nontrivially on $C_c^j(\hat{X})$. It is easy to check that this $\mathcal{O}_G$-module structure and the natural G-action combine to give each $C_c^j(\hat{X})$ the structure of a fine covariant module.
With these preparations the definition of bivariant equivariant cohomology given by Baum and Schneider can be formulated as follows.

Definition 5.2. *Let G be a totally disconnected group and let X and Y be locally compact G-spaces. The (delocalized) bivariant equivariant cohomology of X and Y is*

$$H_G^n(X,Y) = \mathfrak{Ext}_G^n(C_c^\bullet(\hat{X}), C_c^\bullet(\hat{Y}))$$

where $\mathfrak{Ext}_G$ denotes the hyperext functor in the category of fine covariant modules.

The functor $\mathfrak{Ext}_G$ can be viewed as the Hom-functor in the derived category of fine covariant modules. In order to compute the right-hand side in definition 5.2 choose a complex $I^\bullet(\hat{Y})$ consisting of injective fine covariant modules together with a quasiisomorphism $C_c^\bullet(\hat{Y}) \to I^\bullet(\hat{Y})$. Then

$$\mathfrak{Ext}_G^n(C_c^\bullet(\hat{X}), C_c^\bullet(\hat{Y})) = H_n(\mathfrak{Hom}_G(C_c^\bullet(\hat{X}), I^\bullet(\hat{Y})),$$

hence in order to calculate $\mathfrak{Ext}_G$ we have to compute the homology of a certain Hom-complex.
Let us now specialize to group actions on simplicial complexes and give some more definitions.
Recall that a simplicial map between simplicial complexes X and Y is a continuous map $f : X \to Y$ such that the restriction of f to any simplex of X is an affine map into a simplex of Y. We say that the group G acts simplicially on X if every $t \in G$ acts as a simplicial map.
Let G be a totally disconnected group. Assume that G acts simplicially on a simplicial complex X. The action is called type-preserving if for each simplex σ of X the stabilizer G_σ fixes the vertices of σ. In other words, an element of G which fixes a simplex actually acts trivially on this simplex. Passing to the barycentric subdivision one may always achieve that G acts type-preserving. The action of G is called smooth if all isotropy groups are open.
Let us now specify the class of G-spaces we are mainly interested in.

Definition 5.3. *Let G be a totally disconnected group. A G-simplicial complex is a simplicial complex X with a type-preserving smooth simplicial action of G. A morphism of G-simplicial complexes is an equivariant simplicial map $f : X \to Y$.*

Note that every G-simplicial complex is a G-CW-complex. For the definition of a G-CW-complex we refer to [12]. If X is a G-simplicial complex the space X^H of invariants with respect to a subgroup $H \subset G$ is a subcomplex of X. The action of G on X is proper iff the stabilizer of every point is a compact open subgroup of G.
Let X and Y be locally finite G-simplicial complexes. Our goal is to obtain a description of $H_G^n(X,Y)$ which is closer to the definition of equivariant cyclic homology.
Due to lemma 5.1 the Brylinski space $\hat{X}$ associated to a locally finite G-simplicial complex X is a closed subspace of $G \times X$. Hence the formalism of regular smooth

differential developped in section 4 may be applied to $\hat{X}$. According to proposition 4.5 we obtain a c-soft resolution

$$\mathbb{C}_{\hat{X}} \longrightarrow \mathcal{A}^0_{\hat{X}} \xrightarrow{d} \mathcal{A}^1_{\hat{X}} \xrightarrow{d} \mathcal{A}^2_{\hat{X}} \xrightarrow{d} \cdots$$

of the constant sheaf $\mathbb{C}_{\hat{X}}$ by regular smooth differential forms. This is a resolution in the category $\mathrm{Sh}_G(\hat{X})$ of equivariant sheaves on $\hat{X}$. We equip the spaces $\mathcal{A}^p_c(\hat{X})$ of global sections with the fine bornology.

Proposition 5.4. *Let X and Y be locally finite G-simplicial complexes. Then we have a natural isomorphism*

$$H^n_G(X,Y) = \mathfrak{Ext}^n_G(\mathcal{A}^\bullet_c(\hat{X}), \mathcal{A}^\bullet_c(\hat{Y})).$$

Proof. This isomorphism follows from proposition 4.5 and the fact that $\mathfrak{Ext}_G$ does not distinguish between quasiisomorphic complexes. □

Now assume in addition that X is finite dimensional. Then the complex $\mathcal{A}^\bullet_c(\hat{X})$ is not only bounded below but also bounded above. This means that in order to compute $\mathfrak{Ext}^n_G(\mathcal{A}^\bullet_c(\hat{X}), \mathcal{A}^\bullet_c(\hat{Y}))$ we may use a complex $P^\bullet(\hat{X})$ consisting of projective fine covariant modules together with a quasiisomorphism $p : P^\bullet(\hat{X}) \to \mathcal{A}^\bullet_c(\hat{X})$ and obtain

$$\mathfrak{Ext}^n_G(\mathcal{A}^\bullet_c(\hat{X}), \mathcal{A}^\bullet_c(\hat{Y})) = H^n(\mathfrak{Hom}_G(P^\bullet(\hat{X}), \mathcal{A}^\bullet_c(\hat{Y}))).$$

If $D = \dim(\hat{X})$ is the dimension of $\hat{X}$ we can construct a natural projective resolution $P^\bullet(\hat{X})$ of $\mathcal{A}^\bullet_c(\hat{X})$ in such a way that we obtain a commutative diagram of the form

$$\begin{array}{ccccccccc}
\cdots & \xrightarrow{\delta} & P^{D-2}(\hat{X}) & \xrightarrow{\delta} & P^{D-1}(\hat{X}) & \xrightarrow{\delta} & P^D(\hat{X}) & \longrightarrow & 0 \\
 & & \downarrow p & & \downarrow p & & \downarrow p & & \\
\cdots & \xrightarrow{d} & \mathcal{A}^{D-2}_c(\hat{X}) & \xrightarrow{d} & \mathcal{A}^{D-1}_c(\hat{X}) & \xrightarrow{d} & \mathcal{A}^D_c(\hat{X}) & \longrightarrow & 0
\end{array}$$

We may require in addition that $P^j(\hat{X}) = EP^j(\hat{X})$ for all j where E is the canonical projection on the elliptic part of a covariant module. This means in particular that the hyperbolic part of $P^j(\hat{X})$ is zero. In this case we call the projective resolution $P^\bullet(\hat{X})$ regular. Remark that $T = \mathrm{id}$ for the natural operator T on a regular projective resolution $P^\bullet(\hat{X})$.

Let us view $\mathcal{A}^\bullet_c(\hat{X})$ as a mixed complex by setting the Hochschild boundary equal to zero and letting $B = d$ be the exterior differential. To this mixed complex we associate a tower of supercomplexes $\mathcal{A}_c(\hat{X}) = (\mathcal{A}_c(\hat{X})_k)$ as follows. We define

$$\mathcal{A}_c(\hat{X})_k = \bigoplus_{j=0}^{k} \mathcal{A}^j_c(\hat{X})$$

and equip this space with the ordinary grading into even and odd forms and differential $B + b = d$. Observe that $\mathcal{A}_c(\hat{X})_k = \mathcal{A}_c(\hat{X})_D$ for $k \geq D = \dim(\hat{X})$. Hence the tower of supercomplexes $\mathcal{A}_c(\hat{X})$ is isomorphic to the constant supercomplex

$$\mathcal{A}_c(\hat{X}) \cong \bigoplus_{j=0}^{D} \mathcal{A}^j_c(\hat{X}).$$

In a similar way a regular projective resolution $P^\bullet(\hat{X})$ of $\mathcal{A}^\bullet_c(\hat{X})$ satisfies the axioms of a mixed complex. Let us define a tower of supercomplexes $P(\hat{X}) = (P(\hat{X})_k)$ as follows. We set

$$P(\hat{X})_k = P^{-(k+1)}(\hat{X})/\delta(P^{-(k+2)}(\hat{X})) \oplus \bigoplus_{j=-k}^{k} P^j(\hat{X}) \oplus \delta(P^k(\hat{X}))$$

Remark that for $k \geq D$ this becomes

$$P(\hat{X})_k = P^{-(k+1)}(\hat{X})/\delta(P^{-(k+2)}(\hat{X})) \oplus \bigoplus_{j=-k}^{D} P^j(\hat{X}).$$

Clearly we consider the grading into even and odd components on $P(\hat{X})_k$ and equip these spaces with the differential δ. Recall from [19] that a a covariant pro-module is called relatively projective if it has the lifting property with respect to covariant maps between covariant pro-modules having a pro-linear section. Since the covariant modules $P^j(\hat{X})$ are projective for all j it is easy to see that the inverse system $P(\hat{X})$ is relatively projective. The chain map $p : P^\bullet(\hat{X}) \to \mathcal{A}_c^\bullet(\hat{X})$ induces a covariant chain map of supercomplexes $p : P(\hat{X}) \to \mathcal{A}_c(\hat{X})$.

Proposition 5.5. *Let X and Y be finite dimensional locally finite G-simplicial complexes. Then*

$$\bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y) = H_*(\varinjlim_k \mathfrak{Hom}_G(P(\hat{X})_k, \mathcal{A}_c(\hat{Y})))$$
$$= H_*(\mathfrak{Hom}_G(P(\hat{X}), \mathcal{A}_c(\hat{Y})))$$

where in the last expression we take homomorphisms in the pro-category $\mathrm{pro}(G\text{-}\mathfrak{Mod})$ *of covariant modules.*

Proof. The component of degree n in $\mathfrak{Hom}_G(P^\bullet(\hat{X}), \mathcal{A}_c^\bullet(\hat{Y}))$ is

$$\bigoplus_{i\in\mathbb{Z}} \mathfrak{Hom}_G(P^i(\hat{X}), \mathcal{A}_c^{i+n}(\hat{Y})),$$

here a direct sum occurs because $\mathcal{A}_c^\bullet(\hat{Y})$ is a bounded complex. We deduce

$$\bigoplus_{j\in\mathbb{Z}} \mathfrak{Hom}_G^{*+2j}(P^\bullet(\hat{X}), \mathcal{A}_c^\bullet(\hat{Y})) = \bigoplus_{j\in\mathbb{Z}} \bigoplus_{i\in\mathbb{Z}} \mathfrak{Hom}_G(P^i(\hat{X}), \mathcal{A}_c^{i+2j+*}(\hat{Y}))$$

and obtain natural maps

$$\lambda_k : \mathfrak{Hom}_G(P(\hat{X})_k, \mathcal{A}_c(\hat{Y})) \to \bigoplus_{j\in\mathbb{Z}} \mathfrak{Hom}_G^{*+2j}(P^\bullet(\hat{X}), \mathcal{A}_c^\bullet(\hat{Y}))$$

for all $k \geq \dim(\hat{X})$. It is easy to check that each λ_k is a chain map. Moreover the maps λ_k are compatible with the projections in the first variable. The resulting map

$$\lambda : \varinjlim_k \mathfrak{Hom}_G(P(\hat{X})_k, \mathcal{A}_c(\hat{Y})) \to \bigoplus_{j\in\mathbb{Z}} \mathfrak{Hom}_G^{*+2j}(P^\bullet(\hat{X}), \mathcal{A}_c^\bullet(\hat{Y}))$$

is an isomorphism of complexes. □

6. The equivariant Hochschild-Kostant-Rosenberg theorem

The algebra $C_c^\infty(X)$ of regular smooth functions on a G-simplicial complex X is a G-algebra in a natural way. In this section we identify the homology of $\Omega_G(C_c^\infty(X))$ with respect to the equivariant Hochschild boundary. This will be an important ingredient in the proof of theorem 7.1 below.

Recall from section 5 the definition of the extended Brylinski space $\bar{X}$. Let us view $\mathcal{A}_c(\bar{X})$ as a (para-) mixed complex with b-boundary equal to zero and B-boundary equal to the exterior differential d. We define the equivariant Hochschild-Kostant-Rosenberg map $\alpha : \Omega_G(C_c^\infty(X)) \to \mathcal{A}_c(\bar{X})$ by

$$\alpha(f(t) \otimes a_0 da_1 \cdots da_n) = \frac{1}{n!}\, f(t)\, a_0 da_1 \wedge \cdots \wedge da_{n|X^t}$$

where we recall that X^t denotes the set of fixed points under the action of t.

Theorem 6.1. *Let G be a totally disconnected group and let X be a locally finite G-simplicial complex. The equivariant Hochschild-Kostant-Rosenberg map*

$$\alpha : \Omega_G(C_c^\infty(X)) \to \mathcal{A}_c(\bar{X})$$

is a map of paramixed complexes and induces an isomorphism on the homology with respect to the Hochschild boundary.

Proof. Let us first show that α is a map of paramixed complexes. We compute

$$\begin{aligned}\alpha b(f(t)\otimes a_0da_1\cdots da_n) &= \sum_{j=0}^{n-1}(-1)^j\alpha(f(t)\otimes a_0da_1\cdots d(a_ja_{j+1})\cdots da_n)\\ &\quad+(-1)^n\alpha(f(t)\otimes(t^{-1}\cdot a_n)a_0da_1\cdots da_{n-1})\\ &= \frac{1}{(n-1)!}\Big(\sum_{j=0}^{n-1}(-1)^j f(t)\; a_0da_1\wedge\cdots\wedge d(a_ja_{j+1})\cdots\wedge da_{n|X^t}\\ &\quad+(-1)^n f(t)a_na_0da_1\wedge\cdots\wedge da_{n-1|X^t}\Big)=0\end{aligned}$$

where we use $(t^{-1}\cdot a_n)(x) = a_n(t\cdot x) = a_n(x)$ for all $x\in X^t$. Moreover we have

$$\begin{aligned}\alpha B(f(t)\otimes a_0da_1\cdots da_n) &= \sum_{j=0}^{n}(-1)^{nj}\alpha(f(t)\otimes t^{-1}\cdot(da_{n-j+1}\cdots da_n)da_0\cdots da_{n-j})\\ &= \frac{1}{(n+1)!}\sum_{j=0}^{n}(-1)^{nj}f(t)\,da_{n-j+1}\wedge\cdots\wedge da_n\wedge da_0\wedge\cdots\wedge da_{n-j|X^t}\\ &= \frac{1}{n!}\,f(t)\,da_0\wedge\cdots\wedge da_{n|X^t} = d\alpha(f(t)\otimes a_0da_1\cdots da_n)\end{aligned}$$

and hence α commutes with the boundary operators as claimed.
In order to show that α induces an isomorphism in homology it suffices to prove that the localized maps

$$\alpha_t : \Omega_G(C_c^\infty(X))_t \to \mathcal{A}_c(\bar{X})_t = \mathcal{A}_c(X^t)$$

are quasiisomorphisms for all $t\in G$. Let us consider the case that X is an equivariant simplex. By definition, an equivariant simplex is a space of the form $X = G/H\times\Delta^k$ where H is an open subgroup of G and the action on G/H is given by translation. The boundary ∂X of X is defined by $\partial X = G/H\times\partial\Delta^k$. Recall from section 4 that $C_c^\infty(X,\partial X)$ denotes the kernel the restriction map $C_c^\infty(X)\to C^\infty(\partial X)$. Similarly, $\mathcal{A}_c(X^t,\partial X^t)$ is the kernel of the natural map $\mathcal{A}_c(X^t)\to\mathcal{A}_c(\partial X^t)$. The localized equivariant Hochschild-Kostant-Rosenberg map restricts to a chain map

$$\alpha_t : \Omega_G(C_c^\infty(X,\partial X))_t \to \mathcal{A}_c(X^t,\partial X^t).$$

Let us specialize further to the case $G=\mathbb{Z}$ and $t=1$. We write $\Delta^k[n]$ for the $\mathbb{Z}$-equivariant simplex $\mathbb{Z}/n\mathbb{Z}\times\Delta^k$. Remark that the fixed point set $\Delta^k[n]^1$ for the action of $1\in\mathbb{Z}$ is empty for $n=0$ or $n>1$.

Proposition 6.2. *With the notation as above we have for every $k\geq 0$:*

a) The localized equivariant Hochschild-Kostant-Rosenberg map

$$\alpha_1 : \Omega_{\mathbb{Z}}(C^\infty(\Delta^k[1],\partial\Delta^k[1]))_1 \to \mathcal{A}(\Delta^k[1]^1,\partial\Delta^k[1]^1)$$

is a quasiisomorphism.

b) For $n=0$ and $n>1$ the homology of $\Omega_{\mathbb{Z}}(C_c^\infty(\Delta^k[n],\partial\Delta^k[n]))_1$ with respect to the equivariant Hochschild boundary is trivial.

Proof. *a*) By definition we have $\Delta^k[1] = \Delta^k$ and the action is trivial. The algebra $C^\infty(\Delta^k[1], \partial\Delta^k[1])$ can be identified with the algebra $C_c^\infty(\Delta^k \backslash \partial\Delta^k)$ of smooth functions with compact support on $\Delta^k \setminus \partial\Delta^k$. Moreover the space $\mathcal{A}(\Delta^k[1]^1, \partial\Delta^k[1]^1)$ consists of differential forms with compact support on $\Delta^k \setminus \partial\Delta^k$. Hence the assertion follows from the ordinary Hochschild-Kostant-Rosenberg theorem [9], [17].
b) Let $n = 0$ or $n > 1$ and let B be any unital complete bornological algebra. We equip B with the trivial $\mathbb{Z}$-action and consider the $\mathbb{Z}$-algebra $C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B$. An element of this algebra can be written as a linear combination of elements $x[i]$ where $x[i] \in C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B$ for $i \in \mathbb{Z}/n\mathbb{Z}$ denotes the characteristic function located in i with value $x \in B$.
We view the localized Hochschild complex $\Omega_{\mathbb{Z}}(C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B)_1$ as a double complex with two columns. This corresponds to the natural decomposition

$$\Omega_G^n(C) = \mathcal{O}_G\hat{\otimes}C^{\hat{\otimes}n+1} \oplus \mathcal{O}_G\hat{\otimes}C^{\hat{\otimes}n}$$

of the space of equivariant differential forms of a G-algebra C. Since B is unital the algebra $C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B$ has local units. Consequently, the natural inclusion of the first column of $\Omega_{\mathbb{Z}}(C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B)_1$ into the total complex induces an isomorphism in homology. Let us construct a contracting homotopy h for the first column of $\Omega_{\mathbb{Z}}(C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B)_1$ as follows. We define

$$h(x_0[i_0]dx_1[i_1]\cdots dx_p[i_p]) = (-1)^{l+1}x_0[i_0]dx_1[i_1]\cdots d1[i_{l+1}]dx_{l+1}[i_{l+1}]\cdots dx_p[i_p]$$

if $0 \leq l \leq p-1$ is the smallest number such that $i_l \neq i_{l+1}$ and

$$h(x_0[i_0]dx_1[i_1]\cdots dx_p[i_p]) = 1[i_0]dx_0[i_0]\cdots dx_p[i_p]$$

if $i_j = i_{j+1}$ for $0 \leq j \leq p-1$. An easy calculation shows that h is indeed a contracting homotopy. Hence the complexes $\Omega_{\mathbb{Z}}(C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B)_1$ are acyclic for $n = 0$ or $n > 1$.
Due to proposition 4.2 we have an extension of $\mathbb{Z}$-algebras with bounded linear splitting

$$C_c^\infty(\Delta^k[n], \partial\Delta^k[n]) \rightarrowtail C_c^\infty(\Delta^k[n]) \twoheadrightarrow C_c^\infty(\partial\Delta^k[n])$$

The algebras $C_c^\infty(\Delta^k[n])$ and $C_c^\infty(\partial\Delta^k[n])$ are of the form $C_c(\mathbb{Z}/n\mathbb{Z})\hat{\otimes}B$ described above. Hence the complexes $\Omega_{\mathbb{Z}}(C_c^\infty(\Delta^k[n]))_1$ and $\Omega_{\mathbb{Z}}(C_c^\infty(\partial\Delta^k[n]))_1$ are acyclic. Since the algebra $C_c^\infty(\Delta^k[n], \partial\Delta^k[n])$ has local units we obtain a long exact sequence in homology showing that $\Omega_{\mathbb{Z}}(C_c^\infty(\Delta^k[n], \partial\Delta^k[n])_1$ is acyclic as well. This yields the claim. □
Let us come back to the localized Hochschild-Kostant-Rosenberg map for arbitrary G and t and an equivariant simplex $X = G/H \times \Delta^k$. We extend proposition 6.2 to this situation as follows. There is a canonical group homomorphism $\mathbb{Z} \to G$ which maps 1 to t and we may view $X = G/H \times \Delta^k$ as a $\mathbb{Z}$-space in this way. Clearly the localized complexes $\Omega_G(C_c^\infty(X, \partial X))_t$ and $\Omega_{\mathbb{Z}}(C_c^\infty(X, \partial X))_1$ are isomorphic since the equivariant Hochschild boundary in $\Omega_G(C_c^\infty(X, \partial X))_t$ only depends on the action of t. Viewed as a $\mathbb{Z}$-space, X can be written as disjoint union

$$X = \bigcup_{j\in J} \Delta^k[n_j]$$

for some index set J where $\Delta^k[n] = \mathbb{Z}/n\mathbb{Z} \times \Delta^k$ as before. In this decomposition the spaces $\Delta^k[n]$ may appear with multiplicity. Let us determine how $\alpha_t : \Omega_{\mathbb{Z}}(C_c^\infty(X, \partial X))_t \to \mathcal{A}_c(X^t, \partial X^t)$ can be described in terms of the spaces $\Delta^k[n_j]$. On the right hand side the decomposition of X induces a direct sum decomposition

$$\mathcal{A}_c(X^t, \partial X^t) = \bigoplus_{j\in J} \mathcal{A}_c(\Delta^k[n_j]^1, \partial\Delta^k[n_j]^1).$$

Moreover we have an isomorphism

$$C_c^\infty(X) = \bigoplus_{j\in J} C_c^\infty(\Delta^k[n_j])$$

of $\mathbb{Z}$-algebras and a natural inclusion of complexes

$$\iota : \bigoplus_{j\in J} \Omega_{\mathbb{Z}}(C_c^\infty(\Delta^k[n_j], \partial\Delta^k[n_j]))_1 \to \Omega_{\mathbb{Z}}\Big(\bigoplus_{j\in J} C_c^\infty(\Delta^k[n_j], \partial\Delta^k[n_j])\Big)_1$$

on the left hand side. It follows from the existence of local units and an inductive limit argument that the map ι is a quasiisomorphism with respect to the Hochschild boundary.
Hence, up to quasiisomorphism, the map α_t can be decomposed as a direct sum of maps $\alpha_1 : \Omega_{\mathbb{Z}}(C_c^\infty(\Delta^k[n_j], \partial\Delta^k[n_j]))_1 \to \mathcal{A}_c(\Delta^k[n_j]^1, \partial\Delta^k[n_j]^1)$. We apply proposition 6.2 and obtain the following statement.

Proposition 6.3. *Let G be a totally disconnected group and let $X = G/H \times \Delta^k$ be an equivariant simplex. For every $t \in G$ the localized equivariant Hochschild-Kostant-Rosenberg map*

$$\alpha_t : \Omega_G(C_c^\infty(X, \partial X))_t \to \mathcal{A}_c(X^t, \partial X^t)$$

is a quasiisomorphism.

Let us now finish the proof of theorem 6.1. First we assume that the G-simplicial complex X is finite dimensional and use induction on the dimension of X. If $\dim(X) = 0$ the space X is a disjoint union of homogenous spaces G/H. As above it suffices to consider a single equivariant simplex $X = G/H$. Since we have $C_c^\infty(X) = C_c^\infty(X, \partial X)$ in this case the assertion follows from proposition 6.3. Assume that $\dim(X) = k$ and that the assertion is proved for all G-simplicial complexes of dimension $k-1$. We consider the commutative diagram

$$\begin{array}{ccccc}
\Omega_G(C_c^\infty(X, X^{k-1}))_t & \longrightarrow & \Omega_G(C_c^\infty(X))_t & \longrightarrow & \Omega_G(C_c^\infty(X^{k-1}))_t \\
\downarrow{\scriptstyle \alpha_t} & & \downarrow{\scriptstyle \alpha_t} & & \downarrow{\scriptstyle \alpha_t} \\
\mathcal{A}_c(X^t, (X^{k-1})^t) & \longrightarrow & \mathcal{A}_c(X^t) & \longrightarrow & \mathcal{A}_c((X^{k-1})^t)
\end{array}$$

where X^{k-1} denotes the $k-1$-skeleton of X. The algebra $C_c^\infty(X, X^{k-1})$ is a direct sum of algebras of the form $C_c^\infty(\sigma, \partial\sigma)$ where $\sigma = G/H \times \Delta^k$ is an equivariant simplex. In particular $C_c^\infty(X^k, X^{k-1})$ has local units. Hence the upper horizontal sequence induces a long exact sequence in homology. Proposition 6.3 implies that the left vertical map is a quasiisomorphism. The right vertical map is a quasiisomorphism by induction hypothesis. Hence $\alpha_t : \Omega_G(C_c^\infty(X))_t \to \mathcal{A}_c(X^t)$ is a quasiisomorphism as well.
For an arbitrary G-simplicial complex X we take the inductive limit over all finite dimensional subcomplexes in order to obtain the assertion. This completes the proof of theorem 6.1.

7. The comparison theorem

In this section we prove the following theorem which describes the relation between equivariant periodic cyclic homology and bivariant equivariant cohomology in the sense of Baum and Schneider.

Theorem 7.1. *Let G be a totally disconnected group and let X and Y be finite dimensional locally finite G-simplicial complexes. If the action of G on X is proper*

there exists a natural isomorphism

$$HP_*^G(C_c^\infty(X), C_c^\infty(Y)) \cong \bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y).$$

Even if the action on X is not proper the elliptic part of $HP_*^G(C_c^\infty(X), C_c^\infty(Y))$ is naturally isomorphic to $\bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y)$.

It follows immediately from the definitions that the theory defined by Baum and Schneider a priori only has an elliptic part. Hence theorem 7.1 states in particular that the hyperbolic part of $HP_*^G(C_c^\infty(X), C_c^\infty(Y))$ is zero provided the action of G on X is proper. In general the hyperbolic part of $HP_*^G(C_c^\infty(X), C_c^\infty(Y))$ might be different from zero, however, this cannot be detected using the theory of Baum and Schneider.

The proof of theorem 7.1 is divided into several steps. First we shall identify HP_*^G with an auxiliary bivariant theory h_*^G under the assumptions of the theorem. We denote by $\mathfrak{Fine}$ the natural forgetful functor on covariant modules which changes the bornology to the fine bornology. The functor $\mathfrak{Fine}$ is extended to the category pro(G-$\mathfrak{Mod}$) in the obvious way. With this notation we define the bivariant theory $h_*^G(A,B)$ for G-algebras A and B by

$$h_*^G(A,B) = H_*(\mathfrak{Hom}_G(\mathfrak{Fine}(\theta\Omega_G(A\hat{\otimes}\mathcal{K}_G)), \mathfrak{Fine}(\theta\Omega_G(B\hat{\otimes}\mathcal{K}_G)))).$$

This definition is identical to the definition of HP_*^G except that we do not require the covariant maps in the Hom-complex to be bounded. Evidently h_*^G shares many properties with HP_*^G. For our purposes it is important that h_*^G satisfies excision in both variables. This follows immediately from the generalized excision theorem in equivariant periodic cyclic homology [19]. Moreover there is an obvious composition product for h_*^G and a natural transformation

$$v : HP_*^G(A,B) \to h_*^G(A,B)$$

which is obtained by forgetting the bornology. It is clear that v is compatible with the composition product.

Proposition 7.2. *Let X be a finite dimensional locally finite G-simplicial complex and let B be an arbitrary G-algebra. Then the natural map*

$$v : HP_*^G(C_c^\infty(X), B) \to h_*^G(C_c^\infty(X), B)$$

is an isomorphism.

Proof. We use induction on the dimension of X. For $\dim(X) = 0$ the algebra $C_c^\infty(X)$ is equipped with the fine bornology and $\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G)$ is a projective system of fine spaces. Hence the complexes used in the definition of HP_*^G and h_*^G are equal and v is clearly an isomorphism in this case. Now assume that the assertion is true for all G-simplicial complexes of dimension smaller than k and that $\dim(X) = k$. Due to proposition 4.2 we have a linearly split extension of G-algebras of the form

$$\textstyle\bigoplus_{j\in J} C_c^\infty(\sigma_j, \partial\sigma_j) \rightarrowtail C_c^\infty(X) \twoheadrightarrow C_c^\infty(X^{k-1})$$

where each $\sigma_j = G/H_j \times \Delta^k$ is an equivariant simplex. Using the six-term exact sequences for HP_*^G and h_*^G obtained from the excision theorem it suffices to show that

$$v : HP_*^G\Big(\bigoplus_{j\in J} C_c^\infty(\sigma_j, \partial\sigma_j), B\Big) \to h_*^G\Big(\bigoplus_{j\in J} C_c^\infty(\sigma_j, \partial\sigma_j), B\Big)$$

is an isomorphism. Applying excision again we see that in both theories HP_*^G and h_*^G the G-algebras $\bigoplus_{j\in J} C_c^\infty(\sigma_j, \partial\sigma_j)$ and $\bigoplus_{j\in J} C_c(G/H_j)$ are equivalent. Since v

is compatible with products the assertion follows now from the case $\dim(X) = 0$ which we have already proved. □

Corollary 7.3. *For all finite dimensional locally finite G-simplicial complexes X and Y we have a natural isomorphism*

$$HP^G_*(C^\infty_c(X), C^\infty_c(Y)) \cong h^G_*(C^\infty_c(X), C^\infty_c(Y)).$$

We come to the next ingredient in the proof of theorem 7.1. For an arbitrary G-algebra B one defines a map $\mathrm{tr} : \Omega_G(B\hat{\otimes}\mathcal{K}_G) \to \Omega_G(B)$ by

$$\begin{aligned}&\mathrm{tr}(f(t) \otimes (x_0 \otimes k_0)d(x_1 \otimes k_1) \cdots d(x_n \otimes k_n))\\ &\qquad = f(t) \otimes x_0 dx_1 \cdots dx_n \int k_0(r_0, r_1)k_1(r_1, r_2) \cdots k_n(r_n, tr_0)dr_0 \cdots dr_n\end{aligned}$$

and

$$\begin{aligned}&\mathrm{tr}(f(t) \otimes d(x_1 \otimes k_1) \cdots d(x_n \otimes k_n))\\ &\qquad = f(t) \otimes dx_1 \cdots dx_n \int k_1(r_1, r_2) \cdots k_n(r_n, tr_1)dr_1 \cdots dr_n.\end{aligned}$$

It is straightforward to check that tr is a map of paramixed complexes.

Proposition 7.4. *Let X be a locally finite G-simplicial complex. Then the map $\mathrm{tr} : \Omega_G(C^\infty_c(X)\hat{\otimes}\mathcal{K}_G) \to \Omega_G(C^\infty_c(X))$ induces an isomorphism on the homology with respect to the equivariant Hochschild boundary.*

Proof. Consider the natural commutative diagram

$$\begin{array}{ccccc}\Omega_G(C^\infty_c(X)\hat{\otimes}\mathcal{K}_G) & \longrightarrow & \Omega_G(C^\infty_c(X)^+\hat{\otimes}\mathcal{K}_G) & \longrightarrow & \Omega_G(\mathbb{C}\hat{\otimes}\mathcal{K}_G)\\ \downarrow{\scriptstyle \mathrm{tr}} & & \downarrow{\scriptstyle \mathrm{tr}} & & \downarrow{\scriptstyle \mathrm{tr}}\\ \Omega_G(C^\infty_c(X)) & \longrightarrow & \Omega_G(C^\infty_c(X)^+) & \longrightarrow & \Omega_G(\mathbb{C})\end{array}$$

of complexes. According to proposition 4.3 the algebra $C^\infty_c(X)$ has local units. The same is true for $C^\infty_c(X)\hat{\otimes}\mathcal{K}_G$. Hence the horizontal maps induce long exact sequences in homology. Proposition 16.2 in [19] shows that $\mathrm{tr} : \Omega_G(B\hat{\otimes}\mathcal{K}_G) \to \Omega_G(B)$ is a linear homotopy equivalence for every unital G-algebra B. We deduce that the middle and right vertical arrows induce isomorphisms in homology. Consequently, the left vertical arrow induces an isomorphism in homology as well. □

Let X be a G-simplicial complex. In section 6 we have studied the equivariant Hochschild-Kostant-Rosenberg map $\Omega_G(C^\infty_c(X)) \to \mathcal{A}_c(\bar{X})$. We compose this map with the projection onto the elliptic part of $\mathcal{A}_c(\bar{X})$ to obtain a map

$$\alpha : \Omega_G(C^\infty_c(X)) \to \mathcal{A}_c(\bar{X})_{\mathrm{ell}} = \mathcal{A}_c(\hat{X}).$$

By construction, the hyperbolic part of $\Omega_G(C^\infty_c(X))$ is mapped to zero under this map. Let us define a covariant map $q : \mathfrak{Fine}(\theta\Omega_G(C^\infty_c(X)\hat{\otimes}\mathcal{K}_G)) \to \mathcal{A}_c(\hat{X})$ by composing $\mathrm{tr} : \Omega_G(C^\infty_c(X)\hat{\otimes}\mathcal{K}_G) \to \Omega_G(C^\infty_c(X))$ with the map α.

Now let Q be any relatively projective paracomplex of fine covariant pro-modules. Composition with q yields a map

$$f : H_*(\mathfrak{Hom}_G(Q, \mathfrak{Fine}(\theta\Omega_G(C^\infty_c(X)\hat{\otimes}\mathcal{K}_G)))) \to H_*(\mathfrak{Hom}_G(Q, \mathcal{A}_c(\hat{X}))).$$

Moreover, as explained in section 5, we choose a regular projective resolution $P^\bullet(\hat{X})$ of $\mathcal{A}^\bullet_c(\hat{X})$ as above and let $P(\hat{X})$ be the associated supercomplex. Composition with the chain map $p : P(\hat{X}) \to \mathcal{A}_c(\hat{X})$ yields a map

$$g : H_*(\mathfrak{Hom}_G(Q, P(\hat{X}))) \to H_*(\mathfrak{Hom}_G(Q, \mathcal{A}_c(\hat{X})))$$

Remark that the Hom-complexes occuring in the definition of f and g are in fact complexes since all entries are paracomplexes of covariant pro-modules.

Proposition 7.5. *Let X be a finite dimensional locally finite G-simplicial complex. If the action of G on X is proper the map f is an isomorphism.*

Proof. We shall treat the elliptic and the hyperbolic parts separately. Let us abbreviate $\Omega_G(X) = \Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G)$.
First we consider the elliptic part. According to proposition 7.4 and theorem 6.1 the restriction to the elliptic part of the map q defined above induces a quasiisomorphism $\Omega_G(X)_{\mathrm{ell}} \to \mathcal{A}_c(\hat{X})$ on the homology with respect to the Hochschild boundary. Moreover the natural projection E on the elliptic part of covariant modules introduced in section 2 preserves linearly split exact sequences of covariant modules. In particular we obtain a quasiisomorphism $E\Omega_G(X)_{\mathrm{ell}} \to E\mathcal{A}_c(\hat{X}) = \mathcal{A}_c(\hat{X})$ which will be denoted by q again. Since $T = \mathrm{id}$ on $E\Omega_G(X)_{\mathrm{ell}}$ and $\mathcal{A}_c(\hat{X})$ the map q is in fact a map between ordinary mixed complexes. Recall from section 3 that we write $L^j(M) = F^{j-1}(M)/F^j(M)$ for the subquotients of the Hodge filtration of a (para-) mixed complex M. The map q induces chain maps $L^jE\Omega_G(X)_{\mathrm{ell}} \to L^j\mathcal{A}_c(\hat{X})$ for all j. Since q is a quasiisomorphism with respect to the Hochschild boundary it follows easily that these maps are quasiisomorphism of supercomplexes. In particular the corresponding mapping cones are acyclic.
We need the following two auxiliary results.

Lemma 7.6. *Let $\phi : D \to E$ be a morphism of supercomplexes of covariant modules. Assume that $\partial_0 : D_0 \to D_1$ is zero and that $\partial_1 : E_1 \to E_0$ is surjective. Then in the mapping cone C^ϕ we have $\mathrm{im}(\partial_1) = E_0$. Consequently the image of ∂_1 is a direct summand in $C_0^\phi = D_1 \oplus E_0$.*

Proof. By assumption the differential ∂_1 in C^ϕ has the form

$$\begin{pmatrix} 0 & 0 \\ -f & \partial_1 \end{pmatrix}.$$

Since $\partial_1 : E_1 \to E_0$ is surjective the image of ∂_1 in C^ϕ is precisely E_0. □

Lemma 7.7. *Let Q be a relatively projective paracomplex of fine covariant promodules and let $\phi : Q \to C$ be a covariant chain map where C is a constant and acyclic supercomplex. Moreover assume that C_0 admits a direct sum decomposition $C_0 = K \oplus R$ where $K = \mathrm{im}(\partial_1) = \ker(\partial_0)$. Then ϕ is homotopic to zero. Consequently we have $H_*(\mathfrak{Hom}_G(Q, C)) = 0$.*

Proof. The map $\phi_0 : Q_0 \to C_0 = K \oplus R$ may be written as $\phi_0 = k \oplus r$. Since $\partial_1 : C_1 \to K$ is a surjection we find a covariant map $s : Q_0 \to C_1$ such that $\partial_1 s = k$. Hence we may assume without loss of generality that $k = 0$. Now since ϕ is a chain map and the image of $\partial_1\phi_1$ is contained in K we deduce $\partial_1\phi_1 = 0$. Since C is exact we have $\mathrm{im}(\phi_1) \subset \ker(\partial_1) = \mathrm{im}(\partial_0)$. We may thus construct a map $h : Q_1 \to C_0$ such that $\partial_0 h = \phi_1$. Furthermore we may assume that h factorizes over R, that is, $h : Q_1 \to R \to C_0$. Hence, up to chain homotopy, the map ϕ satisfies $\phi_1 = 0$ and $k = 0$ in $\phi_0 = k \oplus r$. Since ϕ is a chain map we now have $0 = \partial_0\phi_0 = \partial_0 r$. But ∂_0 restricted to R is an injection since $\ker(\partial_0) = K$. This implies $\phi_0 = 0$ and hence our original map ϕ is homotopic to zero. Since we have explicitly shown that any chain map $\phi : Q \to C$ is homotopic to zero we obtain $H_0(\mathfrak{Hom}_G(Q, C)) = 0$. By reindexing Q we deduce in the same way that $H_1(\mathfrak{Hom}_G(Q, C)) = 0$. This finishes the proof. □
After possibly reindexing, the map

$$L^jE\Omega_G(X)_{\mathrm{ell}} \to L^j\mathcal{A}_c(\hat{X})$$

satisfies the assumptions of lemma 7.6. It follows that its mapping cone C^j satisfies the assumptions of lemma 7.7. The short exact sequence

$$L^j\mathcal{A}_c(\hat{X}) \rightarrowtail C^j \twoheadrightarrow L^jE\Omega_G(X)_{\mathrm{ell}}[1]$$

of supercomplexes has a covariant splitting and induces a short exact sequence

$$\mathfrak{Hom}_G(Q_{\mathrm{ell}}, L^j\mathcal{A}_c(\hat{X})) \rightarrowtail \mathfrak{Hom}_G(Q_{\mathrm{ell}}, C^j) \twoheadrightarrow \mathfrak{Hom}_G(Q_{\mathrm{ell}}, L^jE\Omega_G(X)_{\mathrm{ell}}[1])$$

of supercomplexes. From lemma 7.7 we deduce $H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, C^j)) = 0$. Since the boundary map in this long exact sequence is the map induced by q we obtain an isomorphism

$$H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, L^jE\Omega_G(X)_{\mathrm{ell}})) \cong H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, L^j\mathcal{A}_c(\hat{X})))$$

for all j.
Since Q is assumed to be relatively projective the Hodge filtrations of $E\theta^n\Omega_G(X)_{\mathrm{ell}}$ and $\theta^n\mathcal{A}_c(\hat{X})$ induce bounded filtrations of the complexes $\mathfrak{Hom}_G(Q_{\mathrm{ell}}, E\theta^n\Omega_G(X)_{\mathrm{ell}})$ and $\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta^n\mathcal{A}_c(\hat{X}))$, respectively. Hence the corresponding spectral sequences converge. The map q induces a map of spectral sequences which gives an isomorphism on the E^1-terms according to the preceeding discussion. Hence we obtain

Proposition 7.8. *With the notation as above the map q induces an isomorphism*

$$H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, E\theta^n\Omega_G(X)_{\mathrm{ell}})) \cong H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta^n\mathcal{A}_c(\hat{X})))$$

for all n.

Let M be a paramixed complex and set $M^n = \mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta^nM)$. Since the structure maps in θM are surjections and Q_{ell} is relatively projective the structure maps in the inverse system $(M^n)_{n\in\mathbb{N}}$ are surjective. This implies $\varprojlim{}^1 M^n = 0$. Therefore we obtain a short exact sequence

$$\varprojlim{}_n M^n \rightarrowtail \textstyle\prod_{n\in\mathbb{N}} M^n \xrightarrow{\mathrm{id}-\sigma} \prod_{n\in\mathbb{N}} M^n$$

of supercomplexes where σ denotes the structure maps in $(M_n)_{n\in\mathbb{N}}$. This induces a long exact sequence

$$\begin{array}{ccccc} H_0(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta M)) & \longrightarrow & H_0(\prod_{n\in\mathbb{N}} M^n) & \longrightarrow & H_0(\prod_{n\in\mathbb{N}} M^n) \\ \uparrow & & & & \downarrow \\ H_1(\prod_{n\in\mathbb{N}} M^n) & \longleftarrow & H_1(\prod_{n\in\mathbb{N}} M^n) & \longleftarrow & H_1(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta M)) \end{array}$$

in homology.
The map q induces a morphism between these exact sequences for $M = E\Omega_G(X)_{\mathrm{ell}}$ and $M = \mathcal{A}_c(\hat{X})$. Hence we obtain an isomorphism

$$H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, E\theta\Omega_G(X)_{\mathrm{ell}} \cong H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \mathcal{A}_c(\hat{X}))).$$

According to proposition 3.4 the canonical projection $\theta\Omega_G(X)_{\mathrm{ell}} \to E\theta\Omega_G(X)_{\mathrm{ell}}$ is a covariant homotopy equivalence. Hence we finally conclude that the map

$$f : H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))_{\mathrm{ell}})) \to H_*(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \mathcal{A}_c(\hat{X}))_{\mathrm{ell}}))$$

is an isomorphism on the elliptic part.
It remains to treat the hyperbolic part. Since X is proper it follows from the equivariant Hochschild-Kostant-Rosenberg theorem 6.1 and proposition 7.4 that the hyperbolic part $\Omega_G(X)_{\mathrm{hyp}}$ of $\Omega_G(X)$ is acyclic with respect to the Hochschild boundary. Hence the associated subquotients $L^j\Omega_G(X)_{\mathrm{hyp}}$ of the Hodge filtration are covariantly contractible paracomplexes. We can now proceed as above to obtain

$$H_*(\mathfrak{Hom}_G(Q_{\mathrm{hyp}}, \theta\Omega_G(X)_{\mathrm{hyp}})) = 0.$$

It follows that the map

$$f : H_*(\mathfrak{Hom}_G(Q_{\mathrm{hyp}}, \mathfrak{Fine}(\theta\Omega_G(X))_{\mathrm{hyp}})) \to H_*(\mathfrak{Hom}_G(Q_{\mathrm{hyp}}, \mathcal{A}_c(\hat{X})_{\mathrm{hyp}}))$$

is an isomorphism because both sides are zero. This shows that f is an isomorphism on the hyperbolic part. □

The assumption on X being proper was only used to prove the isomorphism of the hyperbolic parts in proposition 7.5. Hence we obtain the following general statement for the elliptic part.

Proposition 7.9. *Let X be any finite dimensional locally finite G-simplicial complex. Then the restriction of the map f to the elliptic part is an isomorphism.*

Now we study the map g from above.

Proposition 7.10. *The map g is an isomorphism for any finite dimensional locally finite G-simplicial complex X.*

Proof. Recall from section 5 that $P(\hat{X})$ can be viewed as an unbounded mixed complex with b-boundary equal to zero and B-boundary equal to the differential δ. By definition, the supercomplex $P(\hat{X})$ is the projective system of supercomplexes $\xi P(\hat{X}) = (\xi^n P(\hat{X}))$ given by

$$\xi^n P(\hat{X}) = P(\hat{X})_{-(n+1)}/BP(\hat{X})_{-(n+2)} \oplus \bigoplus_{i=-n}^{n} P(\hat{X})_i \oplus B(P(\hat{X})_i).$$

Since $P(\hat{X})$ is bounded above in the sense that $P(\hat{X})_n = 0$ for $n > D = \dim(\hat{X})$ we obtain

$$\xi^n P(\hat{X}) = P(\hat{X})_{-(n+1)}/BP(\hat{X})_{-(n+2)} \oplus \bigoplus_{i=-n}^{D} P(\hat{X})_i$$

for $n > D$. We define the Hodge filtration F^j of $\xi^n P(\hat{X})$ for $n > D$ by

$$F^j\xi^n P(\hat{X}) = P(\hat{X})_{-(n+1)}/B(P(\hat{X})_{-(n+2)}) \oplus \bigoplus_{i=-n}^{j} P(\hat{X})_i \oplus B(P(\hat{X})_j).$$

Hence $F^j\xi^n P(\hat{X})$ is a finite increasing filtration such that $F^{-(n+2)}\xi^n P(\hat{X}) = 0$ and $F^n\xi^n P(\hat{X}) = \xi^n P(\hat{X})$. If we proceed in the same way for $\mathcal{A}_c(\hat{X})$ we see that the map $p : P^\bullet(\hat{X}) \to \mathcal{A}_c^\bullet(\hat{X})$ induces chain maps $\xi^n P(\hat{X}) \to \xi^n \mathcal{A}_c(\hat{X})$ which are compatible with the filtrations. By construction, the map p is a quasiisomorphism with respect to the boundary B. If we denote again by L^j the subquotients of the Hodge filtration it follows that $p : L^j\xi^n P(\hat{X}) \to L^j\xi^n \mathcal{A}_c(\hat{X})$ is a quasiisomorphism for each j and $n > D$. Hence the corresponding mapping cone C^j is acyclic. Since Q is relatively projective we see in the same way as in the proof of proposition 7.5 that the map

$$H_*(\mathfrak{Hom}_G(Q, \xi^n P(\hat{X}))) \to H_*(\mathfrak{Hom}_G(Q, \xi^n \mathcal{A}_c(\hat{X})))$$

is an isomorphism for $n > D$.

For an unbounded mixed complex M we set $M^n = \mathfrak{Hom}_G(Q, \xi^n M)$. Since for $M = P(\hat{X})$ and $M = \mathcal{A}_c(\hat{X})$ the projective system $\xi M = (\xi^n M)_{n\in\mathbb{N}}$ is isomorphic to the projective systems $(\xi^n M)_{n>D}$ we obtain as in the proof of proposition 7.5 long exact sequences

$$\begin{array}{ccccc} H_0(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta M)) & \longrightarrow & H_0(\prod_{n\in\mathbb{N}} M^n) & \longrightarrow & H_0(\prod_{n\in\mathbb{N}} M^n) \\ \uparrow & & & & \downarrow \\ H_1(\prod_{n\in\mathbb{N}} M^n) & \longleftarrow & H_1(\prod_{n\in\mathbb{N}} M^n) & \longleftarrow & H_1(\mathfrak{Hom}_G(Q_{\mathrm{ell}}, \theta M)) \end{array} .$$

Comparing these exact sequences for $P(\hat{X})$ and $\mathcal{A}_c(\hat{X})$ proves the claim. □
Propositions 7.5, 7.9 and 7.10 yield the following theorem.

Theorem 7.11. *Let X be a finite dimensional locally finite G-simplicial complex. If the action of G on X is proper the paracomplexes $\mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))$ and $P(\hat{X})$ are covariantly homotopy equivalent.*
The elliptic parts of $\mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))$ and $P(\hat{X})$ are covariantly homotopy equivalent even if the action of G on X is not necessarily proper.

Proof. We consider only the case that X is proper since the second assertion is proved in the same way. Denote by f_1 respectively f_2 the isomorphism f for $Q = P(\hat{X})$ and $Q = \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))$. Similarly, denote by g_1 respectivly g_2 the isomorphism g for $Q = \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))$ and $Q = P(\hat{X})$. Now let x be the preimage of $[p]$ under the isomorphism f_1 and let y be the preimage of $[q]$ under the isomorphism g_1. Then we have $f_1(x) = x \cdot [q] = [p]$ and $g_1(y) = y \cdot [p] = [q]$. Hence $g_2(x{\cdot}y) = x{\cdot}y{\cdot}[p] = [p]$ and $f_2(y{\cdot}x) = y{\cdot}x{\cdot}[q] = [q]$. Since g_2 and f_2 are isomorphisms we obtain $x \cdot y = \mathrm{id}$ and $y \cdot x = \mathrm{id}$. This implies that $\mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G))$ and $P(\hat{X})$ are covariantly homotopy equivalent. □
Now we finish the proof of theorem 7.1. Again we will restrict ourselves to the case that X is a proper G-simplicial complex.
Using proposition 5.5 and proposition 7.9 we obtain an isomorphism

$$\bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y) \cong H_*(\mathfrak{Hom}_G(P(\hat{X}), \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(Y)\hat{\otimes}\mathcal{K}_G)))).$$

Remark that the hyperbolic part of the Hom-complex vanishes independently of the fact that the action of G on Y may not be proper. We apply theorem 7.11 to deduce

$$\begin{aligned} H_*(\mathfrak{Hom}_G(P(\hat{X}), \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(Y)\hat{\otimes}\mathcal{K}_G)))) \\ \cong H_*(\mathfrak{Hom}_G(\mathfrak{Fine}(\theta\Omega_G(C_c^\infty(X)\hat{\otimes}\mathcal{K}_G)), \mathfrak{Fine}(\theta\Omega_G(C_c^\infty(Y)\hat{\otimes}\mathcal{K}_G)))). \end{aligned}$$

Consequently we have an isomorphism

$$\bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X,Y) \cong h_*^G(C_c^\infty(X), C_c^\infty(Y))$$

where h_*^G denotes the bivariant homology theory introduced before proposition 7.2. Combining this with corollary 7.3 we obtain the desired identification of equivariant periodic cyclic homology with the theory of Baum and Schneider. This finishes the proof of theorem 7.1.
Remark that throughout this discussion we used frequently the assumption on X being a simplicial complex. In contrast, it is mainly for convenience to require Y to be a simplicial complex. As soon as an analogue of the equivariant Hochschild-Kostant-Rosenberg theorem for Y is available, the proof presented above can be easily adapted to other classes of spaces. We consider briefly the following situation. Let G be a discrete group and let M be a G-manifold, that is, a smooth manifold on which G acts by diffeomorphisms. Let $\mathcal{A}_c(M)$ be the space of smooth differential forms with compact supports on M in the usual sense. Then the equivariant Hochschild-Kostant-Rosenberg map $\alpha : \Omega_G(C_c^\infty(M))_{\mathrm{ell}} \to \mathcal{A}_c(\hat{M})$ can be defined as in section 6 and one has the following result.

Theorem 7.12. *Let G be a discrete group and let M be a G-manifold. The equivariant Hochschild-Kostant-Rosenberg map*

$$\alpha : \Omega_G(C_c^\infty(M))_{\mathrm{ell}} \to \mathcal{A}_c(\hat{M})$$

induces an isomorphism on homology with respect to the Hochschild boundary.

We obtain the following variant of theorem 7.1 in the case of discrete groups.

Theorem 7.13. *Let G be a discrete group and let X be a proper finite dimensional and locally finite G-simplicial complex. Moreover let M be a G-manifold. Then there exists a natural isomorphism*

$$HP_*^G(C_c^\infty(X), C_c^\infty(M)) \cong \bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(X, M).$$

Let us define the equivariant cohomology with G-compact supports of a locally compact G-space Y by

$$H^*(\underline{E}G; Y) = \varinjlim_{K\subset \underline{E}G} \bigoplus_{j\in\mathbb{Z}} H_G^{*+2j}(K, Y)$$

where the limit is taken over all G-finite subcomplexes K of $\underline{E}G$. Here $\underline{E}G$ is the universal example for proper actions [3] which can be chosen to be a simplicial complex. In [4] Baum and Schneider show that there is a canonical isomorphism

$$\bigoplus_{j\in\mathbb{Z}} H^{*+2j}(\underline{E}G; M) \cong H^*(M, G)$$

for all discrete groups G and G-manifolds M. Here

$$H^*(M, G) = \bigoplus_{j\in\mathbb{Z}} H_{2j+*}(EG \times_G T\hat{M}, (EG \times_G T\hat{M}) \setminus \{0\}; \mathbb{C})$$

are the equivariant cohomology groups introduced by Baum and Connes [2]. Using theorem 7.13 we see that the theory of Baum and Connes can be expressed in terms of equivariant cyclic homology.

8. Bredon homology and cosheaf homology

In this section we review the definitions of equivariant Bredon homology [7], [13] and cosheaf homology [3] and compare these theories. Throughout we work with coefficients in the complex numbers.

Let us begin with Bredon homology. The smooth orbit category $\mathsf{Or}(G)$ of a totally disconnected group G has as objects all homogenous spaces G/H where H is an open subgroup of G. The morphisms in $\mathsf{Or}(G)$ are all G-equivariant maps. One can also consider subcategories of $\mathsf{Or}(G)$ by restricting the class of subgroups. We are interested in the class $\mathcal{F}$ of all compact open subgroups of G. The corresponding full subcategory $\mathsf{Or}(G, \mathcal{F})$ of $\mathsf{Or}(G)$ consists of all homogeneous spaces G/H where H is compact open.

If $\mathcal{C}$ is a small category a covariant (contravariant) $\mathcal{C}$-vector space is a covariant (contravariant) functor from $\mathcal{C}$ to the category of vector spaces. Morphisms of $\mathcal{C}$-vector spaces are natural transformations. More generally one defines covariant and contravariant $\mathcal{C}$-objects as functors with values in arbitrary target categories. If G is a discrete group viewed as a category with one object then a G-vector space is simply a complex representation of G. In the sequel we will have to work with certain $\mathsf{Or}(G, \mathcal{F})$-vector-spaces and $\mathsf{Or}(G, \mathcal{F})$-chain complexes.

Given a contravariant $\mathcal{C}$-vector space M and a covariant $\mathcal{C}$-vector space N the tensor product $M \otimes_{\mathcal{C}} N$ is the direct sum of $M(c) \otimes N(c)$ over all objects $c \in \mathcal{C}$ divided by all tensor relations $mf \otimes n - m \otimes fn$ for $m \in M(d), n \in N(c)$ and morphisms $f : c \to d$ in $\mathcal{C}$.

A natural domain of definition for equivariant Bredon homology is the category of G-CW-complexes. Throughout we will assume that all G-CW-complexes are smooth in the sense that isotropy groups are open. The basic building blocks for these spaces are equivariant cells $G/H \times D^k$ where H is an open subgroup of G. For detailed information see [12].

Let X be a proper G-CW-complex. If $H \subset G$ is an open subgroup we have a canonical identification $X^H = \mathrm{Map}_G(G/H, X)$ of the H-fixed point set where Map_G denotes the space of all equivariant continuous maps. Using this description of fixed point sets we see that one obtains a contravariant functor from $\mathsf{Or}(G,\mathcal{F})$ to the category of CW-complexes which associates to G/H the fixed point set X^H. Composition with the covariant functor from CW-complexes to chain complexes which associates to a CW-complex Y the cellular chain complex $C_*(Y)$ with complex coefficients yields a contravariant $\mathsf{Or}(G,\mathcal{F})$-chain complex $C_*^{\mathsf{Or}(G,\mathcal{F})}(X)$.
Next we define a covariant $\mathsf{Or}(G,\mathcal{F})$-vector space $\mathcal{R}_q$ as follows. For a compact open subgroup H of G set

$$\mathcal{R}_q(G/H) = K_q(C^*(H)) \otimes_{\mathbb{Z}} \mathbb{C}$$

where K_* denotes topological K-theory and $C^*(H)$ is the group C^*-algebra of H. Note that $K_0(C^*(H)) = R(H)$ is the representation ring of H and $K_1(C^*(H)) = 0$. For a compact totally disconnected group H the character map induces an isomorphism

$$K_0(C^*(H)) \otimes_{\mathbb{Z}} \mathbb{C} \cong \mathcal{R}(H)$$

where $\mathcal{R}(H)$ is the ring of conjugation invariant smooth functions on H.
We define a chain complex $C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0)$ by equipping

$$C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0) = \bigoplus_{p=0}^{\infty} C_p^{\mathsf{Or}(G,\mathcal{F})}(X) \otimes_{\mathsf{Or}(G,\mathcal{F})} \mathcal{R}_0$$

with the differential induced from $C_*^{\mathsf{Or}(G,\mathcal{F})}(X)$. The Bredon homology of X with coefficients in $\mathcal{R}_0$ is the homology

$$H_n^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0) = H_n(C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0))$$

for $n\ in\mathbb{Z}$. Similarly, we define a $\mathbb{Z}_2$-graded chain complex $C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R})$ by considering

$$C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}) = \bigoplus_{p+q=*} C_p^{\mathsf{Or}(G,\mathcal{F})}(X) \otimes_{\mathsf{Or}(G,\mathcal{F})} \mathcal{R}_q$$

again with the differential coming from $C_*^{\mathsf{Or}(G,\mathcal{F})}(X)$.

Definition 8.1. *Let G be a totally disconnected group and let X be a proper G-CW-complex. The equivariant Bredon homology of X (with coefficients in $\mathcal{R}$) is*

$$\mathcal{B}H_*^G(X) = H_*(C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R})).$$

Next we recall the definition of cosheaf homology. Let X be a simplicial complex. We view X as a category whose objects are the simplices of X and whose morphisms are inclusions of simplices. A cosheaf $\mathcal{A}$ on X is a contravariant functor from X to the category of complex vector spaces. More concretely, a cosheaf $\mathcal{A}$ is specified by vector spaces $\mathcal{A}(\sigma)$ for every simplex $\sigma \subset X$ and linear maps $\alpha_\sigma^\eta : \mathcal{A}(\sigma) \to \mathcal{A}(\eta)$ for every inclusion $\eta \subset \sigma$. These maps are required to satisfy $\alpha_\sigma^\tau = \alpha_\eta^\tau \alpha_\sigma^\eta$ whenever $\tau \subset \eta \subset \sigma$ and $\alpha_\sigma^\sigma = \mathrm{id}$ for every simplex σ.
Now let G be a totally disconnected group and let X be a proper G-simplicial complex. We are interested in the following cosheaf $\mathcal{R}_X$ on X. For a simplex $\sigma \subset X$ we define $\mathcal{R}_X(\sigma) = R(G_\sigma) \otimes_{\mathbb{Z}} \mathbb{C} \cong \mathcal{R}(G_\sigma)$ where G_σ denotes the stabilizer of σ. If $\eta \subset \sigma$ is a face the map $\iota_\sigma^\eta : \mathcal{R}(G_\sigma) \to \mathcal{R}(G_\eta)$ is given by induction.
Let us define a complex $S_*(X;\mathcal{R}_X)$ as follows. We set

$$S_n(X;\mathcal{R}_X) = \left(\bigoplus_{\dim(\sigma)=n} \mathcal{R}_X(\sigma) \right) / (f[\sigma] - f[-\sigma] \text{ for } f \in \mathcal{R}_X(\sigma))$$

where the sum is taken over all oriented simplices of X and $f[\sigma]$ is our notation for $f \in \mathcal{R}_X(\sigma)$ viewed as element in the summand corresponding to σ. Moreover $-\sigma$ denotes the oriented simplex σ equipped with the opposite orientation. The boundary $\partial : S_n(X;\mathcal{R}_X) \to S_{n-1}(X;\mathcal{R}_X)$ is defined by

$$\partial(f[\sigma]) = \sum_{\substack{\eta\subset\sigma \\ \dim(\eta)=\dim(\sigma)-1}} \iota_\sigma^\eta(f)[\eta].$$

The group G acts on $S_*(X;\mathcal{R}_X)$ in a natural way. We let $S_*^G(X;\mathcal{R}_X)$ denote the complex of coinvariants obtained from $S_*(X;\mathcal{R}_X)$.

Definition 8.2. *Let G be a totally disconnected group and let X be a proper G-simplicial complex. The cosheaf homology of X (with values in $\mathcal{R}_X$) is*

$$\mathcal{C}H_*^G(X) = H_*(C_*^G(X;\mathcal{R}_X)).$$

Let X be a G-simplicial complex. Retaining the notation from above we define a map $\phi : S_*^G(X;\mathcal{R}_X) \to C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0)$ by

$$\phi(f[\sigma]) = [\sigma](G_\sigma) \otimes f(G_\sigma)$$

where we indicate by the brackets on the right hand side of this equation that the tensor is viewed as an element in $C_*^{\mathsf{Or}(G,\mathcal{F})}(X^{G_\sigma}) \otimes \mathcal{R}(G_\sigma)$ which maps canonically into the target of ϕ.

Proposition 8.3. *Let G be a totally disconnected group and let X be a proper G-simplicial complex. The map $\phi : S_*^G(X;\mathcal{R}_X) \to C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0)$ defined above is an isomorphism of chain complexes.*

Proof. First of all one checks that ϕ vanishes on coinvariants and hence yields a well-defined map. We compute

$$\begin{aligned}\phi\partial(f[\sigma]) = \phi\left(\sum_{\eta\subset\sigma} \iota_\sigma^\eta(f)\right) &= \sum_{\eta\subset\sigma} \eta(G^\eta) \otimes \iota_\sigma^\eta(f)(G^\eta) \\ &= \sum_{\eta\subset\sigma} \eta(G^\sigma) \otimes f(G^\sigma) = \partial([\sigma](G^\sigma) \otimes f(G^\sigma)) = \partial\phi(f[\sigma])\end{aligned}$$

for a simplex $\sigma \subset X$ and $f \in \mathcal{R}(G^\sigma)$. Here we use the fact that the simplicial and the cellular chain complex of a simplicial complex can be identified. It follows that ϕ is a chain map. Let us define a map $\psi : C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0) \to S_*^G(X;\mathcal{R}_X)$ by

$$\psi([\sigma](H) \otimes f(H)) = \mathrm{ind}_H^{G_\sigma}(f)[\sigma].$$

It is straightforward to check that ψ is well-defined and by definition we have $\psi\phi = \mathrm{id}$. We calculate

$$\phi\psi([\sigma](H) \otimes f(H)) = [\sigma](G^\sigma) \otimes \mathrm{ind}_H^{G^\sigma}(f)(G^\sigma) = [\sigma](H) \otimes f(H)$$

and conclude $\phi\psi = \mathrm{id}$. This finishes the proof. □

Corollary 8.4. *Let G be a totally disconnected group and let X be a proper G-simplicial complex. Then the Bredon homology $H_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}_0)$ of X is naturally isomorphic to the cosheaf homology $\mathcal{C}H_*^G(X)$.*

Combining this with theorem 7.1 and results from [4] we obtain the following statement.

Theorem 8.5. *Let X be a proper G-simplicial complex. Then there is a natural isomorphism*

$$\varinjlim_{K\subset X} HP_*^G(C_c^\infty(K),\mathbb{C}) \cong \bigoplus_{j\in\mathbb{Z}} \mathcal{C}H_{*+2j}^G(X) \cong \mathcal{B}H_*^G(X)$$

where the limit on the left hand side is taken over all G-finite subcomplexes of X.

References

[1] Baum, P., Connes, A., Geometric K-theory for Lie groups and foliations, Preprint IHES (1982)

[2] Baum, P., Connes, A., Chern character for discrete groups, in: A fête of topology, 163 - 232, Academic Press, 1988

[3] Baum, P., Connes, A., Higson, N., Classifying space for proper actions and K-theory of group C^*-algebras, in C^*-algebras: 1943 - 1993 (San Antonio, TX, 1993), 241 - 291, Contemp. Math. 167, 1994

[4] Baum, P., Schneider, P., Equivariant bivariant Chern character for profinite groups, K-theory 25 (2002), 313 - 353

[5] Bernstein, J., Lunts, V., Equivariant sheaves and functors, Lecture Notes in Mathematics 1578, Springer, 1994

[6] Bernstein, J., Zelevinskii, A., Representations of the group $GL(n, F)$ where F is a local non-archimedian field, Russian Math. Surveys 31 (1976), 1 - 68

[7] Bredon, G., Equivariant cohomology theories, Lecture Notes in Mathematics 34, Springer, 1967

[8] Bredon, G., Sheaf theory, second edition, Graduate Texts in Mathematics 170, Springer, 1997

[9] Connes, A., Noncommutative differential geometry, Publ. Math. IHES 39 (1985), 257 - 360

[10] Connes, A., Noncommutative Geometry, Academic Press, 1994

[11] Higson, N., Nistor, V., Cyclic homology of totally disconnected groups acting on buildings, J. Funct. Anal. 141 (1996) no. 2, 466 - 495

[12] Lück, W., Transformation groups and algebraic K-theory, Lecture Notes in Mathematics 1408, Springer, 1989

[13] Lück, W., Chern characters for proper equivariant homology theories and applications to K- and L-theory, J. Reine Angew. Math. 543 (2002), 193 - 234

[14] Meyer, R., Analytic cyclic cohomology, Preprintreihe SFB 478, Geometrische Strukturen in der Mathematik, Münster, 1999

[15] Meyer, R., Smooth group representations on bornological vector spaces, Bull. Sci. Math. 128 (2004), 127 - 166

[16] Spanier, E. H., Algebraic Topology, McGraw-Hill, 1966

[17] Teleman, N., Microlocalisation de l'homologie de Hochschild, C. R. Acad. Sci. Paris 326 (1998), 1261 - 1264

[18] Voigt, C., Equivariant cyclic homology, Preprintreihe SFB 478, Geometrische Strukturen in der Mathematik, Münster, 2003

[19] Voigt, C., Equivariant periodic cyclic homology, arXiv:math.KT/0412021 (2004), to appear in J. Inst. Math. Jussieu

[20] Voigt, C., Chern character for totally disconnected groups, arXiv:math.KT/0608626 (2006)

[21] Weibel, C. A., An introduction to homological algebra, Cambridge Studies in Advanced Mathematics 38, Cambridge University Press, 1994

[22] Willis, G. A., Totally disconnected groups and proofs of conjectures of Hofmann and Mukherjea, Bull. Austral. Math. Soc. 51 (1995), 489 - 494

[23] Wodzicki, M., The long exact sequence in cyclic homology associated with an extension of algebras, C. R. Acad. Sci. Paris 306 (1988), 399 - 403

[24] Wodzicki, M., Excision in cyclic homology and in rational algebraic K-Theory, Ann. of Math. 129 (1989), 591 - 639

Mathematisches Institut, Westfälische Wilhelms-Universität Münster, Einsteinstr. 62, 48149 Münster, Germany

E-mail address: cvoigt@math.uni-muenster.de