CHERN CHARACTER FOR TOTALLY DISCONNECTED GROUPS

CHRISTIAN VOIGT

ABSTRACT. In this paper we construct a bivariant Chern character for the equivariant KK-theory of a totally disconnected group with values in bivariant equivariant cohomology in the sense of Baum and Schneider. We prove in particular that the complexified left hand side of the Baum-Connes conjecture for a totally disconnected group is isomorphic to cosheaf homology. Moreover, it is shown that our transformation extends the Chern character defined by Baum and Schneider for profinite groups.

1. INTRODUCTION

Let G be a second countable locally compact group. The Baum-Connes conjecture [4] asserts that the K-theory of the reduced group C^*-algebra $C^*_{\mathsf{red}}(G)$ of G is isomorphic to the equivariant K-homology of the universal proper G-space $\underline{E}G$. More precisely, the assembly map

$$\mu : K^G_*(\underline{E}G) \to K_*(C^*_{\mathsf{red}}(G))$$

is conjectured to be an isomorphism. In this paper we construct an equivariant Chern character for the left hand side of the assembly map in the case that the group G is totally disconnected. We show in particular that the complexified equivariant K-homology of $\underline{E}G$ for a totally disconnected group G is given by cosheaf homology [4].

The corresponding result for discrete groups has been obtained independently by Baum and Connes [3] and Lück [18]. The approach of Lück actually yields Chern characters for arbitrary equivariant homology theories with source given by associated Bredon homologies. For equivariant K-homology and group actions on simplicial complexes, the corresponding Bredon homology with complex coefficients is naturally isomorphic to cosheaf homology. However, it seems to be unclear wether the case of totally disconnected groups can still be handled in the framework of Lück. We remark that the result for totally disconnected groups stated above has also been obtained by Baum, Block and Higson in an unpublished paper [1].

In fact we prove a more general statement. The target of our Chern character is bivariant equivariant cohomology in the sense of Baum and Schneider [5]. One virtue of this cohomology theory is that it unifies and generalizes previous constructions in the literature. In particular, the theory of Baum and Schneider contains the cosheaf homology groups mentioned above as a special case.

The main result of this paper is the construction of a bivariant Chern character

$$ch^G_* : KK^G_*(C_0(X), C_0(Y)) \to \bigoplus_{j\in\mathbb{Z}} H^{*+2j}_G(X, Y)$$

from equivariant KK-theory with values in the theory of Baum and Schneider. Here G is a totally disconnected group, X and Y are finite dimensional locally finite G-simplicial complexes in the sense of [32] and X is assumed to be proper

and G-finite. This Chern character becomes an isomorphism after tensoring the left hand side with the complex numbers. We also discuss how the character can be extended to G-CW-complexes. The restriction to simplicial complexes is necessary in order to apply the machinery of equivariant cyclic homology [30]. In fact, equivariant cylic homology is the main ingredient in our construction.

We remark that Baum and Schneider conjectured the existence of such a bivariant Chern character in [5] and provided a construction in the case of profinite groups. However, this construction does not extend to noncompact groups. Thus, in some sense, the present paper completes the work of Baum and Schneider.

An important ingredient in the construction of Lück is the induction structure for an equivariant homology theory [18]. As already mentioned above, the approach presented here is based on equivariant cyclic homology. In turn, for the construction of our Chern character we need some information on the compatibility of equivariant cyclic homology with induction. From a technical point of view the corresponding considerations constitute the main part of this paper. Apart from this, the constructions and results from [31] play a central role. In fact, our Chern character is obtained by combining the equivariant Chern-Connes character from [31] with the description of bivariant equivariant cohomology in terms of cyclic homology given in [32].

Let us explain how the paper is organized. In section 2 we recall some facts about smooth representations of totally disconnected groups and anti-Yetter-Drinfeld modules. Smooth representations and anti-Yetter-Drinfeld modules are a basic ingredient in the definition of equivariant cyclic homology and the theory of Baum and Schneider. Section 3 contains a brief survey of the different variants of equivariant cyclic homology involved in the construction of the Chern character. The basic theory of restriction and induction of G-modules, G-algebras and anti-Yetter-Drinfeld modules is discussed in section 4. In section 5 this is used to study induction in equivariant cyclic homology. Section 6 contains the construction of the equivariant Chern character for G-simplicial complexes. This yields the main result stated above. In section 7 we show that our Chern character restricts to the natural transformation defined by Baum and Schneider in the case of profinite groups. Finally, in section 8 we explain how to extend the homological Chern character arising from our constructions to proper G-CW-complexes and discuss the relation to the Chern character constructed by Lück.

I would like to thank W. Lück for some helpful comments.

2. Smooth representations and anti-Yetter-Drinfeld modules

Throughout this paper let G be a second countable locally compact and totally disconnected group. As in [32] we call an element $t \in G$ elliptic if it is contained in a compact subgroup. The set of all elliptic elements of G is denoted by G_{ell}. In contrast we shall say that an element $t \in G$ is hyperbolic if it is not elliptic. Let G_{hyp} be the set of all hyperbolic elements of G. Hence, according to these definitions, we obtain a disjoint union decomposition

$$G = G_{\mathrm{ell}} \cup G_{\mathrm{hyp}}$$

of the space G. It follows from the structure theory developed in [33] that G_{ell} is a closed subset of G and that G_{ell} is the union of all compact open subgroups of G. Consequently G_{hyp} is again an open and closed subset of G.

In this section we review some facts about smooth representations of totally disconnected groups on bornological vector spaces and anti-Yetter-Drinfeld modules. For more information on bornological vector spaces and smooth representations we

refer to [11], [12], [20], [23]. Concerning anti-Yetter-Drinfeld modules further details can be found in [30], [32] where these objects were called covariant modules instead. All bornological vector spaces in this paper are assumed to be convex and separated. The completion of a bornological vector space V is denoted by V^c.

A representation of G on a bornological vector space V is a group homomorphism $\pi : G \to \mathrm{Aut}(V)$ where $\mathrm{Aut}(V)$ denotes the group of bounded linear automorphisms of V. A bounded linear map $f : V \to W$ between representations of G is called equivariant if $f(s \cdot v) = s \cdot f(v)$ for all $s \in G, v \in V$. To a representation $\pi : G \to \mathrm{Aut}(V)$ one associates the linear map $[\pi] : V \to F(G, V)$ defined by $[\pi](v)(t) = \pi(t)(v)$ where $F(G, V)$ denotes the space of all functions from G to V. We denote by $\mathcal{E}(G, V)$ the space of all locally constant functions on G with values in V.

Definition 2.1. *Let V be a separated bornological vector space. A representation π of G on V is called smooth if $[\pi]$ defines a bounded linear map from V into $\mathcal{E}(G, V)$. A separated (complete) G-module is a smooth representation of V on a separated (complete) bornological vector space.*

If V is a separated G-module the stabilizers of small subsets of V are open subgroups of G. Conversely, if π is a representation on a bornological vector space V such that the stablizers of all small subsets are open it follows that π is a G-module. In particular, if V carries the fine bornology one recovers the usual notion of a smooth representation on a complex vector space.

The Hecke algebra of the group G is the space $\mathcal{D}(G)$ of locally constant functions with compact support on G equipped with the convolution product with respect to a fixed Haar measure. Every separated G-module V becomes a module over $\mathcal{D}(G)$ using integration. Moreover, the $\mathcal{D}(G)$-module V obtained in this way is essential in the sense that the canonical map $\mathcal{D}(G) \otimes_{\mathcal{D}(G)} V \to V$ is an isomorphism. Conversely, every essential $\mathcal{D}(G)$-module is the integrated form of a smooth representation.

For an arbitrary representation of G on a separated bornological vector space V there exists a smoothing $\mathfrak{Smooth}(V)$ which is a smooth representation. This construction yields a functor $\mathfrak{Smooth}$ that is right adjoint to the natural forgetful functor from the category of G-modules to the category of arbitrary representations on bornological vector spaces.

Let us now recall the concept of an anti-Yetter-Drinfeld module. We write $\mathcal{O}_G$ for the vector space $\mathcal{D}(G)$ equipped with pointwise multiplication and the action of G by conjugation.

Definition 2.2. *A separated (complete) anti-Yetter-Drinfeld module M is a separated (complete) G-module which is at the same time an essential $\mathcal{O}_G$-module. The G-module structure and the $\mathcal{O}_G$-module structure are required to be compatible in the sense that*

$$s \cdot (f \cdot m) = (s \cdot f) \cdot (f \cdot m)$$

for all $s \in G, f \in \mathcal{O}_G$ and $m \in M$.

A homomorphism $\xi : M \to N$ of anti-Yetter-Drinfeld modules is a bounded linear map which is both G-equivariant and $\mathcal{O}_G$-linear. In the sequel we will use the abbreviations AYD-module and AYD-map for anti-Yetter-Drinfeld modules and their homomorphisms. We write $\mathfrak{Hom}_G(M, N)$ for the space of AYD-maps between AYD-modules M and N.

The category of AYD-modules is isomorphic to the category of essential $\mathsf{A}(G)$-modules where $\mathsf{A}(G) = \mathcal{O}_G \rtimes G$ is the smooth crossed product of $\mathcal{O}_G$ with respect to the adjoint action. In particular, for any bornological vector space V the space $\mathsf{A}(G) \otimes V$ becomes an AYD-module. The modules arising in this way are projective,

and every projective AYD-module is a direct summand in an AYD-module of this form. An important property of AYD-modules is the fact that they are equipped with a natural automorphism T. In this way the category of AYD-modules becomes a para-additive category in the sense of [30]. For equivariant differential forms we will describe the automorphism T explicitly in section 3 below.

Let us discuss some features of AYD-modules which are specific to the setting of totally disconnected groups. For a totally disconnected group there is a natural decomposition of an AYD-module into an elliptic and a hyperbolic part. Since the group G is the disjoint union of its elliptic and hyperbolic elements one has two natural multipliers P_{ell} and P_{hyp} of the algebra $\mathcal{O}_G$. By definition, the multiplier P_{ell} is the characteristic function of the set G_{ell} whereas $P_{\mathrm{hyp}} = 1 - P_{\mathrm{ell}}$ is the characteristic function of G_{hyp}. For every AYD-module M this yields a natural direct sum decomposition of AYD-modules

$$M = M_{\mathrm{ell}} \oplus M_{\mathrm{hyp}}$$

where $M_{\mathrm{ell}} = P_{\mathrm{ell}} \cdot M$ and $M_{\mathrm{hyp}} = P_{\mathrm{hyp}} \cdot M$. Usually, the behaviour of the operator T is quite different on the two summands in this decomposition. For instance, for every element $m \in M_{\mathrm{ell}}$ there exists a natural number n such that $T^n(m) = m$. Using this fact one constructs an AYD-map $E : M_{\mathrm{ell}} \to M_{\mathrm{ell}}$ which yields a projection onto the T-invariant elements in M_{ell}. In contrast, if M is a projective AYD-module then the operator $\mathrm{id} - T$ is injective on the hyperbolic part M_{hyp} of M. Moreover $\mathrm{id} - T : M_{\mathrm{hyp}} \to (\mathrm{id} - T) M_{\mathrm{hyp}}$ is an isomorphism in this case.

If M is an AYD-module we write $M \otimes_T \mathbb{C}$ for the space of T-coinvariants, that is, the quotient of M by the closed subspace generated by all elements $T(m) - m$ for $m \in M$. Note that the action of T on M can be viewed as a module structure of the algebra $R = \mathbb{C}[z, z^{-1}]$ of Laurent polynomials. Then $M \otimes_T \mathbb{C} = M \otimes_R \mathbb{C}$ where $\mathbb{C}$ is viewed as a module over R using the character given by evaluation at 1. Remark that $M \otimes_T \mathbb{C}$ is again an AYD-module in a natural way. Every AYD-map $M \to N$ induces an AYD-map $M \otimes_T \mathbb{C} \to N \otimes_T \mathbb{C}$ on T-coinvariants.

Finally, for local cyclic homology the concept of a primitive AYD-module is important [31]. An AYD-module P is called primitive if there is a small disk $S \subset P$ such that the natural map $\mathsf{A}(G) \otimes \langle S \rangle \to P$ is a quotient map. Every AYD-module M can be written in a canonical way as direct limit of the primitive modules $\mathsf{A}(G)\langle S \rangle$ generated by small disks S in M. The relation between inductive systems of primitive modules and general AYD-modules is analogous to the one between inductive systems of normed spaces and separated bornological vector spaces.

3. Equivariant periodic, analytic and local cyclic homology

In this section we review the definition of equivariant periodic, analytic and local cyclic homology. These theories are studied in detail in [30], [31].

A separated G-algebra is a separated bornological algebra A which is also a G-module such that the multiplication map $A \otimes A \to A$ is equivariant. The unitarization of a G-algebra A is denoted by A^+ and again a G-algebra in a natural way where the group acts trivially on scalar multiples of the unit element $1 \in A^+$.

Let A be a separated G-algebra. The equivariant n-forms of A are defined by $\Omega_G^n(A) = \mathcal{O}_G \otimes \Omega^n(A)$ where $\Omega^n(A) = A^+ \otimes A^{\otimes n}$ is the space of noncommutative differential forms over A. The group G acts diagonally on $\Omega_G^n(A)$ and there is an obvious $\mathcal{O}_G$-module structure given by multiplication. In this way $\Omega_G^n(A)$ becomes a separated AYD-module. We write $\Omega_G(A)$ for the direct sum of the spaces $\Omega_G^n(A)$. On equivariant differential forms there are two important boundary operators. The

equivariant Hochschild boundary $b : \Omega_G^n(A) \to \Omega_G^{n-1}(A)$ is defined by

$$b(f(s) \otimes x_0 dx_1 \cdots dx_n) = f(t) \otimes x_0 x_1 dx_2 \cdots dx_n$$

$$+ \sum_{j=1}^{n-1} (-1)^j f(s) \otimes x_0 dx_1 \cdots d(x_j x_{j+1}) \cdots dx_n$$

$$+ (-1)^n f(s) \otimes (s^{-1} \cdot x_n) x_0 dx_1 \cdots dx_{n-1}.$$

Moreover we have the equivariant Connes operator $B : \Omega_G^n(A) \to \Omega_G^{n+1}(A)$ which is given by

$$B(f(s) \otimes x_0 dx_1 \cdots dx_n) = \sum_{i=0}^{n} (-1)^{ni} f(s) \otimes s^{-1} \cdot (dx_{n+1-i} \cdots dx_n) dx_0 \cdots dx_{n-i}.$$

Both b and B are AYD-maps and the natural isomorphism T for AYD-modules has the form

$$T(f(s) \otimes \omega) = f(s) \otimes s^{-1} \cdot \omega$$

on $\Omega_G^n(A)$. The space of equivariant differential forms together with the operators b and B forms a paramixed complex [30] which means that the relations $b^2 = 0$, $B^2 = 0$ and $[b, B] = bB + Bb = \mathrm{id} - T$ hold.

Equivariant differential forms are used to define the equivariant X-complex of a separated G-algebra.

Definition 3.1. *Let A be a separated G-algebra. The equivariant X-complex $X_G(A)$ of A is*

$$X_G(A): \ \Omega_G^0(A) \ \underset{b}{\overset{B}{\rightleftarrows}} \ \Omega_G^1(A)/b(\Omega_G^2(A)).$$

If ∂ denotes the boundary operator in $X_G(A)$ then $\partial^2 = \mathrm{id} - T$ is in general not zero. Hence the equivariant X-complex is not a chain complex. Instead, it is a paracomplex in the following sense [30].

Definition 3.2. *A paracomplex $C = C_0 \oplus C_1$ is a given by AYD-modules C_0 and C_1 in $\mathcal{C}$ together with AYD-maps $\partial_0 : C_0 \to C_1$ and $\partial_1 : C_1 \to C_0$ such that*

$$\partial^2 = \mathrm{id} - T.$$

A chain map $\phi : C \to D$ between two paracomplexes is an AYD-map from C to D that commutes with the differentials.

In the same way as for ordinary chain complexes one defines homotopies, mapping cones and suspensions. Together with this additional structure the homotopy category of paracomplexes becomes a triangulated category.

In order to obtain the paracomplexes needed for the definition of equivariant cyclic homology one has to insert certain tensor algebras into the equivariant X-complex. Consider the space $\Omega(A)$ of ordinary differential forms over a G-algebra A. By definition, the analytic bornology on $\Omega(A)$ is the bornology generated by the sets

$$S \cup \bigcup_{n=1}^{\infty} S(dS)^n \cup (dS)^n$$

for all small sets $S \subset A$. Unless explicitly stated otherwise, we will always equip $\Omega(A)$ with the analytic bornology. The space $\Omega(A)$ becomes a separated G-algebra with the Fedosov product

$$\omega \circ \eta = \omega \eta - (-1)^{|\omega|} d\omega d\eta$$

for homogenous forms ω and η and the diagonal action of G. By definition, the analytic tensor algebra $\mathcal{T}A$ of A is the subalgebra of differential forms of even degree. The basic ingredient in the definition of periodic, analytic and local cyclic homology is the equivariant X-complex of the tensor algebra.

To obtain interesting homology groups one has to complete this paracomplex in some way. The most evident way of completion, namely the completion of the underlying bornological vector space, leads to the analytic theory. Before giving the general definition of equivariant analytic cyclic homology let us first discuss the completions needed to define the local and the periodic theory, respectively.

For the local theory the ordinary completion is replaced by the derived completion. The derived completion is the left derived functor of the completion functor with respect to the localization of the homotopy category of paracomplexes at the class of locally contractible paracomplexes. A paracomplex C is called locally contractible provided all chain maps $P \to C$ are homotopic to zero whenever P is a primitive paracomplex. By definition, a paracomplex P is called primitive if its underlying AYD-module is primitive. The class of locally contractible paracomplexes forms a null system in the homotopy category of paracomplexes and the corresponding localization is called the local derived category. The derived completion of a paracomplex C is given by

$$C^{\mathbb{L}c} \cong P(C)^c$$

where $P(C) \to C$ is a projective resolution with respect to the class of locally contractible paracomplexes. For the explicit construction of a projective resolution functor we refer to [31]. We remark that there is a natural map $C^{\mathbb{L}c} \to C^c$ for every paracomplex C.

The completion needed for the periodic theory is obtained using quotients of the analytic tensor algebra. More precisely, if A is a separated G-algebra then the natural homomorphism $\tau_A : \mathcal{T}A \to A$ given by the projection onto differential forms of degree zero fits into an extension

$$\mathcal{J}A \rightarrowtail \mathcal{T}A \twoheadrightarrow A$$

of separated G-algebras where $\mathcal{J}A$ denotes the kernel of the map τ_A. The n-th power $(\mathcal{J}A)^n$ of the ideal $\mathcal{J}A$ can be identified with the space of even differential forms of degree greater or equal $2n$. By definition, the periodic tensor algebra of A is the projective system of G-algebras

$$\mathcal{T}A/(\mathcal{J}A)^\infty = (\mathcal{T}A/(\mathcal{J}A)^n)_{n \in \mathbb{N}}$$

which can be viewed as a pro-G-algebra in a natural way. Accordingly, the equivariant X-complex of the periodic tensor algebra is a pro-paracomplex. In the context of pro-modules and pro-paracomplexes, morphisms are always understood in the sense of pro-categories.

We shall now give the definition of equivariant periodic, analytic and local cyclic homology.

Definition 3.3. *Let G be a totally disconnected group and let A and B be separated G-algebras. The equivariant analytic cyclic homology of A and B is*

$$HA_*^G(A, B) = H_*(\mathfrak{Hom}_G(X_G(\mathcal{T}(A \otimes \mathcal{K}_G))^c, X_G(\mathcal{T}(B \otimes \mathcal{K}_G))^c)).$$

The equivariant local cyclic homology $HL_*^G(A, B)$ is obtained by replacing the completion with the left derived completion. For equivariant periodic cyclic homology $HP_*^G(A, B)$ one has to replace instead the analytic tensor algebras by periodic tensor algebras.

Here $\mathcal{K}_G$ is a certain subalgebra of the algebra of compact operators on $L^2(G)$. By definition, it consists of integral operators with kernels in $\mathcal{D}(G \times G)$. We point

out that, despite of the fact that both entries are only paracomplexes, the Hom-complexes occuring on the right hand side are complexes in the usual sense.

It is sometimes useful to extend the previous definition to bornological algebras A and B with an action of G by bounded automorphisms which is not necessarily smooth. This is done simply by replacing A and B on the right hand side by their smoothings $\mathfrak{Smooth}(A)$ and $\mathfrak{Smooth}(B)$, respectively.

For our considerations below it is crucial to compare the paracomplex $X_G(\mathcal{T}A)$ with a certain $B + b$-complex. More precisely, let A be a separated G-algebra and consider the bornology on $\Omega_G(A)$ generated by the sets

$$ D \otimes S \cup D \otimes [S]dS \cup \bigcup_{n=1}^{\infty} n! \, D \otimes [S][dS](dS)^{2n} $$

for small subsets $D \subset \mathcal{O}_G$ and $S \subset A$. Here we write $[R]$ for the set $R \cup \{1\}$ where 1 is the unit in the unitarization. This bornology is called the transposed analytic bornology. We denote by $\Omega_G^{\mathsf{an}}(A)$ the space $\Omega_G(A)$ equipped with the transposed analytic bornology. This space is an AYD-module in a natural way and there is a bornological isomorphism $X_G(\mathcal{T}A) \cong \Omega_G^{\mathsf{an}}(A)$ for every G-algebra A. Moreover the operators b and B induce bounded maps on $\Omega_G^{\mathsf{an}}(A)$. In particular, together with the boundary $B + b$ the space $\Omega_G^{\mathsf{an}}(A)$ becomes a paracomplex. The following theorem is proved in [31].

Theorem 3.4. *For every separated G-algebra A there exists a natural homotopy equivalence between the paracomplexes $X_G(\mathcal{T}A)$ and $\Omega_G^{\mathsf{an}}(A)$.*

As a consequence, in the definition of the analytic and local theory one may replace the equivariant X-complexes by the corresponding $B + b$-complexes. An analogous result holds for the periodic theory [30].

Finally, we remark that according to the decomposition of an AYD-module into an elliptic and a hyperbolic part discussed in section 2 we obtain corresponding decompositions of the paracomplexes defining the different cyclic theories. This will be used in section 5 when we study induction in equivariant cyclic homology.

4. RESTRICTION AND INDUCTION OF MODULES AND ALGEBRAS

In this section we discuss restriction and compact induction of G-modules, G-algebras and AYD-modules. We only consider these constructions in the particular cases which are needed in the sequel. For a detailed account to induction and restriction of smooth representations of locally compact groups we refer to [23]. We remark that in the context of equivariant sheaves for totally disconnected groups these constructions are treated in [26].

We begin with compact induction of smooth representations. Let K be a compact open subgroup of a totally disconnected group G and let V be a separated K-module. The compactly induced module $\mathrm{ind}_K^G(V)$ of V is given by

$$ \mathrm{ind}_K^G(V) = \{f \in \mathcal{D}(G, V) \mid f(ts) = s^{-1} \cdot f(t) \text{ for all } t \in G, s \in K\} $$

where G acts by left translations. Since K is open there exists a natural K-equivariant map $\theta_V : V \to \mathrm{ind}_K^G(V)$ given by $\theta_V(v)(t) = \chi(t)t^{-1} \cdot v$ where χ is the characteristic function of the set K.

One may equivalently describe the compactly induced module as follows. Since K is an open subgroup of G there is an evident map $\mathcal{D}(K) \to \mathcal{D}(G)$ given by extending functions by zero. We may thus view $\mathcal{D}(K)$ as a subalgebra of $D(G)$ if the Haar measure on K is chosen to be the restriction of the Haar measure on G. Then there

is a natural isomorphism $\Phi : \mathcal{D}(G) \otimes_{\mathcal{D}(K)} V \to \mathrm{ind}_K^G(V)$ given by

$$\Phi(f)(t) = \int_K s \cdot f(ts) ds$$

for the class of $f \in \mathcal{D}(G, V) = \mathcal{D}(G) \otimes V$.

If W is a separated G-module then restriction of the action to K yields the structure of a K-module on W. We write $\mathrm{res}_K^G(W)$ for the K-module obtained this way. Compact induction and restriction are related as follows.

Proposition 4.1. *Let K be a compact open subgroup of the totally disconnected group G. Then the map θ_V induces a natural isomorphism*

$$\mathrm{Hom}_G(\mathrm{ind}_K^G(V), W) \cong \mathrm{Hom}_K(V, \mathrm{res}_K^G(W))$$

for all separated K-modules V and separated G-modules W.

Proof. The map $\Psi : \mathrm{Hom}_K(V, \mathrm{res}_K^G(W)) \to \mathrm{Hom}_G(\mathrm{ind}_K^G(V), W)$ defined by

$$\Psi(\phi)(f) = \sum_{t \in G/K} t \cdot \phi(f(t))$$

is inverse to the canonical map $\mathrm{Hom}_G(\mathrm{ind}_K^G(V), W) \to \mathrm{Hom}_K(V, \mathrm{res}_K^G(W))$ induced by θ_V. $\qquad\square$

Remark that induction is compatible with completion in the sense that the natural map $\mathrm{ind}_K^G(V)^c \to \mathrm{ind}_K^G(V^c)$ is an isomorphism for every separated K-module V.

Next we consider induction and restriction for algebras. Let again K be a compact open subgroup of G and let B be a separated K-algebra. By definition, the induced G-algebra is $\mathrm{ind}_K^G(B)$ with pointwise multiplication. The natural map $\theta_B : B \to \mathrm{ind}_K^G(B)$ is a K-equivariant algebra homomorphism. Moreover there is a natural K-equivariant homomorphism $\pi_B : \mathrm{ind}_K^G(B) \to B$ given by $\pi_B(f) = f(e)$ and satisfying $\pi_B \theta_B = \mathrm{id}$. On the other hand every separated G-algebra A is a K-algebra in a natural way by restriction of the action. We will frequently denote this K-algebra again by A instead of $\mathrm{res}_K^G(A)$.

Let us now discuss induction and restriction of AYD-modules. If K is a compact open subgroup of G there is a natural K-equivariant inclusion homomorphism $\mathcal{O}_K \to \mathcal{O}_G$. Moreover we have an algebra homomorphism $\mathcal{D}(K) \to \mathcal{D}(G)$ as explained above. These maps yield an algebra homomorphism $\iota : \mathsf{A}(K) \to \mathsf{A}(G)$. The induced module $\mathrm{ind}_K^G(M)$ of a separated K-AYD-module M is the G-AYD-module defined by

$$\mathrm{ind}_K^G(M) = \mathsf{A}(G) \otimes_{\mathsf{A}(K)} M$$

with the obvious left $\mathsf{A}(G)$-module structure. Conversely, let M be a separated G-AYD-module. We define the restriction $\mathrm{res}_K^G(M)$ of M to K by

$$\mathrm{res}_K^G(M) = \mathcal{O}_K \otimes_{\mathcal{O}_G} M$$

where $\mathcal{O}_K$ is viewed as an $\mathcal{O}_G$-module by restriction of functions. The space $\mathrm{res}_K^G(M)$ becomes a K-module using the diagonal action. Moreover it is a nondegenerate $\mathcal{O}_K$-module by multiplication of functions in the first tensor factor. In this way $\mathrm{res}_K^G(M)$ becomes a K-AYD-module.

The following proposition describes the relation between induction and restriction of AYD-modules.

Proposition 4.2. *Let G be a totally disconnected group and let $K \subset G$ be a compact open subgroup. Then there exists a natural isomorphism*

$$\mathfrak{Hom}_G(\mathrm{ind}_K^G(M), N) \cong \mathfrak{Hom}_K(M, \mathrm{res}_K^G(N))$$

for all separated K-AYD-modules M and G-AYD-modules N.

Proof. We define a map $\alpha : \mathfrak{Hom}_K(M, \mathrm{res}_K^G(N)) \to \mathfrak{Hom}_G(\mathrm{ind}_K^G(M), N)$ by

$$\alpha(\phi)(f \otimes m) = f \cdot \mu(\phi(m))$$

where $\mu : \mathcal{O}_K \otimes_{\mathcal{O}_G} N \to N$ is the multiplication map. Conversely, we construct a map $\beta : \mathfrak{Hom}_G(\mathrm{ind}_K^G(M), N) \to \mathfrak{Hom}_K(M, \mathrm{res}_K^G(N))$ as follows. Given a G-AYD-map $\phi : \mathrm{ind}_K^G(M) \to N$ we let $\beta(\phi)$ be the composition

$$M \cong \mathsf{A}(K) \otimes_{\mathsf{A}(K)} M \xrightarrow{\iota \otimes \mathrm{id}} \mathsf{A}(G) \otimes_{\mathsf{A}(K)} M \xrightarrow{\phi} N \longrightarrow \mathrm{res}_K^G(N)$$

where the last maps sends $n \in N$ to $1 \otimes n \in \mathcal{O}_K \otimes_{\mathcal{O}_G} N = \mathrm{res}_K^G(N)$. For the first isomorphism we use the fact that M is a nondegenerate $\mathsf{A}(K)$-module. Explicitly, we have

$$\beta(\phi)(f \cdot m) = 1 \otimes \phi(\iota(f) \otimes m)$$

for $f \in \mathsf{A}(K)$ and $m \in M$. It is straightforward to check that α and β are inverse isomorphisms. $\square$

We will mainly have to work with G-AYD-modules of the form $M = \mathcal{O}_G \otimes V$ for some separated G-module V. In this case the reduced module $\mathrm{res}_K^G(M)$ is equal to $\mathcal{O}_K \otimes \mathrm{res}_K^G(V)$. In particular, we may identify $\mathrm{res}_K^G(X_G(A))$ with $X_K(\mathrm{res}_K^G(A))$ for every separated G-algebra A.

Similarly, there is a useful description of the induced modules for K-AYD-modules of the form $\mathcal{O}_K \otimes V$ for a separated K-module V. Let $\chi \in \mathcal{O}_G$ be the characteristic function of the set K. Note that every element in $\mathcal{O}_G$ determines a multiplier of $\mathsf{A}(G)$ in a natural way.

Lemma 4.3. *Let G be a totally disconnected group and let $K \subset G$ be a compact open subgroup. For every separated K-module V we have a natural isomorphism*

$$\mathrm{ind}_K^G(\mathcal{O}_K \otimes V) \cong I \otimes_{\mathcal{D}(K)} V$$

where $I \subset \mathsf{A}(G)$ denotes the left ideal generated by the multiplier χ.

Proof. First observe that the ideal I is indeed right K-invariant. We define a map $\alpha : \mathrm{ind}_K^G(\mathcal{O}_K \otimes V) \to I \otimes_{\mathcal{D}(K)} V$ by

$$\alpha(f \otimes h \otimes v) = f \cdot h \otimes v$$

for $f \in \mathsf{A}(G)$ and $h \otimes v \in \mathcal{O}_K \otimes V$. Conversely, let $\beta : I \otimes_{\mathcal{D}(K)} V \to \mathrm{ind}_K^G(\mathcal{O}_K \otimes V)$ be given by

$$\beta(f \otimes v) = f \otimes \chi \otimes v.$$

It is easy to check that α and β define inverse isomorphisms. $\square$

Finally, remark that if M is a paracomplex of K-AYD-modules then the induced module $\mathrm{ind}_K^G(M)$ is a paracomplex of G-AYD-modules in a natural way. Similarly, if N is a paracomplex of G-AYD-modules then its restriction $\mathrm{res}_K^G(N)$ is a paracomplex of K-AYD-modules. Observe moreover that the isomorphism obtained in proposition 4.2 is a chain map in this case.

5. Induction in equivariant cyclic homology

In this section we study the compatibility of equivariant cyclic homology with induction. We will restrict attention to the situation which arises in the construction of the equivariant Chern character later on.

The main result of this section is the following theorem.

Theorem 5.1. *Let K be a compact open subgroup of G. For every separated K-algebra A and every separated G-algebra B there is a natural homotopy equivalence*

$$\mathrm{ind}_K^G(X_K(\mathcal{T}(A \otimes \mathrm{res}_K^G(B))))^{\mathbb{L}c} \simeq X_G(\mathcal{T}(\mathrm{ind}_K^G(A) \otimes B))^{\mathbb{L}c}$$

of paracomplexes of G-AYD-modules. There are analogous homotopy equivalences if the derived completion is replaced by the ordinary completion or if one considers periodic tensor algebras instead.

The proof of theorem 5.1 is divided into several steps. We will carry out the details for the analytic tensor algebra and the derived completion, the other assertions are obtained in a similar way. For simplicity we assume that the Haar measure on G is normalized such that the measure of the compact set K is 1.

Consider the natural K-equivariant homomorphism $\theta_A : A \to \mathrm{ind}_K^G(A)$ given by $\theta_A(a)(t) = \chi(t)t^{-1} \cdot a$ where χ denotes the characteristic function of the set K. Similarly, we have the K-equivariant homomorphism $\pi_A : \mathrm{ind}_K^G(A) \to A$ given by $\pi_A(\alpha) = \alpha(e)$. Tensoring the previous maps with the identity on B yields K-equivariant homomorphisms $\theta : A \otimes B \to \mathrm{ind}_K^G(A) \otimes B$ and $\pi : \mathrm{ind}_K^G(A) \otimes B \to A \otimes B$, respectively. In particular, θ induces a K-equivariant homomorphism $\Theta : \mathcal{T}(A \otimes B) \to \mathcal{T}(\mathrm{ind}_K^G(A) \otimes B)$ and an $\mathsf{A}(K)$-linear chain map $X_K(\mathcal{T}(A \otimes B)) \to X_G(\mathcal{T}(\mathrm{ind}_K^G(A) \otimes B))$. According to proposition 4.2 this map corresponds to an $\mathsf{A}(G)$-linear chain map

$$i : \mathrm{ind}_K^G(X_K(\mathcal{T}(A \otimes B))) \to X_G(\mathcal{T}(\mathrm{ind}_K^G(A) \otimes B))$$

and using lemma 4.3 we obtain the explicit formula

$$i(F \otimes x_0)(s) = \int_G F(s,t)t \cdot \Theta(x_0)dt$$

$$i(F \otimes x_0 Dx_1)(s) = \int_G F(s,t)(t \cdot \Theta(x_0))D(t \cdot \Theta(x_1))dt$$

for i where F is an element in the left ideal $I \subset \mathsf{A}(G)$ generated by the multiplier χ and $x_i \in \mathcal{T}(A \otimes B)$.

We construct a chain map p which is going to be the homotopy inverse of i. Using lemma 4.3 we define $p : X_G(\mathcal{T}(\mathrm{ind}_K^G(A) \otimes B)) \to \mathrm{ind}_K^G(X_K(\mathcal{T}(A \otimes B)))$ by

$$p(f \otimes \alpha_0)(s,t) = f(s)(t \cdot \chi)(s)\Pi(t^{-1} \cdot \alpha_0)$$

$$p(f \otimes \alpha_0 D\alpha_1)(s,t) = f(s)(t \cdot \chi)(s)\Pi(t^{-1} \cdot \alpha_0)D\Pi(t^{-1} \cdot \alpha_1)$$

for $\alpha_i \in \mathcal{T}(\mathrm{ind}_K^G(A) \otimes B)$ where $\Pi : \mathcal{T}(\mathrm{ind}_K^G(A) \otimes B)) \to \mathcal{T}(B \otimes A)$ denotes the homomorphism induced by $\pi : \mathrm{ind}_K^G(A) \otimes B \to A \otimes B$. Remark that $\Pi(t \cdot \alpha)$ has compact support as a function of t for every $\alpha \in \mathcal{T}(\mathrm{ind}_K^G(A) \otimes B)$. It is easy to check that p is an AYD-map, and by definition p commutes with the boundary operator d. A straightforward computation shows that p commutes with the boundary b as well. It follows that p is a chain map. We compute

$$pi(F \otimes x_0)(s,t) = \int_G F(s,r)(t \cdot \chi)(s)\Pi(t^{-1}r \cdot \Theta(x_0))dr$$

$$= \int_K F(s,tr)(t \cdot \chi)(s)\Pi(r \cdot \Theta(x_0))dr$$

where the second equality follows from left invariance of the Haar measure and the fact that $\Pi(r \cdot \Theta(x_0))$ vanishes for r outside K. Using that Π is K-equivariant we obtain

$$\int_K F(s,tr)(t \cdot \chi)(s)\Pi(r \cdot \Theta(x_0))dr = \int_K F(s,t)(t \cdot \chi)(s)\Pi\Theta(x_0)dr$$

$$= F(s,t)(t \cdot \chi)(s)x_0 = F(s,t)x_0$$

taking into account that the Haar measure on K is normalized and that F is contained in the ideal I. In the same way one calculates

$$pi(F \otimes x_0 Dx_1)(s,t) = F(s,t)x_0 Dx_1$$

in degree one. Hence we have proved the relation $pi = \mathrm{id}$.

Let us now assume that A and B are unital and show that ip is homotopic to the identity. For this it is convenient to use theorem 3.4 and to work with equivariant differential forms instead of equivariant X-complexes. Rewriting the formula for i yields

$$i(F \otimes x_0 dx_1 \cdots dx_n)(s) = \int_G F(s,t)(t \cdot \theta(x_0)) d(t \cdot \theta(x_1)) \cdots d(t \cdot \theta(x_n)) dt$$

on $\mathrm{ind}_K^G(\Omega_K^n(A \otimes B))$. For the map p we obtain

$$p(f \otimes \alpha_0 d\alpha_1 \cdots d\alpha_n)(s,t) = f(s)(t \cdot \chi)(s)\pi(t^{-1} \cdot \alpha_0) d\pi(t^{-1} \cdot \alpha_1) \cdots d\pi(t^{-1} \cdot \alpha_n)$$

on $\Omega_G^n(\mathrm{ind}_K^G(A) \otimes B)$. Given $\alpha = \beta \otimes b \in \mathrm{ind}_K^G(A) \otimes B$ let us denote by $\alpha(t)$ the element $\beta(t) \otimes b$ in $A \otimes B$. Moreover, for $t \in G$ we write $\alpha[t]$ for the element in $\mathrm{ind}_K^G(A) \otimes B$ given by $\alpha[t](s) = \alpha(s)\chi(t^{-1}s)$. That is, $\alpha[t]$ is equal to the function α on the coset tK and zero elsewhere. Using this notation we obtain

$$ip(f \otimes \alpha_0 d\alpha_1 \cdots d\alpha_n)(s)$$
$$= \int_G f(s)(t \cdot \chi)(s)(t \cdot (\theta\pi(t^{-1} \cdot \alpha_0))) d(t \cdot \theta\pi(t^{-1} \cdot \alpha_1)) \cdots d(t \cdot \theta\pi(t^{-1} \cdot \alpha_n)) dt$$
$$= \int_G f(s)(t \cdot \chi)(s)\, \alpha_0[t] d\alpha_1[t] \cdots d\alpha_n[t] dt.$$

This implies

$$(\mathrm{id} - ip)(f \otimes \alpha_0 d\alpha_1 \cdots d\alpha_n)(s) = \int (1 - \chi(t_1) \cdots \chi(t_n)(t_0 \cdot \chi)(s))f(s)$$
$$\alpha_0[t_0] d\alpha_1[t_0 t_1] \cdots d\alpha_n[t_0 \cdots t_n]\, dt_0 dt_1 \cdots dt_n$$

and

$$(\mathrm{id} - ip)(f \otimes d\alpha_1 \cdots d\alpha_n)(s) = \int (1 - \chi(t_2) \cdots \chi(t_n)(t_1 \cdot \chi)(s))f(s)$$
$$d\alpha_1[t_1] \cdots d\alpha_n[t_1 \cdots t_n]\, dt_1 \cdots dt_n.$$

Let us abbreviate $\mathcal{C} = \Omega_G(\mathrm{ind}_K^G(A) \otimes B)$ and define a map $h : \mathcal{C} \to \mathcal{C}$ by

$$h(f \otimes \alpha_0 d\alpha_1 \cdots d\alpha_n)(s) = \sum_{j=0}^{n-1}(-1)^j \int \chi(t_1) \cdots \chi(t_j)\eta(t_{j+1})\, f(s)\, \alpha_0[t_0] d\alpha_1[t_0 t_1] \cdots$$
$$\cdots d\alpha_j[t_0 \cdots t_j] d1[t_0 \cdots t_j] d\alpha_{j+1}[t_0 \cdots t_{j+1}] \cdots d\alpha_n[t_0 \cdots t_n]\, dt_0 \cdots dt_n$$
$$+ (-1)^n \int \chi(t_1) \cdots \chi(t_n)\, f(s)(t_0 \cdot \eta)(s)\, \alpha_0[t_0] d\alpha_1[t_0 t_1] \cdots$$
$$\cdots d\alpha_n[t_0 \cdots t_n] d1[t_0 \cdots t_n]\, dt_0 \cdots dt_n$$

where $\eta(t) = 1 - \chi(t)$ as well as

$$h(f \otimes d\alpha_1 \cdots d\alpha_n)(s) = \sum_{j=1}^{n-1}(-1)^j \int \chi(t_2) \cdots \chi(t_j)\eta(t_{j+1})\, f(s)\, d\alpha_1[t_1] d\alpha_2[t_1 t_2] \cdots$$
$$\cdots d\alpha_j[t_1 \cdots t_j] d1[t_1 \cdots t_j] d\alpha_{j+1}[t_1 \cdots t_{j+1}] \cdots d\alpha_n[t_1 \cdots t_n]\, dt_1 \cdots dt_n$$

$$+ \sum_{j=1}^{n-1} (-1)^{j+n} \int \chi(t_2) \cdots \chi(t_j) \eta(t_{j+1}) \, f(s) \eta(t_1^{-1} s^{-1} t_1 \cdots t_n)$$

$$s^{-1} \cdot \alpha_n[t_1 \cdots t_n] ds^{-1} \cdot 1[t_1 \cdots t_n] d\alpha_1[t_1] \cdots$$

$$\cdots d\alpha_j[t_1 \cdots t_j] d1[t_1 \cdots t_j] d\alpha_{j+1}[t_1 \cdots t_{j+1}] \cdots d\alpha_{n-1}[t_1 \cdots t_{n-1}] \, dt_1 \cdots dt_n$$

$$+ (-1)^n \int \chi(t_2) \cdots \chi(t_n) \, f(s)(t_1 \cdot \eta)(s) \, d\alpha_1[t_1] d\alpha_2[t_1 t_2] \cdots$$

$$\cdots d\alpha_n[t_1 \cdots t_n] d1[t_1 \cdots t_n] \, dt_1 \cdots dt_n$$

$$+ \int \chi(t_2) \cdots \chi(t_n) \, f(s)(t_1 \cdot \eta)(s) \, ds^{-1} \cdot 1[t_1] d\alpha_1[t_1] \cdots d\alpha_n[t_1 \cdots t_n] \, dt_1 \cdots dt_n.$$

It is easy to check that h is an AYD-map and bounded with respect to the analytic bornology. A lengthy but straightforward computation yields the relation

$$bh + hb = \mathrm{id} - ip$$

on $\mathcal{C}$ which shows that ip is homotopic to the identity with respect to the Hochschild operator. Moreover we have $hi = 0$ and $ph = 0$.

Recall that $M \otimes_T \mathbb{C}$ denotes the space of T-coinvariants of an AYD-module M. We are interested in the space $\mathcal{C} \otimes_T \mathbb{C}$ of coinvariants of $\mathcal{C}$. The transposed analytic bornology on this space is generated by sets of the form

$$D \otimes S \cup D \otimes [S] dS \cup \bigcup_{n=1}^{\infty} n! \, D \otimes [S][dS](dS)^{2n}$$

where $D \subset \mathcal{O}_G$ and $S \subset \mathrm{ind}_K^G(A) \otimes B$ are small. Actually it suffices to consider sets D and S of a special form. Let us fix a K-invariant small disk R of $A \otimes B$ containing the identity element. We may assume that S consists of all α in $\mathrm{ind}_K^G(A) \otimes B$ with support in a K-invariant compact subset L of G such that the element $\alpha(t)$ is contained in R for all $t \in G$. For the set D we may assume that it is closed under multiplication by all functions $t \cdot \eta$ for $t \in L$. Let us call the pair D and S special if these conditions are satisfied.

We define endomorphisms T_j of $\Omega_G^n(\mathrm{ind}_K^G(A) \otimes B) \otimes_T \mathbb{C}$ by the formula

$$T_j(f \otimes \alpha_0 d\alpha_1 \cdots d\alpha_n)(s) = f(s) \, (s^{-1} \cdot \alpha_0) d(s^{-1} \cdot \alpha_1) \cdots d(s^{-1} \cdot \alpha_{j-1}) d\alpha_j \cdots d\alpha_n,$$

and for small subsets D and S as before we set

$$D\Omega^n(S) = \bigcup_{j=0}^{n} T_j(D \otimes [S](dS)^n)$$

for $n > 0$ as well as $D\Omega^0(S) = D \otimes S$. Using this notation we obtain the following estimate.

Lemma 5.2. *Let D and S be a pair of special small sets. There exists a positive constant m dependent on S such that $(hB)^j(x)$ can be written as a sum of at most*

$$m^{n+1}(n+1)(n+3) \cdots (n+2j-1)2$$

terms in $D\Omega^{n+2j}(S)$ for any $x \in D\Omega^n(S)$ and $j \in \mathbb{N}$.

Proof. Let us fix an element $x \in D\Omega^n(S)$. Then, by definition, the element $B(x)$ is given as a sum of $n+1$ terms contained in $D\Omega^{n+1}(S)$. Similarly, $h(x)$ is given as a finite sum of elementary terms. Let m be the number of elements in the image of L in G/K under the canonical projection where $L \subset G$ is the compact subset occuring in the definition of S. Using that D and S are special it is easily seen that each term in the formula for $h(x)$ can be written as a sum of at most m^{n+1} elements belonging to $D\Omega^{n+1}(S)$. It suffices to estimate the number of nonzero summands in $(hB)^j(x)$ for all j. In the definition of h for elements in the image of the map d only

the terms in the second sum are not contained in the kernel of d. It follows that $(hB)^j(x)$ can be written as a sum of at most $m^{n+1}(n+1)(n+3)\cdots(n+2j-1)2$ terms in $D\Omega^{n+2j}(S)$ as claimed. $\qquad\square$

With the notation as above we let $D\Omega(S)$ be the circled convex hull of the set

$$\bigcup_{n=0}^{\infty} n!\, D\Omega^{2n}(S) \cup \bigcup_{n=0}^{\infty} n!\, D\Omega^{2n+1}(S)$$

in $\mathcal{C} \otimes_T \mathbb{C}$. If D and S are small then the set $D\Omega(S)$ is small in the transposed analytic bornology. Using lemma 5.2 we construct a map on the coinvariants of the derived completion $\mathcal{C}^{\mathbb{L}c}$ as follows.

Proposition 5.3. *On the space of coinvariants $\mathcal{C}^{\mathbb{L}c} \otimes_T \mathbb{C}$ of the derived completion of the paracomplex $\mathcal{C}$ the series*

$$\sum_{j=0}^{\infty} (-1)^j (hB)^j$$

determines an endomorphism of AYD*-modules.*

Proof. Every small subset of $\mathcal{C} \otimes_T \mathbb{C}$ is contained in a set of the form $D\Omega(S)$ where D and S are a special pair of small sets in the sense above. Using lemma 5.2 we see that the partial sums in the above series satisfy the Cauchy criterion if they are viewed as bounded linear maps from the normed space $\langle D\Omega(S)\rangle$ into the normed space $\langle D\Omega(\lambda S)\rangle$ provided λ is chosen such that $\lambda^{n+2j} \geq 2m^{n+1}4^j$ for all n and j. It follows that the series converges to a bounded AYD-map from $\mathsf{A}(G)\langle D\Omega(S)\rangle$ into the completion of $\mathsf{A}(G)\langle D\Omega(\lambda S)\rangle$. In this way one obtains an endomorphism of $(\mathcal{C} \otimes_T \mathbb{C})^{\mathbb{L}c}$. Inspecting the construction of the derived completion yields a natural isomorphism $\mathcal{C}^{\mathbb{L}c} \otimes_T \mathbb{C} \cong (\mathcal{C} \otimes_T \mathbb{C})^{\mathbb{L}c}$. This proves the assertion. $\qquad\square$

Let us write $C = \mathcal{C}^{\mathbb{L}c}$ and consider the map $k : C \otimes_T \mathbb{C} \to C \otimes_T \mathbb{C}$ obtained in proposition 5.3. Using this map we are able to apply the perturbation lemma [15]. More precisely, together with the previously established properties of the maps i, p and h we obtain the relation

$$ip - \mathrm{id} = [\partial, k]$$

on $C \otimes_T \mathbb{C}$ where ∂ denotes the boundary map $B + b$. For the precise formulation and a proof of the perturbation lemma in the context of paramixed complexes we refer to [30].

Let us denote the natural projection $C \to C \otimes_T \mathbb{C}$ by q. It is not hard to check that there exists an AYD-map $K : C \to C$ lifting k. As a consequence we see in particular that the image of

$$\delta = \mathrm{id} - ip + [\partial, K]$$

is contained in the kernel $(\mathrm{id} - T)C$ of q.

Lemma 5.4. *The chain map $\delta : C \to C$ is homotopic to zero. Consequently, the chain map $ip : C \to C$ is homotopic to the identity.*

Proof. We have to consider the elliptic and hyperbolic part of C separately. On the elliptic part we have $(\mathrm{id} - T)C_{\mathrm{ell}} = (\mathrm{id} - E)C_{\mathrm{ell}}$ where E is the canonical projection on the T-invariant part. The argument given in the proof of proposition 3.4 in [32] shows that the natural inclusion $(\mathrm{id} - E)C_{\mathrm{ell}} \to C_{\mathrm{ell}}$ is homotopic to zero. Hence the chain map δ is homotopic to zero on the elliptic part.

Now consider the hyperbolic part. According to proposition 4.1 the G-module $\mathrm{ind}_K^G(A)$ is projective. Hence the G-module $\mathrm{ind}_K^G(A) \otimes B$ is projective as well, and it follows that $\mathcal{C}$ is a projective AYD-module [30]. It is then easy to check that

$$(\mathrm{id} - T) : C_{\mathrm{hyp}} \to (\mathrm{id} - T)C_{\mathrm{hyp}}$$

is an isomorphism. Hence there exists a chain map $\Delta : C_{\mathrm{hyp}} \to C_{\mathrm{hyp}}$ such that $\delta = (\mathrm{id} - T)\Delta$ on C_{hyp}. Since $\mathrm{id} - T$ is homotopic to zero this shows that the restriction of δ to the hyperbolic part is homotopic to zero. We conclude that ip is homotopic to the identity on C. $\qquad\square$

Lemma 5.4 completes the proof of theorem 5.1 in the case that the algebras A and B are unital. Now let A and B be arbitrary and consider the canonical split extension $0 \to A \to A^+ \to \mathbb{C} \to 0$ of K-algebras. The corresponding extension

$$0 \to \mathrm{ind}_K^G(A) \to \mathrm{ind}_K^G(A^+) \to \mathrm{ind}_K^G(\mathbb{C}) \to 0$$

of G-algebras has an equivariant splitting as well, and similarly we have the split extension $0 \to B \to B^+ \to \mathbb{C} \to 0$ of G-algebras. Using the result in the unital case we obtain the assertion for general A and B by considering tensor products of the algebras in these extensions and excision for K-equivariant and G-equivariant local cyclic homology [31]. This finishes the proof of theorem 5.1.

Let us discuss some consequences of theorem 5.1. First of all, we deduce that induction from compact open subgroups descends to the level of equivariant cyclic homology.

Theorem 5.5. *Let $K \subset G$ be a compact open subgroup. Then there exists a transformation*

$$\mathrm{ind}_K^G : HL_*^K(A, B) \to HL_*^G(\mathrm{ind}_K^G(A), \mathrm{ind}_K^G(B))$$

which is compatible with the composition product. Analogous transformations exist for the analytic and periodic theories.

Proof. Using lemma 4.3 one checks that induction is compatible with derived completion in the sense that there is a natural isomorphism

$$\mathrm{ind}_K^G(X_K(\mathcal{T}(C \otimes \mathcal{K}_G))^{\mathbb{L}c}) \cong \mathrm{ind}_K^G(X_K(\mathcal{T}(C \otimes \mathcal{K}_G)))^{\mathbb{L}c}$$

for every K-algebra C. Hence theorem 5.1 yields a natural homotopy equivalence

$$\mathrm{ind}_K^G(X_K(\mathcal{T}(C \otimes \mathcal{K}_K))^{\mathbb{L}c}) \simeq X_G(\mathcal{T}(\mathrm{ind}_K^G(C) \otimes \mathcal{K}_G))^{\mathbb{L}c}$$

where we take into account stability of K-equivariant cyclic homology [31]. As a consequence one obtains easily the desired induction homomorphism for the local theory. The assertions for the analytic and the periodic theories are proved in the same way. $\qquad\square$

Furthermore we obtain the following induction isomorphism.

Theorem 5.6. *Let $K \subset G$ be a compact open subgroup. For every separated K-algebra A and every separated G-algebra B there exists a natural isomorphism*

$$HL_*^G(\mathrm{ind}_K^G(A), B) \cong HL_*^K(A, \mathrm{res}_K^G(B)).$$

Analogous isomorphisms hold for the analytic and periodic theories.

Proof. Using the fact that induction is compatible with derived completion this follows again by the homotopy equivalence

$$\mathrm{ind}_K^G(X_K(\mathcal{T}(A \otimes \mathcal{K}_G)))^{\mathbb{L}c} \simeq X_G(\mathcal{T}(\mathrm{ind}_K^G(A) \otimes \mathcal{K}_G))^{\mathbb{L}c}$$

obtained in theorem 5.1 together with proposition 4.2 and stability. In the same way one obtains the assertions for the analytic and the periodic theories. $\qquad\square$

6. The equivariant Chern character

In this section we construct the equivariant Chern character with values in bivariant equivariant cohomology in the sense of Baum and Schneider.

The first ingredient in this construction is the equivariant Chern-Connes character into equivariant local cyclic homology obtained in [31].

Theorem 6.1. *Let A and B be separable G-C^*-algebras. Then there exists a transformation*
$$\mathrm{ch}^G_* : KK^G_*(A, B) \to HL^G_*(A, B)$$
which is compatible with the Kasparov product in KK^G_* and the composition product in HL^G_*. This transformation maps elements in $KK^G_0(A, B)$ induced by equivariant $*$-homomorphisms from A to B to the corresponding elements in $HL^G_0(A, B)$.

We remark that in the context of local cyclic homology all G-C^*-algebras are equipped with the precompact bornology. Moreover, it is understood that the smoothing functor $\mathfrak{Smooth}$ is applied to the resulting bornological algebras since, unless G is discrete, the group action on a G-C^*-algebra is not smooth in general. For more information we refer to [31].

We shall apply theorem 6.1 in the case that A and B are algebras of functions on G-simplicial complexes in the sense of [32]. By definition, a G-simplicial complex is a simplicial complex equipped with smooth, simplicial and type-preserving action of the group G. In the sequel we will only consider G-simplicial complexes that have at most countably many simplices. In order to determine the right hand side of the Chern-Connes character further in this situation we have to pass to smooth functions. A suitable notion of smooth functions on simplicial complexes was introduced in [32]. For a locally finite G-simplicial complex Z the resulting algebra $C^\infty_c(Z)$ of regular smooth functions with compact support is a G-algebra in a natural way. Moreover, it is shown in [31] that the inclusion $C^\infty_c(Z) \to \mathfrak{Smooth}(C_0(Z))$ induces an invertible element in the local theory provided Z is finite dimensional. It follows that there is an isomorphism
$$HL^G_*(C_0(X), C_0(Y)) \cong HL^G_*(C^\infty_c(X), C^\infty_c(Y))$$
for all finite dimensional locally finite G-simplicial complexes X and Y which is compatible with the composition product.

The next aim is to determine the equivariant local cyclic homology of algebras of regular smooth functions. In order to do this we shall use two auxiliary homology theories. Firstly, we define $HLA^G_*(A, B)$ by
$$HLA^G_*(A, B) = H_*(\mathfrak{Hom}_G(X_G(\mathcal{T}(A \otimes \mathcal{K}_G))^{\mathbb{L}c}, X_G(\mathcal{T}(B \otimes \mathcal{K}_G))^c))$$
for all G-algebras A and B. By construction, this theory is a combination of equivariant local and analytic cyclic homology. There is a homomorphism
$$HL^G_*(A, B) \to HLA^G_*(A, B)$$
induced by the canonical map from the derived completion to the ordinary completion in the second variable. According to a result from [31] the natural chain map $X_G(\mathcal{T}(B \otimes \mathcal{K}_G))^{\mathbb{L}c} \to X_G(\mathcal{T}(B \otimes \mathcal{K}_G))^c$ is an isomorphism in the local derived category provided B is a Schwartz space satisfying the approximation property. This yields the following assertion.

Proposition 6.2. *Assume that B is a G-algebra whose underlying bornological vector space is a Schwartz space satisfying the approximation property. Then the natural map*
$$HL^G_*(A, B) \to HLA^G_*(A, B)$$
is an isomorphism for every separated G-algebra A.

In particular, proposition 6.2 applies in the case that $B = C_c^\infty(Y)$ for a finite dimensional locally finite G-simplicial complex Y.

Let us define another homology theory $HPA_*^G(A, B)$ by

$$HPA_*^G(A, B) = H_*(\mathfrak{Hom}_G(X_G(\mathcal{T}(A \otimes \mathcal{K}_G)/\mathcal{J}(A \otimes \mathcal{K}_G)^\infty), X_G(\mathcal{T}(B \otimes \mathcal{K}_G))^c))$$

for all G-algebras A and B. This theory provides a link between equivariant periodic cyclic homology and HAL_*^G. In particular, there is a natural transformation

$$HPA_*^G(A, B) \to HLA_*^G(A, B)$$

induced by the canonical map from the derived completion to the ordinary completion and the projection from the analytic tensor algebra to the periodic tensor algebra in the first variable.

Proposition 6.3. *Let X be a G-finite proper G-simplicial complex and let B be any G-algebra. Then the natural map*

$$HPA_*^G(C_c^\infty(X), B) \to HLA_*^G(C_c^\infty(X), B)$$

is an isomorphism.

Proof. Observe that the transformation $HPA_*^G(A, B) \to HLA_*^G(A, B)$ is compatible with the boundary maps in the six-term exact sequences associated to an extension of algebras. Using induction on the dimension of X and excision [30], [31] it suffices to prove the assertion in the case that $X = G/K$ is a homogenous space for a compact open subgroup K of G. Remark that we have $C_c^\infty(X) = \mathrm{ind}_K^G(\mathbb{C})$ in this situation and consider the commutative diagram

$$
\begin{array}{ccc}
X_G(\mathcal{T}(\mathrm{ind}_K^G(\mathbb{C}) \otimes \mathcal{K}_G))^{\mathbb{L}c} & \longrightarrow & X_G(\mathcal{T}(\mathrm{ind}_K^G(\mathbb{C}) \otimes \mathcal{K}_G)/\mathcal{J}(\mathrm{ind}_K^G(\mathbb{C}) \otimes \mathcal{K}_G)^\infty) \\
\uparrow \simeq & & \uparrow \simeq \\
\mathrm{ind}_K^G(X_K(\mathcal{T}(\mathcal{K}_G)))^{\mathbb{L}c} & \longrightarrow & \mathrm{ind}_K^G(X_K(\mathcal{T}(\mathcal{K}_G)/\mathcal{J}(\mathcal{K}_G)^\infty) \\
\uparrow \simeq & & \uparrow \simeq \\
\mathrm{ind}_K^G(\mathcal{O}_K[0])^{\mathbb{L}c} & \xrightarrow{\;\cong\;} & \mathrm{ind}_K^G(\mathcal{O}_K[0])
\end{array}
$$

where the upper vertical homotopy equivalences are obtained using theorem 5.1 and the lower vertical homotopy equivalences follow from stability in equivariant cyclic homology [30], [31]. According to the definition of the derived completion the lower horizontal map is an isomorphism. Hence the remaining horizontal arrows in this diagram are homotopy equivalences. This yields the assertion for $X = G/K$ and finishes the proof. $\square$

Applying the natural projection from the analytic to the periodic tensor algebra in the second variable we obtain a transformation

$$HPA_*^G(A, B) \to HP_*^G(A, B)$$

for all separated G-algebras A and B.

Proposition 6.4. *Let X and Y be G-simplicial complexes where X is G-finite and proper and Y is finite dimensional and locally finite. Then the natural map*

$$HPA_*^G(C_c^\infty(X), C_c^\infty(Y)) \to HP_*^G(C_c^\infty(X), C_c^\infty(Y))$$

is an isomorphism.

Proof. Using excision in the first variable we can reduce the assertion to the case that $X = G/K$ is a homogenous space where K is a compact open subgroup of G. Due to theorem 5.1 it thus suffices to consider the case that G is compact and that X is a point. Observe that for a compact group G the chain complexes

defining $HPA^G_*(\mathbb{C}, B)$ and $HP^G_*(\mathbb{C}, B)$ for a G-algebra B are obtained by taking the G-invariant part of $X_G(\mathcal{T}(B))^c$ and $X_G(\mathcal{T}(B)/\mathcal{J}(B)^\infty)$, respectively.

We will now use induction on the dimension of Y. If $\dim(Y) = 0$ the space Y is the disjoint union of a family of homogenous spaces. Accordingly, we have

$$C^\infty_c(Y) = \bigoplus_{j \in J} C(G/K_j)$$

for some countable index set J in this case where each K_j is an open subgroup of the compact group G. If the set J is finite it follows in the same way as in the proof of proposition 6.3 that the natural map

$$X_G(\mathcal{T}(C^\infty(Y)))^c \to X_G(\mathcal{T}(C^\infty(Y))/\mathcal{J}(C^\infty(Y))^\infty)$$

is a homotopy equivalence. Observe that due to stability we may drop the algebra $\mathcal{K}_G$ in our arguments since G is compact. In order to treat the case of an infinite index set J we use a direct limit argument. More precisely, since completion commutes with direct limits the natural map

$$\bigoplus_{j \in J} HPA^G_*(\mathbb{C}, C(G/K_j)) \to HPA^G_*(\mathbb{C}, C^\infty_c(Y))$$

is an isomorphism. For the periodic theory there is an analogous isomorphism according to the results obtained in [32]. Using the case of finite J treated before it follows that the claim is true for arbitrary Y of dimension 0. As in the proof of proposition 6.3 the induction step is carried out using excision. $\square$

The link between equivariant cyclic homology and the theory of Baum and Schneider is provided by the main result of [32].

Theorem 6.5. *Let G be a totally disconnected group and let X and Y be finite dimensional locally finite G-simplicial complexes. If the action of G on X is proper there exists an isomorphism*

$$HP^G_*(C^\infty_c(X), C^\infty_c(Y)) \cong \bigoplus_{j \in \mathbb{Z}} H^{*+2j}_G(X, Y).$$

Combining theorem 6.1 with propositions 6.2, 6.3, 6.4 and theorem 6.5 we obtain the following result.

Theorem 6.6. *Let G be a totally disconnected group and let X and Y be finite dimensional locally finite G-simplicial complexes. If X is proper and G-finite there exists an equivariant Chern character*

$$ch^G_* : KK^G_*(C_0(X), C_0(Y)) \to \bigoplus_{j \in \mathbb{Z}} H^{*+2j}_G(X, Y)$$

which becomes an isomorphism after tensoring the left hand side with $\mathbb{C}$.

Proof. It is not very hard to verify that the character ch^G_* is natural with respect to proper equivariant simplicial maps in both variables. In order to show that it becomes an isomorphism after tensoring the left hand side with $\mathbb{C}$ we proceed in the same way as in the proof of proposition 6.3 and proposition 6.4 to reduce to the case that G is compact and X and Y are one-point spaces. In this case the assertion follows from proposition 12.4 in [31]. $\square$

Our arguments above show in fact that for $A = C_0(X)$ and $B = C_0(Y)$ satisfying the assumptions of theorem 6.6 the Chern-Connes character with values in equivariant local cyclic homology is an isomorphism after tensoring the left hand side with $\mathbb{C}$. Moreover, all the different variants of equivariant cyclic homology for the corresponding algebras of smooth functions agree in this case.

Let us discuss some consequences of theorem 6.6. The equivariant K-homology of a proper G-simplicial complex X is defined by

$$K_*^G(X) = \varinjlim_{F \subset X} KK_*^G(C_0(F), \mathbb{C})$$

where the limit is taken over all G-finite subcomplexes F of X. Note that every G-finite subcomplex of X is locally finite and hence locally compact. In a similar way we define the equivariant homology of X by

$$H_*^G(X) = \varinjlim_{F \subset X} HP_*^G(C_c^\infty(F), \mathbb{C}).$$

if X is a proper G-simplicial complex. We remark that, taking into account the above considerations, one could as well take equivariant analytic or local cyclic homology in this definition. Moreover, using theorem 6.5 and the work of Baum and Schneider [5] it follows that the groups $H_*^G(X)$ are naturally isomorphic to the cosheaf homology of X as defined in [4].

Taking direct limits in the first variable in theorem 6.6 we obtain the following statement.

Theorem 6.7. *Let G be a totally disconnected group and let X be a proper G-simplicial complex. Then there exists a natural transformation*

$$K_*^G(X) \to H_*^G(X)$$

which becomes an isomorphism after tensoring the left hand side with the complex numbers.

For totally disconnected groups the universal space for proper actions $\underline{E}G$ can be chosen to be a G-simplicial complex. Applying theorem 6.7 to this particular space yields the following result.

Theorem 6.8. *Let G be a totally disconnected group. Then there exists a Chern character*

$$ch_*^G : K_*^G(\underline{E}G) \to H_*^G(\underline{E}G)$$

from the topological K-theory of G to the cosheaf homology of $\underline{E}G$ which is an isomorphism after tensoring the left hand side with $\mathbb{C}$.

7. The character of Baum and Schneider

In this section we compare the Chern character obtained in section 6 with the bivariant equivariant Chern character for profinite groups defined by Baum and Schneider in [5]. The construction of Baum and Schneider is based on universal coefficient theorems for equivariant KK-theory and bivariant equivariant cohomology. Roughly speaking, we show that in the case of profinite groups the equivariant Chern character ch_*^G reduces to the character defined by Baum and Schneider. In this way we obtain in particular a convenient description of ch_*^G for profinite groups. For simplicity we consider only finite G-simplicial complexes in this section.

Let G be a profinite group. To every locally compact G-space X one associates the Brylinski space

$$\hat{X} = \{(t, x) \in G \times X \,|\, t \cdot x = x\} \subset G \times X$$

which is again a G-space by considering the action given by

$$s \cdot (t, x) = (sts^{-1}, s \cdot x)$$

for $s \in G$ and $(t, x) \in \hat{X}$. Note that if G is a finite group one may view $\hat{X}$ as the disjoint union of the fixed point sets $X^t = \{x \in X \,|\, t \cdot x = x\}$ of elements $t \in G$.

Now let X and Y be finite G-simplicial complexes. The Chern character constructed by Baum and Schneider is of the form

$$\phi_*^G : KK_*^G(C(X), C(Y)) \to \mathfrak{Hom}_G(K^*(\hat{X})_{\mathbb{C}}, K^*(\hat{Y})_{\mathbb{C}}) \to \mathfrak{Hom}_G(H^*(\hat{X}), H^*(\hat{Y}))$$

where the subscript $\mathbb{C}$ stands for tensoring with $\mathbb{C}$ over the integers and the second arrow is induced by the classical Chern character for K-theory. Similarly, we may view ch_*^G as a map

$$ch_*^G : KK_*^G(C(X), C(Y)) \to HP_*^G(C^\infty(X), C^\infty(Y)) \cong \mathfrak{Hom}_G(H^*(\hat{X}), H^*(\hat{Y}))$$

using the universal coefficient formula for bivariant equivariant cohomology obtained in [5].

In order to compare these transformations we consider first the case that X is a point. Let us review the construction of Baum and Schneider in this situation. Given a G-equivariant vector bundle E over a compact G-space Y one considers the pull-back bundle $\pi^* E$ along the natural projection $\pi : \hat{Y} \to Y$. The bundle $\pi^* E$ is again G-equivariant and carries in addition an action of the profinite completion $\hat{\mathbb{Z}}$ of the integers. To describe this action note that the space of sections of $\pi^* E$ is an AYD-module in a natural way. On the level of sections, the action of $\hat{\mathbb{Z}}$ is determined by the natural automorphism T of this AYD-module. In fact pull-back along π yields a homomorphism

$$\pi^* : K_G^0(Y) \to K_{\hat{\mathbb{Z}}}^0(\hat{Y})^G$$

of equivariant K-groups where the superscript G on the right hand side denotes taking the invariant part under the action of G. Taking the trace of the element $1 \in \hat{\mathbb{Z}}$ in a representation of $\hat{\mathbb{Z}}$ defines a ring homomorphism

$$\tau : R(\hat{\mathbb{Z}}) \to \mathbb{C}.$$

Since the action of $\hat{\mathbb{Z}}$ on $\hat{Y}$ is trivial one obtains a homomorphism

$$\Pi : \quad K_G^0(Y) \xrightarrow{\pi^*} K_{\hat{\mathbb{Z}}}^0(\hat{Y})_{\mathbb{C}}^G \xrightarrow{\cong} (K^0(\hat{Y}) \otimes_{\mathbb{Z}} R(\hat{\mathbb{Z}})_{\mathbb{C}})^G \xrightarrow{\mathrm{id} \otimes \tau} K^0(\hat{Y})_{\mathbb{C}}^G$$

where, as above, the subscript $\mathbb{C}$ stands for tensoring with $\mathbb{C}$. The transformation ϕ_0 is obtained by composing the map Π with the map induced by the ordinary Chern character $\mathrm{ch} : K^0(\hat{Y}) \to H^*(\hat{Y})$. We shall assume that the latter is normalized as in [6] in order to be compatible with cyclic homology. In the odd case the character of Baum and Schneider is defined using suspension.

Proposition 7.1. *Let G be a finite group and let Y be a finite G-simplicial complex. The equivariant Chern character*

$$ch_*^G : K_*^G(C(Y)) \to \bigoplus_{j \in \mathbb{Z}} H^{*+2j}(\hat{Y})^G$$

agrees with the transformation ϕ_*^G constructed by Baum and Schneider in this case.

Proof. The algebra $C^\infty(Y)$ is a dense subalgebra of $C(Y)$ which is closed under holomorphic functional calculus. As in the nonequivariant situation the natural inclusion $C^\infty(Y) \to C(Y)$ induces an isomorphism $K_0^G(C^\infty(Y)) \cong K_0^G(C(Y))$. Consequently, to obtain the assertion in the even case it suffices to compare the characters $ch_0^G(p)$ and $\phi_0(p)$ of a G-invariant idempotent $p \in C^\infty(Y) \otimes \mathbb{L}(V)$ where V is a finite dimensional Hilbert space equipped with a unitary representation λ of G. Here the algebra $\mathbb{L}(V)$ of linear operators on V is equipped with the natural

action $s \cdot T = \lambda(s)T\lambda(s^{-1})$ induced by λ. Under the Chern character ch_0^G the element in $HP_0^G(\mathbb{C}, C^\infty(Y))$ corresponding to p is given by the cycle

$$\mathrm{ch}_0^G(p)(s) = \mathrm{tr}(\lambda(s)p) + \sum_{j=1}^\infty (-1)^j \frac{(2j)!}{j!} \mathrm{tr}\left(\lambda(s)\left(p - \frac{1}{2}\right)(dpdp)^j\right)$$

in the equivariant periodic cyclic complex of $C^\infty(Y)$ where tr denotes the ordinary trace. Here we use the analogue of theorem 3.4 for the periodic theory and work with noncommutative differential forms instead of the X-complex in the second variable. The above formula for the Chern character appears also in the work of Block and Getzler on actions of compact Lie groups [8].

Since G is finite the space $\hat{Y}$ is the disjoint union of the fixed point sets Y^s for all elements $s \in G$. The equivariant Hochschild-Kostant-Rosenberg theorem [32] shows that $ch_0^G(p)$ is given by the class

$$\sum_{s \in G} \sum_{j=0}^\infty \frac{(-1)^j}{j!} \mathrm{tr}(\lambda(s)p(dpdp)^j)_{|Y^s}$$

in the even cohomology of $\hat{Y}$. It is straightforward to check that this class is equal to the Chern character of the element $\Pi(p) \in K^0(\hat{Y})_{\mathbb{C}}$. Note that the occurence of the operator $\lambda(s)$ arises from the map τ above. As a consequence, we see that the cohomology classes $ch_0^G(p)$ and $\phi_0(p)$ are equal.

In the order to treat the odd case consider the space S^1 with trivial G-action and the split extension

$$C_0(Y \times (0,1)) \rightarrowtail C(Y \times S^1) \xrightarrow{\iota^*} C(Y)$$

of G-C^*-algebras where ι^* is the transpose of the inclusion $\iota : Y \to Y \times S^1$ given by $\iota(y) = (y,1)$. One obtains a corresponding commutative diagram

$$
\begin{array}{ccc}
K_0^G(C_0(Y \times (0,1))) \rightarrowtail K_0^G(C(Y \times S^1)) \longrightarrow K_0^G(C(Y)) \\
\Big\downarrow{\phi_0^G} \qquad\qquad \Big\downarrow{\phi_0^G} \qquad\qquad \Big\downarrow{\phi_0^G} \\
H_c^*(\hat{Y} \times (0,1))^G \rightarrowtail H^*(\hat{Y} \times S^1)^G \longrightarrow H^*(\hat{Y})^G
\end{array}
$$

for the transformation defined by Baum and Schneider. In order to apply the character ch_0^G in this situation we choose an equivariant triangulation of $Y \times S^1$ such that the inclusion $\iota : Y \to Y \times S^1$ becomes a map of G-simplicial complexes. Similarly, we choose an equivariant triangulation of $Y \times (0,1)$. Remark that the inclusion $Y \times (0,1) \to Y \times S^1$ will not be a simplicial map with respect to these triangulations. Still, it is easily verified that the analogue of the previous diagram for ch_0^G is commutative.

According to our previous considerations the transformations ϕ_0^G and ch_0^G agree for $Y \times S^1$ and Y. It follows that ϕ_0^G and ch_0^G agree also for the suspension $Y \times (0,1)$ of Y. As already mentioned above, the character ϕ_1^G is obtained from the even case using suspension. Assuming that ϕ_1^G is normalized correctly it follows that ch_*^G and ϕ_*^G agree in the odd case as well. $\qquad\square$

One can verify directly that the assertion of proposition 7.1 holds also for profinite groups. We shall obtain this as a special case of the following result.

Theorem 7.2. *Let G be a profinite group. On the category of finite G-simplicial complexes the equivariant Chern character*

$$ch_*^G : KK_*^G(C(X), C(Y)) \to \bigoplus_{j \in \mathbb{Z}} H_G^{*+2j}(X, Y)$$

agrees with the natural transformation ϕ_*^G constructed by Baum and Schneider.

Proof. Let us first assume that G is finite. As before we use the identification

$$\bigoplus_{j \in \mathbb{Z}} H_G^{*+2j}(X, Y) \cong \mathfrak{Hom}_G(H^*(\hat{X}), H^*(\hat{Y}))$$

for the bivariant equivariant cohomology of X and Y. It suffices to show for every $t \in G$ that the transformations ch_*^G and ϕ_*^G agree after localization in t on the right hand side. If Z_t denotes the centralizer of t in G and $f \in KK_*^G(C(X), C(Y))$ is an arbitrary class we shall thus prove that the elements in $\mathrm{Hom}_{Z_t}(H^*(X^t), H^*(Y^t))$ induced by $ch_*^G(f)$ and $\phi_*^G(f)$ are equal.

The localized Chern character ch_*^G can be written as the composition

$$ch_*^G : KK_*^G(C(X), C(Y)) \to KK_*^{Z_t}(C(X), C(Y)) \to \mathrm{Hom}_{Z_t}(H^*(X^t), H^*(Y^t))$$

where the first arrow is the obvious forgetful map and the second arrow is the localized Chern character $ch_*^{Z_t}$ for Z_t. There is an analogous factorization for the localized character ϕ_*^G due to its construction. Hence it suffices to consider the group $G = Z_t$ and localization in t.

Let us moreover specialize to $X = Z_t/(t)$ where (t) denotes the subgroup of Z_t generated by t. The cohomology $H^*(Z_t/(t)) \cong C(Z_t/(t))[0]$ is located in degree zero and we have a commutative diagram

$$
\begin{array}{ccc}
KK_*^{Z_t}(C(Z_t/(t)), C(Y)) & \xrightarrow{\cong} & KK_*^{(t)}(\mathbb{C}, C(Y)) \\
\downarrow{ch_*^{Z_t}} & & \downarrow{ch_*^{(t)}} \\
\mathrm{Hom}_{Z_t}(C(Z_t/(t))[0], H^*(Y^t)) & \xrightarrow{\cong} & \mathrm{Hom}_{(t)}(\mathbb{C}[0], H^*(Y^t))
\end{array}
$$

obtained by induction. Note that $\mathrm{Hom}_{(t)}(\mathbb{C}[0], H^*(Y^t)) = H^*(Y^t)$ since the action of t on Y^t is trivial. Inspecting the construction in [5] we see that there is a corresponding commutative diagram where the vertical maps are replaced by the transformations $\phi_*^{Z_t}$ and $\phi_*^{(t)}$, respectively. According to proposition 7.1 the right vertical arrows in these diagrams coincide. It follows that the localized characters for Z_t are equal in the case $X = Z_t/(t)$.

Both transformations $\phi_*^{Z_t}$ and $ch_*^{Z_t}$ are multiplicative with respect to the Kasparov product and the composition product. Let $f \in KK_*^{Z_t}(C(X), C(Y))$ be an arbitrary element and consider $v \in KK_*^{Z_t}(C(Z_t/(t)), C(X))$. After localization in t we obtain

$$ch_*^{Z_t}(v) \cdot \phi_*^{Z_t}(f) = \phi_*^{Z_t}(v) \cdot \phi_*^{Z_t}(f) = \phi_*^{Z_t}(v \cdot f) = ch_*^{Z_t}(v \cdot f) = ch_*^{Z_t}(v) \cdot ch_*^{Z_t}(f)$$

according to our previous discussion. Since the images of elements of the form $ch_*^{Z_t}(v)$ in $\mathrm{Hom}_{(t)}(\mathbb{C}[0], H^*(X^t))$ generate $H^*(X^t)$ it follows that $\phi_*^{Z_t}(f)$ and $ch_*^{Z_t}(f)$ define the same element in $\mathrm{Hom}_{Z_t}(H^*(X^t), H^*(Y^t))$. We conclude that the characters ϕ_*^G and ch_*^G agree in the case that G is finite.

For an arbitrary profinite group G the construction of ϕ_*^G in section 3 of [5] shows that one has a commutative diagram

$$
\begin{array}{ccc}
\varinjlim_{H \subset G} KK_*^{G/H}(C(X), C(Y/H)) & \xrightarrow{\cong} & KK_*^G(C(X), C(Y)) \\
\downarrow{\varinjlim \phi_*^{G/H}} & & \downarrow{\phi_*^G} \\
\varinjlim_{H \subset G} \bigoplus_{j \in \mathbb{Z}} H_{G/H}^{*+2j}(X, Y/H) & \xrightarrow{\cong} & \bigoplus_{j \in \mathbb{Z}} H_G^{*+2j}(X, Y)
\end{array}
$$

where the limits are taken over all open normal subgroups H of G which act trivially on X. It is easily verified that there is an analogous commutative diagram where the vertical arrows are replaced by $\varinjlim ch_*^{G/H}$ and ch_*^G, respectively. According to our discussion for finite groups the left vertical maps in these diagrams are equal. Hence the same is true for the right vertical maps ϕ_*^G and ch_*^G. $\qquad\square$

8. The homological Chern character and equivariant Bredon homology

In this section we consider the homological Chern character otained by inserting the complex numbers in the second variable of the equivariant KK-groups. The target of this transformation can be identified with equivariant Bredon homology. We show how to extend the homological Chern character obtained in section 6 appropriately from proper G-simplicial complexes to proper G-CW-complexes.
Let G be a totally disconnected group. We call a G-space X smooth if all isotropy groups G_x for $x \in X$ are open subgroups of G. A G-space X is obtained from the G-space A by attaching smooth equivariant k-dimensional cells if there is a G-equivariant pushout

$$
\begin{array}{ccc}
\coprod_{i \in I} G/H_i \times S^{k-1} & \longrightarrow & A \\
\downarrow & & \downarrow \\
\coprod_{i \in I} G/H_i \times D^k & \longrightarrow & X
\end{array}
$$

where $(H_i)_{i \in I}$ is a family of open subgroups of G. A smooth G-CW-complex is a G-space X together with a G-invariant filtration

$$
\emptyset = X_{-1} \subset X_0 \subset X_1 \subset \cdots \subset \bigcup_{k=0}^{\infty} X_k = X
$$

such that X carries the weak topology with respect to this filtration and for every k the space X_k is obtained from X_{k-1} by attaching smooth equivariant k-dimensional cells. In the sequel all G-CW-complexes are assumed to be smooth. This means in particular that every G-CW-complex can be viewed as an ordinary CW-complex in a natural way. Note that every G-simplicial complex is a G-CW-complex. A G-CW-complex X is called G-finite if the quotient X/G is compact. It is proper iff all isotropy groups G_x for $x \in X$ are compact open subgroups of G. For more detailed information about G-CW-complexes we refer to [17].
We denote by $sd^n(X)$ the nth iterated barycentric subdivision of a G-simplicial complex X. Note that the barycentric subdivision of a G-simplical complex is again a G-simplicial complex. The following equivariant simplicial approximation theorem is a straightforward extension of ordinary simplicial approximation as it can be found, for instance, in [28].

Proposition 8.1. *Let $f : X \to Y$ be an equivariant continuous map between G-simplicial complexes. If X is G-finite there exists a natural number n and an equivariant simplicial map $F : sd^n(X) \to Y$ equivariantly homotopic to f.*

The next proposition contains the basic ingredient needed to extend the equivariant Chern character ch_*^G to G-CW-complexes.

Proposition 8.2. *Let X be a G-finite G-CW complex. Then there exists a G-finite G-simplicial complex which is equivariantly homotopy equivalent to X.*

Proof. We use induction on the dimension of X, the case $\dim(X) = 0$ being clear. Using lemma 2.13 in [18] it suffices to show that the space obtained by attaching a finite number of smooth equivariant k-dimensional cells to a $(k-1)$-dimensional G-simplicial complex Y is equivariantly homotopy equivalent to a G-simplicial complex.

Hence let us consider a G-pushout diagram

$$
\begin{array}{ccc}
\coprod_{i\in I} G/H_i \times \partial\Delta^k & \xrightarrow{\ f\ } & Y \\
\downarrow & & \downarrow \\
\coprod_{i\in I} G/H_i \times \Delta^k & \longrightarrow & X
\end{array}
$$

where Δ^k denotes the standard k-simplex and I is finite. According to proposition 8.1 there exists $n \in \mathbb{N}$ such that the upper horizontal map f is equivariantly homotopic to an equivariant simplicial map $g : \coprod_{i\in I} G/H_i \times sd^n(\partial\Delta^k) \to Y$. Using again lemma 2.13 in [18] it follows that the G-pushouts corresponding to f and g are equivariantly homotopy equivalent. Let us abbreviate $Y_0 = \coprod_{i\in I} G/H_i \times sd^n(\partial\Delta^k)$ and $Y_1 = \coprod_{i\in I} G/H_i \times sd^n(\Delta^k)$ and consider the map $g : Y_0 \to Y$. After replacing Y_0 by an equivariantly homotopy equivalent G-simplicial complex Z_0 we can achieve that neighbouring vertices, that is, vertices which are connected by an edge, are mapped to different vertices in Y. If we also replace Y_1 appropriately by an equivariantly homotopy equivalent G-simplicial complex Z_1 we obtain a G-pushout diagram

$$
\begin{array}{ccc}
Z_0 & \xrightarrow{\ h\ } & Y \\
\downarrow & & \downarrow \\
Z_1 & \longrightarrow & Z
\end{array}
$$

of G-finite G-simplicial complexes and equivariant simplicial maps. Moreover, by construction, in the commutative diagram

$$
\begin{array}{ccccc}
Y_1 & \longleftarrow & Y_0 & \xrightarrow{\ g\ } & Y \\
\downarrow & & \downarrow & & \| \\
Z_1 & \longleftarrow & Z_0 & \xrightarrow{\ h\ } & Y
\end{array}
$$

the left horizontal arrows are G-cofibrations and the vertical maps are equivariant homotopy equivalences. Using once more lemma 2.13 in [18] we conclude that Z and X are equivariantly homotopy equivalent. This proves the assertion. $\square$

Let k^G and h^G be homotopy invariant functors on the category of G-finite proper G-CW-complexes and equivariant maps. Moreover assume that $\phi : k^G \to h^G$ is a natural transformation on the subcategory of G-finite proper G-simplicial complexes and equivariant simplicial maps. We shall show how ϕ can be extended to a natural transformation on the whole category. This is similar to the techniques explained in the appendix of [10].

For a proper G-finite G-CW-complex X let $\mathcal{S}_X$ be the following category. The objects in $\mathcal{S}_X$ are the equivariant homotopy equivalences $S \to X$ where S is a proper G-finite G-simplicial complex. Note that there exist such equivariant homotopy equivalences due to proposition 8.2. A morphism from $R \to X$ to $S \to X$ in $\mathcal{S}_X$ is an equivariant simplicial map $f : R \to S$ such that the diagram

$$
\begin{array}{ccc}
R & \longrightarrow & X \\
\downarrow{\scriptstyle f} & & \| \\
S & \longrightarrow & X
\end{array}
$$

is commutative up to equivariant homotopy. To be precise, we shall rather work with isomorphism classes of G-simplicial complexes in order to achieve that the

category $\mathcal{S}_X$ is small. Let us define

$$Sk^G(X) = \varinjlim_{S \to X} k^G(S)$$

where the limit is taken over $\mathcal{S}_X$. Using proposition 8.1 one checks that $Sk^G(X)$ is a functor on the category of G-finite proper G-CW-complexes. There is a canonical natural transformation $Sk^G \to k^G$ which is an isomorphism since k^G was supposed to be homotopy invariant. In a similar way we obtain a functor Sh^G and a natural isomorphism $Sh^G \to h^G$. For every $S \to X$ in $\mathcal{S}_X$ let us define $\Phi_S(X) : k^G(S) \to Sh^G(X)$ to be the composition $k^G(S) \to h^G(S) \to Sh^G(X)$ of $\phi(S)$ with the canonical map. Passing to the limit we obtain a family of maps $\Phi(X) : Sk^G(X) \to Sh^G(X)$. Using again proposition 8.1 this family is easily seen to be a natural transformation on the category of G-finite proper G-CW-complexes. Due to the natural isomorphisms $Sk^G \cong k^G$ and $Sh^G \cong h^G$ we can view Φ as a natural transformation from k^G to h^G. By construction one obtains $\Phi(X) = \phi(X)$ for every G-simplicial complex X. Moreover, the extended transformation Φ is uniquely determined by ϕ. We have thus proved the following statement.

Proposition 8.3. *Let k^G and h^G be homotopy invariant functors on the category of G-finite proper G-CW-complexes. Every natural transformation $k^G \to h^G$ defined on the subcategory of G-simplicial complexes and equivariant simplicial maps can be uniquely extended to a natural transformation $k^G \to h^G$ on the whole category of G-finite proper G-CW-complexes and equivariant continuous maps.*

Next we recall the definition of Bredon homology [18], [32]. The smooth orbit category $\mathsf{Or}(G)$ of a totally disconnected group G has as objects all homogenous spaces G/H where H is an open subgroup of G. The morphisms in $\mathsf{Or}(G)$ are all G-equivariant maps. One obtains subcategories of $\mathsf{Or}(G)$ by restricting the class of subgroups. We are interested in the class $\mathcal{F}$ of all compact open subgroups of G. The corresponding full subcategory $\mathsf{Or}(G, \mathcal{F})$ of $\mathsf{Or}(G)$ consists of all homogeneous spaces G/H where H is compact open.

We will work over the complex numbers in the sequel. If $\mathcal{C}$ is a small category a covariant (contravariant) $\mathcal{C}$-vector space is a covariant (contravariant) functor from $\mathcal{C}$ to the category of vector spaces. Morphisms of $\mathcal{C}$-vector spaces are natural transformations. More generally one defines covariant and contravariant $\mathcal{C}$-objects as functors with values in arbitrary target categories.

Given a contravariant $\mathcal{C}$-vector space M and a covariant $\mathcal{C}$-vector space N the tensor product $M \otimes_{\mathcal{C}} N$ is the direct sum of $M(c) \otimes N(c)$ over all objects $c \in \mathcal{C}$ divided by the tensor relations $mf \otimes n - m \otimes fn$ for $m \in M(d), n \in N(c)$ and morphisms $f : c \to d$ in $\mathcal{C}$.

Let X be a proper G-CW-complex. There is a contravariant functor from $\mathsf{Or}(G, \mathcal{F})$ to the category of CW-complexes which associates to G/H the fixed point set X^H. Composition with the covariant functor from CW-complexes to chain complexes which associates to a CW-complex Y the cellular chain complex $C_*(Y)$ with complex coefficients yields a contravariant $\mathsf{Or}(G, \mathcal{F})$-chain complex $C_*^{\mathsf{Or}(G,\mathcal{F})}(X)$.

We define a covariant $\mathsf{Or}(G, \mathcal{F})$-vector space $\mathcal{R}_q$ as follows. For a compact open subgroup H of G set

$$\mathcal{R}_q(G/H) = K_q(C^*(H)) \otimes_{\mathbb{Z}} \mathbb{C}$$

where K_* denotes topological K-theory and $C^*(H)$ is the group C^*-algebra of H. Note that $K_0(C^*(H)) = R(H)$ is the representation ring of H and $K_1(C^*(H)) = 0$. The character map induces an isomorphism

$$\mathcal{R}_0(G/H) = K_0(C^*(H)) \otimes_{\mathbb{Z}} \mathbb{C} \cong \mathcal{R}(H)$$

where $\mathcal{R}(H)$ is the ring of conjugation invariant smooth functions on H.
We define a chain complex $C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R})$ by equipping

$$C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R}) = \bigoplus_{p+q=*} C_p^{\mathsf{Or}(G,\mathcal{F})}(X) \otimes_{\mathsf{Or}(G,\mathcal{F})} \mathcal{R}_q$$

with the differential induced from $C_*^{\mathsf{Or}(G,\mathcal{F})}(X)$.

Definition 8.4. *Let G be a totally disconnected group. The equivariant Bredon homology of a proper G-CW-complex X (with coefficients in $\mathcal{R}$) is*

$$\mathcal{B}H_*^G(X;\mathcal{R}) = H_*(C_*^{\mathsf{Or}(G,\mathcal{F})}(X;\mathcal{R})).$$

Let us recall the relation between Bredon homology and cosheaf homology [32]. In section 6 the latter was described in terms of equivariant cyclic homology.

Proposition 8.5. *Let G be a totally disconnected group and let X be a proper G-simplicial complex. Then there is a natural isomorphism*

$$H_*^G(X) \cong \mathcal{B}H_*^G(X;\mathcal{R})$$

where H_*^G denotes cosheaf homology.

We shall now extend the homological Chern character obtained as a special case of theorem 6.7 to a natural transformation on the category of proper G-CW-complexes.

Theorem 8.6. *Let X be a proper G-CW-complex. There exists a natural transformation*

$$ch_*^G : K_*^G(X) \to \mathcal{B}H_*^G(X;\mathcal{R})$$

which is an isomorphism after tensoring the left hand side with $\mathbb{C}$.

Proof. Set $k_*^G = K_*^G$ and let h_*^G be equivariant Bredon homology. According to proposition 8.3 and proposition 8.5, the homological Chern character obtained in theorem 6.7 can be extended to the category of G-finite proper G-CW-complexes. Since both k_*^G and h_*^G are theories with G-compact supports, this transformation extends uniquely to the whole category of proper G-CW-complexes. Moreover theorem 6.7 implies that the extended transformation becomes an isomorphism after tensoring the left hand side with the complex numbers. $\square$

In [18] Lück has constructed Chern characters for proper equivariant homology theories in the context of discrete groups. A proper equivariant homology theory is an assignment which associates to every group G a proper G-homology theory such that the theories for different groups are related by an induction structure. As a special case of this construction Lück obtains a natural isomorphism

$$\lambda_*^G : \mathcal{B}H_*^G(X;\mathcal{R}) \to K_*^G(X) \otimes_{\mathbb{Z}} \mathbb{C}$$

on the category of proper G-CW-complexes for a discrete group G. The construction is compatible with the induction structures of the corresponding equivariant homology theories. In particular, induction from H to G yields a commutative diagram

$$\begin{array}{ccc}
\mathcal{B}H_*^G(G/H;\mathcal{R}) & \xrightarrow{\lambda_*^G} & K_*^G(G/H) \otimes_{\mathbb{Z}} \mathbb{C} \\
\downarrow{\cong} & & \downarrow{\cong} \\
\mathcal{B}H_*^H(\star;\mathcal{R}) & \xrightarrow{\lambda_*^H} & K_*^H(\star) \otimes_{\mathbb{Z}} \mathbb{C}
\end{array}$$

where $\star$ denotes the trivial H-space consisting of a single point. Our Chern character ch_* is compatible with induction from H to G in the same way. Moreover, the bottom horizontal arrow λ_*^H in the above diagram is given by the identity map

$\mathcal{R}(H) \to \mathcal{R}(H) \cong R(H) \otimes_{\mathbb{Z}} \mathbb{C}$. On the other hand, the equivariant Chern character ch_*^H for the space $\star$ is induced by the character map $R(H) \to \mathcal{R}(H)$. This is precisely the map used for the identification $R(H) \otimes_{\mathbb{Z}} \mathbb{C} \cong \mathcal{R}(H)$. Hence the complexified character ch_*^H and the map λ_*^H for the one-point space are inverse to each other.

Let us assume that we are given a natural transformation $\alpha_*^G : \mathcal{B}H_*^G \to \mathcal{B}H_*^G$ of equivariant Bredon homology for every group G. Suppose moreover that each α_*^G is compatible with the boundary maps and $\alpha_*^H(\star) = \mathrm{id}$ for all finite groups H. As explained in [19], if the natural transformations α_*^G are in addition compatible with induction from finite subgroups, then $\alpha_*^G = \mathrm{id}$ on the category of proper G-CW-complexes for all G.

Despite of the above observations, for $\alpha_*^G = ch_*^G \circ \lambda_*^G$ we cannot conclude $\alpha_*^G = \mathrm{id}$ since ch_*^G commutes with the boundary maps only up to a factor $\sqrt{2\pi i}$ and possibly a sign. However, in view of the constructions, the complexification of ch_*^G and the character λ_*^G of Lück are as close as possible to being mutually inverse.

References

[1] Baum, P., Block, J. Higson, N., Chern character for totally disconnected groups, unpublished

[2] Baum, P., Connes, A., Geometric K-theory for Lie groups and foliations, Preprint IHES (1982)

[3] Baum, P., Connes, A., Chern character for discrete groups, in: A fête of topology, 163 - 232, Academic Press, 1988

[4] Baum, P., Connes, A., Higson, N., Classifying space for proper actions and K-theory of group C^*-algebras, in C^*-algebras: 1943 - 1993 (San Antonio, TX, 1993), 241 - 291, Contemp. Math. 167, 1994

[5] Baum, P., Schneider, P., Equivariant bivariant Chern character for profinite groups, K-theory 25 (2002), 313 - 353

[6] Berline, N., Getzler, E., Vergne, M., Heat kernels and Dirac operators, Grundlehren der mathematischen Wissenschaften 298, Springer-Verlag, Berlin, 1992

[7] Blackadar, B., K-theory for operator algebras. Second edition. Cambridge University Press, Cambridge, 1998

[8] Block, J., Getzler, E., Equivariant cyclic homology and equivariant differential forms, Ann. Sci. École. Norm. Sup. 27 (1994), 493 - 527

[9] Connes, A., Noncommutative Geometry, Academic Press, 1994

[10] Dold, A., Lectures on algebraic topology. Second edition. Grundlehren der mathematischen Wissenschaften 200, Springer-Verlag, Berlin - New York, 1980

[11] Hogbe-Nlend, H., Bornologies and functional analysis, North-Holland Publishing Co., 1977

[12] Hogbe-Nlend, H., Moscatelli, V. B., Nuclear and conuclear spaces, North-Holland Publishing Co., Amsterdam-New York, 1981

[13] Kasparov, G. G., The operator K-functor and extensions of C^*-algebras, Izv. Akad. Nauk SSSR Ser. Mat. 44 (1980), 571 - 636

[14] Kasparov, G. G., Equivariant KK-theory and the Novikov conjecture, Invent. Math. 91 (1988), 147 - 201

[15] Kassel, C., Homologie cyclique, caractère de Chern et lemme de perturbation, J. Reine Angew. Math. 408 (1990), 159 - 180

[16] Loday, J-L., Cyclic Homology, Grundlehren der mathematischen Wissenschaften 301, Springer-Verlag, Berlin, 1992

[17] Lück, W., Transformation groups and algebraic K-theory, Lecture Notes in Mathematics 1408, Springer, 1989

[18] Lück, W., Chern characters for proper equivariant homology theories and applications to K- and L-theory, J. Reine Angew. Math. 543 (2002), 193 - 234

[19] Meyer, M., Chern characters for topological groups, PhD thesis, Münster, 2006

[20] Meyer, R., Analytic cyclic homology, Preprintreihe SFB 478, Geometrische Strukturen in der Mathematik, Münster, 1999

[21] Meyer, R., Equivariant Kasparov theory and generalized homomorphisms, K-theory 21 (2000), 201 - 228

[22] Meyer, R., Some analysis and homological algebra related to local cyclic cohomology, preprint

[23] Meyer, R. Smooth group representations on bornological vector spaces, Bulletin des Sciences Mathématiques 128 (2004), 127 - 166

[24] Meyer, R. Bornological versus topological analysis in metrizable spaces. Banach algebras and their applications, 249 - 278, Contemp. Math. 363, 2004

[25] Puschnigg, M., Diffeotopy functors of ind-algebras and local cyclic cohomology, Doc. Math. 8 (2003), 143 - 245

[26] Schneider, P., Equivariant homology for totally disconnected groups, J. Algebra 203 (1998), 50 - 68

[27] Segal, G., Equivariant K-theory, Publ. Math. IHES 34 (1968), 129 - 151

[28] Spanier, E. H., Algebraic topology, McGraw-Hill Book Co., New York-Toronto, 1966

[29] Switzer, R. M., Algebraic topology - homotopy and homology, Grundlehren der mathematischen Wissenschaften 212, Springer-Verlag, New York-Heidelberg, 1975

[30] Voigt, C., Equivariant periodic cyclic homology, J. Inst. Math. Jussieu 6 (2007), 689 - 763

[31] Voigt, C., Equivariant local cyclic homology and the equivariant Chern-Connes character, Doc. Math. 12 (2007), 313 - 359

[32] Voigt, C., A new description of equivariant cohomology for totally disconnected groups, Journal of K-Theory 1 (2008), 431 - 472

[33] Willis, G. A., Totally disconnected groups and proofs of conjectures of Hofmann and Mukherjea, Bull. Austral. Math. Soc. 51 (1995), 489 - 494

Institut for Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark

E-mail address: `cvoigt@math.ku.dk`