The 60th British Applied Mathematics Colloquium

BAMC 2018

Programme

University of St Andrews

26th - 29th March 2018

Edinburgh Mathematical Society
Institute of mathematics & its applications
London Mathematical Society
Science & Technology Facilities Council
Royal Astronomical Society
SIAM UKIE

Conference Programme

<table>
<tr><th></th><th>Monday 26th March</th><th>Tuesday 27th March</th><th>Wednesday 28th March</th><th>Thursday 29th March</th></tr>
<tr><td>8:00 – 8:45</td><td>Registration
(Physics Foyer)</td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>8:45 – 9:00</td><td>Conference Opening
(Physics A)</td></tr>
<tr><td>9:00 – 9:30</td><td rowspan="2">Plenary :
Philippa Browning
(Physics A)</td><td rowspan="2">Plenary:
Coralia Cartis
(Physics A)</td><td rowspan="2">Plenary:
David Abrahams
(Physics A)</td><td rowspan="4">Minisymposia and Contributed Talks</td></tr>
<tr><td>9:30 – 10:00</td></tr>
<tr><td>10:00 – 10:30</td><td>Morning Coffee
(Physics/Medicine)</td><td>Morning Coffee
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(Physics/Medicine)</td></tr>
<tr><td>10:30 – 11:00</td><td rowspan="4">Minisymposia and Contributed Talks</td><td rowspan="4">Minisymposia and Contributed Talks</td><td rowspan="4">Minisymposia and Contributed Talks</td></tr>
<tr><td>11:00 – 11:30</td><td>Morning Coffee
(Physics/Medicine)</td></tr>
<tr><td>11:30 – 12:00</td><td rowspan="2">Plenary:
Luigi Preziosi
(Physics A)</td></tr>
<tr><td>12:00 – 12:30</td></tr>
<tr><td>12:30 – 13:00</td><td rowspan="3">Lunch
(Physics/Medicine)</td><td rowspan="3">Lunch
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(Physics/Medicine)</td><td>Wrap Up and Prizes
(Physics A)</td></tr>
<tr><td>13:00 – 13:30</td><td rowspan="3">Lunch
(Physics/Medicine)</td></tr>
<tr><td>13:30 – 14:00</td></tr>
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<tr><td>14:30 – 15:00</td><td rowspan="8"></td></tr>
<tr><td>15:00 – 15:30</td></tr>
<tr><td>15:30 – 16:00</td></tr>
<tr><td>16:00 – 16:30</td><td>Afternoon Tea
(Physics/Medicine)</td><td>Afternoon Tea
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<tr><td>16:30 – 17:00</td><td rowspan="2">Stewartson Memorial Lecture:
Alfio Quarteroni
(Physics A)</td><td rowspan="2">EMS Public Lecture:
Carlos Frenk
(Physics A)</td><td rowspan="2">IMA
Lighthill Lecture:
Reidun Twarock
(Physics A)</td></tr>
<tr><td>17:00 – 17:30</td></tr>
<tr><td>17:30 – 18:00</td><td></td><td rowspan="2">EMS Wine Reception and Poster Session
(Physics Foyer)</td><td rowspan="2">Poster Session
(Physics Foyer)</td></tr>
<tr><td>18:00 – 19:00</td><td>Reception
(Lower College Hall)</td></tr>
<tr><td>19:00 –</td><td></td><td></td><td>Conference Dinner
(Old Course Hotel)</td></tr>
</table>

Monday 26th March 2018

	Physics A	Physics B	Physics C	Maths B	Maths C	Maths D
10:30 — 12:30	MS 1	MS 2	MS 3	UKMHD 1	CT 1	CT 2
14:00 — 16:00	MS 4	MS 5	MS 6	UKMHD 2 (13:30 start)	CT 3	CT 4

Tuesday 27th March 2018

	Physics A	Physics B	Physics C	Maths B	Maths C
10:30 — 12:30	MS 7	MS 8	MS 9	UKMHD 3	CT 5
14:00 — 16:00	MS 10	MS 11	MS 12	UKMHD 4 (13:30 start)	CT 6

Wednesday 28th March 2018

	Physics A	Physics B	Physics C	Maths B	Maths C
10:30 — 12:30	MS 13	MS 14	CT 7	UKMHD 5	CT 8
14:00 — 16:00	MS 15	MS 16	CT 9	CT 10	CT 11

Thursday 29th March 2018

	Physics A	Physics B	Physics C	Maths B	Maths C
9:00 — 11:00	MS 17	MS 18	CT 12	CT 13	CT 14

Plenary Talks and Named Lectures

Room: Physics A

Monday 26th March 2018

TIME		
9:00	**Philippa Browning** *Relaxation, reconnection and avalanches in solar and laboratory plasma magnetic flux ropes* Chair: Alan Hood	Manchester
16:30	**Alfio Quarteroni** *Numerical models for the heart function* Chair: David Abrahams	Politecnico di Milano

Tuesday 27th March 2018

TIME		
9:00	**Coralia Cartis** *Optimization with expensive and uncertain data - challenges and successes* Chair: Charles Elliott	Oxford
16:30	**Carlos Frenk** *Clues to the identity of the dark matter in our local neighbourhood* Chair: Thomas Neukirch	Durham

Wednesday 28th March 2018

TIME		
9:00	**David Abrahams** *How to Build an Acoustic Metamaterial in Three Days Flat and Other Tall Stories* Chair: Karen Page	Isaac Newton Institute
16:30	**Reidun Twarock** *Geometry as a Key to the Virosphere: New Insights into Virus Structure, Assembly, Evolution & Therapy* Chair: John King	York

Thursday 29th March 2018

TIME		
11:30	**Luigi Preziosi** *Multi-level mathematical models for cell migration in dense fibrous environments* Chair: Mark Chaplain	Politecnico di Torino

Minisymposia

Session MS 1

Monday 26th March 2018, 10:30-12:30

Room: Physics A

From Cell to Tissues: Multiscale Mathematical Approaches for Collective Cell Migration in Cancer Growth and Spread

Organiser and Chair: Dr D Trucu (Dundee)

TIME		
10:30	**Talal Alzahrani** *Multiscale modelling of cancer response to viral therapy*	Dundee
11:00	**Victoria Ponce Bobadilla** *A quantitative framework for understanding cancer cell invasion through in vitro scratch assays*	Heidelberg
11:30	**Arran Hodgkinson** *Spatio-metabolic modelling elucidates resistance and re-sensitisation to treatment in heterogeneous melanoma*	Montpellier
12:00	**Fiona Macfarlane** *Modelling the immune response to cancer: an individual-based approach accounting for the difference in movement between inactive and activated T cells*	St Andrews

Session MS 2

Monday 26th March 2018, 10:30-12:30

Room: Physics B

Waves in Fluids

Organisers and Chairs: Dr A Athanassoulis (Dundee) & Dr M Carr (St Andrews)

TIME		
10:30	**Ricardo Barros** *Strongly nonlinear effects on mode-2 ISW*	Loughborough
10:54	**Ton van den Bremer** *The wave-induced flow of internal gravity wavepackets with arbitrary aspect ratio*	Edinburgh
11:18	**Bertrand Kibler** *Ubiquitous modulation instability: complex breathing scenarios in optics and hydrodynamics*	Bourgogne
11:42	**Miguel Onorato** *On the origin of non Gaussian statistics in equations of the Nonlinear Schrdinger type*	Torino
12:06	**Danielle Wain** *Lateral transport by mode-two internal waves in an enclosed basin*	Bath

Session MS 3

Monday 26th March 2018, 10:30-12:30

Room: Physics C

Numerical Analysis Meets Mathematical Biology

Organisers and Chairs: Dr I Kyza (Dundee) & Dr M Ptashnyk (Heriot-Watt)

TIME		
10:30	**Charles Elliott** *ABC for surface PDEs*	Warwick
10:54	**Ping Lin** *A consistency study of coarse-grained models for a large dynamical particle system*	Dundee
11:18	**Anotida Madzvamuse** *Coupled bulk-surface reaction-diffusion systems: modelling, analysis and simulations*	Sussex
11:42	**Björn Stinner** *Finite element approximation of geometric PDEs coupled with surface PDEs*	Warwick
12:06	**Chandrasekhar Venkataraman** *Modelling receptor-ligand interactions*	St Andrews

Session MS 4

Monday 26th March 2018, 14:00-16:00

Room: Physics A

Multi-scale Soft Tissue Modelling: Upscaling from Cell to Tissue

Organisers: Dr P Stewart (Glasgow), Prof R Ogden (Glasgow), Prof S McDougall (Heriot-Watt) & Dr P Watton (Sheffield)

TIME		
14:00	**Hanadi Alzubadi** *A travelling wave analysis of a prolfierating cell population*	Dundee
14:20	**Roxanna Barry** *Discrete-to-continuum modelling of hyperelastic cells*	Glasgow
14:40	**John King** *Multiphase modelling of tissue growth*	Nottingham
15:00	**Giulia Pederzani** *A mathematical model of cerebral vasospasm and comparison*	Sheffield
15:20	**Raimondo Penta** *Homogenized modeling for vascularized poroelastic materials*	Glasgow
15:40	**Vasiliki Voulgaridou** *Ultrasound response to tumour induced angiogenesis*	Heriot-Watt

Session MS 5

Monday 26th March 2018, 14:00-16:00

Room: Physics B

Constitutive Modelling in Biomechanics

Organisers and Chairs: Dr V Balbi (Galway) & Dr G Zurlo (Galway)

TIME		
14:00	**Martine Ben Amar** *What do we learn about embryo-genesis with nano-ablation: the case of C-elegans elongation?*	LPS ENS
14:20	**Ilaria Cinelli** *Head-to-nerve analysis of electro-mechanical impairments of diffuse axonal injury*	NUI Galway
14:40	**Alexander Erlich** *Morphoelastic dynamics of one-dimensional bio-networks*	Manchester
15:00	**Daniel Garcia Gonzalez** *A viscous-hyperelastic constitutive model for transverse isotropic soft tissues*	Oxford
15:20	**Andrey Melnik** *The generalised structure tensor approach for the mixed invariant I8 and its application to constitutive modelling of passive myocardium*	Glasgow
15:40	**Giuseppe Zurlo** *The constitutive response of bodies undergoing surface growth.*	NUI Galway

Session MS 6

Monday 26th March 2018, 14:00-16:00

Room: Physics C

Multiscale Analysis of Porous Media

Organisers and Chairs: Dr K Daly (Southampton) & Dr D McKay-Fletcher (Southampton)

TIME		
14:00	**Laura Cooper** *Multiphase flow in porous media*	Warwick
14:30	**Keith Daly** *Combining homogenisation theory and image based modelling to predict the poro-elastic properties of multi-constituent soils*	Southampton
15:00	**Simon Duncan** *Multiple scale asymptotic homogenisation of nutrient movement and crop growth in partially saturated soil*	Southampton
15:30	**Daniel McKay-Fletcher** *A multi-image based approach for modelling plant-fertiliser interaction*	Southampton

Session MS 7

Tuesday 27th March 2018, 10:30-12:30

Room: Physics A

Optimal Control and Dynamic Games: Theory and Application

Organisers and Chairs: Prof V Turetsky (Ort Braude College) & Prof A Tsourdos (Cranfield)

TIME		
10:30	**Moshe Idan** *Control strategies for linear systems with Cauchy distributed noises*	Technion, Israel
10:50	**Stéphane Le Menéc** *Linear differential game capture zones in the case of measurement errors*	MBDA, France
11:10	**Eric Rogers** *Optimal control and estimation for long-range AUV missions*	Southampton
11:30	**Martin Weiss** *The minimize-effort-specify-performance approach to guidance algorithm design*	Technion, Israel
11:50	**Chang-Hun Lee** *New insights into optimal control in guidance applications*	Cranfield
12:10	**Vladimir Turetsky** *Tracking error and control effort trade-off in a robust trajectory tracking problem*	Ort Braude College

Session MS 8

Tuesday 27th March 2018, 10:30-12:30

Room: Physics B

Conflicting Attitudes in the History of Mathematical Science

Organiser and Chair: Dr I Falconer (St Andrews)

TIME		
10:30	**Alex Craik** *The hydrostatics of George Sinclair and Robert Boyle*	St Andrews
11:00	**Isobel Falconer** *Maxwell, Kelvin, and the inverse square law of electrostatics*	St Andrews
11:30	**Ben Marsden** *William John Macquorn Rankine and the making of engineering science in nineteenth-century Glasgow*	Aberdeen
12:00	**Mark McCartney** *'Graecum est legi non potest': James Thomson Snr and the teaching of arithmetic, trigonometry and calculus in early 19th century Belfast.*	Ulster

Session MS 9

Tuesday 27th March 2018, 10:30-12:30

Room: Physics C

Frontiers and Challenges in Pattern Formation

Organisers and Chairs: Dr A Krause (Oxford) & Dr T Woolley (Cardiff)

TIME		
10:30	**Andrew Krause** *Emergent dynamics due to spatial heterogeneity in reaction-diffusion systems*	Oxford
10:54	**Anotida Madzvamuse** *Cross-diffusion-driven instability for reaction-diffusion systems on evolving domains and surfaces: models, analysis and simulations*	Sussex
11:18	**Stephen Watson** *Lorentzian symmetry predicts universality beyond power laws*	Glasgow
11:42	**Matthias Winter** *Spike clusters for the Gierer-Meinhardt system*	Brunel
12:06	**Thomas Woolley** *Pattern production through a chiral chasing mechanism*	Cardiff

Session MS 10

Tuesday 27th March 2018, 14:00-16:00

Room: Physics A

Multi-scale Soft Tissue Modelling: Cardiac Electrophysiology and Active Contraction

Organiser and Chair: Dr R Simitev (Glasgow)

TIME		
14:00	**Muhamad Hifzhudin Bin Noor Aziz** *A generic model of single-cell cardiac electrophysiology*	Glasgow
14:30	**Irina Biktasheva** *Cardiac re-entry evolution in MRI-based models of the heart*	Liverpool
15:00	**Peter Mortensen** *Numerical simulations of action potential propagation in cardiac tissues with myocardial infarction scars*	Glasgow
15:30	**Radostin Simitev** *Ignition of waves in excitable systems*	Glasgow

Session MS 11

Tuesday 27th March 2018, 14:00-16:00

Room: Physics B

Public Understanding of Maths; Theory and Practice

Organiser and Chair: Prof C Budd (Bath)

TIME		
14:00	**Chris Budd** *What have mathematicians done for us?*	Bath
14:30	**Alina Loth** *Public engagement as a career – from engaging professionally to an engagement profession*	St Andrews
15:00	**Madeleine Shepherd** *Maths by the back door – engaging new audiences with cross-discipline activities*	ICMS
15:30	**Katie Steckles** *The many faces of mathematical engagement*	Freelance

Session MS 12

Tuesday 27th March 2018, 14:00-16:00

Room: Physics C

Fluctuating Complex Dynamical Systems

Organisers and Chairs: Professor S Kalliadasis (Imperial), Dr P Yatsyshin (Imperial) & Dr M Duran-Olivencia (Imperial)

TIME		
14:00	**Selcuk Atalay** *Systematic and reliable multiscale modelling of lithium batteries*	Heriot-Watt
14:30	**Sergio P. Perez** *Well-balanced finite volume schemes for hydrodynamic equations with general free energy*	Imperial
15:00	**Marc Pradas** *Convergent Chaos*	Open Univeristy
15:30	**Antonio Russo** *Computational challenges in fluctuating hydrodynamics*	Imperial

Session MS 13

Wednesday 28th March 2018, 10:30-12:30

Room: Physics A

Nonlinear Dispersive Waves

Organisers and Chairs: Dr K Khusnutdinova (Loughborough) & Dr E Parau (UEA)

TIME		
10:30	**Lyuba Chumakova** *Leaky GFD problems*	Edinburgh
10:54	**Thibault Congy** *Scattering of linear waves by dispersive hydrodynamic states*	Loughborough
11:18	**Karima Khusnutdinova** *Scattering of long weakly-nonlinear waves in bi-layers with delamination*	Loughborough
11:42	**Emilian Parau** *Fully dispersive equations for hydroelastic waves*	East Anglia
12:16	**Jean-Marc Vanden-Broeck** *New family of gravity solitary of waves*	UCL

Session MS 14

Wednesday 28th March 2018, 10:30-12:30

Room: Physics B

A Snapshot of Scottish Mathematical Biology

Organisers and Chairs: Dr T Lorenzi (St Andrews) & Dr C Venkataraman (St Andrews)

TIME		
10:30	**Mark Chaplain** *Spatial and spatial-stochastic modelling of gene regulatory networks*	St Andrews
11:00	**Jozsef Farkas** *Net reproduction functions for nonlinear structured population models*	Stirling
11:30	**John Mackenzie** *Models and numerics: tools for the dissection of cell Migration and chemotaxis*	Strathclyde
12:00	**Dumitru Trucu** *Novel multiscale modelling approaches in cancer invasion*	Dundee

Session MS 15

Wednesday 28th March 2018, 14:00-16:00

Room: Physics A

Multiscale Modelling in Mathematical Biology

Organisers and Chairs: Dr S Merino-Aceituno (Imperial) & Dr E Zatorska (UCL)

TIME		
14:00	**Jon Chapman** *A mean-field approach to evolving spatial networks, with an application to osteocyte network formation*	Oxford
14:30	**Karen M. Page** *Mesoscopic description of a morphogen-controlled bistable switch*	UCL
15:00	**Anotida Madzvamuse** *A robust and efficient adaptive multigrid solver for the optimal control of phase field formulations of geometric evolution laws with applications to cell migration*	Sussex
15:30	**Mariya Ptashnyk** *Stochastic homogenization for a chemotaxis system*	Heriot-Watt

Session MS 16

Wednesday 28th March 2018, 14:00-16:00

Room: Physics B

Multiscale Soft Tissue Modelling: Parameter Inference

Organisers and Chairs: Prof D Husmeier (Glasgow) & Dr H Gao (Glasgow)

TIME		
14:00	**Vinny Davies** *Fast parameter inference in a computational model of the left-ventricle using emulation*	Leeds
14:24	**Hao Gao** *Mathematical modelling acute myocardial infarction based on magnetic resonance imaging*	Glasgow
14:48	**Alan Lazarus** *Investigating left ventricular geometries from diseased patients and healthy volunteers*	Glasgow
15:12	**Benn Macdonald** *Multiscale soft tissue modelling: parameter inference using gradient matching*	Glasgow
15:36	**Mihaela Paun** *MCMC using Gaussian processes for inference in a partial differential equations model of pulmonary circulation*	Glasgow

Session MS 17

Thursday 29th March 2018, 9:00-11:00

Room: Physics A

Multiscale Soft Tissue Modelling: Circulation and Fluid-Structure Interaction

Organisers and Chairs: Prof X Luo (Glasgow) & Prof N Hill (Glasgow)

TIME		
9:00	**Ankush Aggarwal** *Parameter identification, uncertainty quantification, and design of experiments for soft tissues with exponential nonlinearity*	Swansea
9:24	**Jordi Alastruey-Arimon** *Arterial blood flow modelling using 1D/0D formulations*	KCL
9:48	**Liuyang Feng** *On the chordae structure and dynamic behaviour of the mitral valve*	Glasgow
10:12	**Xiaoyu Luo** *Three-dimensional flows in a hyperelastic tube under external pressure*	Glasgow
10:36	**Jay Mackenzie** *A mathematical model for blood flow in the coronary circulation*	Glasgow

Session MS 18

Thursday 29th March 2018, 9:00-11:00

Room: Physics B

Multiscale Modelling, Analysis and Simulation in Biology and Medicine

Organiser: Prof M Owen (Nottingham)

Chair: Prof M Chaplain (St Andrews)

TIME		
9:00	**Robin Thompson** *Connecting within-host and population-level models of HIV: untangling the mechanisms underlying transmission using data from both scale*	Oxford
9:24	**Nathan Mellor** *Modelling the effect of intercellular plasmodesmata on auxin dynamics at the Arabidopsis root tip*	Nottingham
9:48	**Philip Pearce** *Physical determinants of bacterial biofilm architectures*	MIT
10:12	**Mariya Ptashnyk** *Multiscale modelling and analysis of auxin transport in plant tissues*	Heriot-Watt
10:36	**Alexander Erlich** *Blood flow and solute transfer in feto-placental capillary networks*	Manchester

UKMHD 2018

Session UKMHD 1

Monday 26th March 2018, 10:30-12:30

Room: Maths B

Chair: Dr L Silvers (City)

TIME		
10:30	**Abrar Ali** *The effect of time-dependent γ-pumping on buoyant magnetic structures*	City
10:45	**Daniela Weston** *The influence of turbulent pumping and turbulent diffusion on magnetic buoyancy instability*	Leeds
11:00	**James Hollins** *Mean fields and fluctuations in a compressible, random medium from Gaussian smoothing*	Newcastle
11:15	**Mouloud Kessar** *Scale selection in the stratified convection of the solar photosphere*	Leeds
11:30	**Fryderyk Wilczynski** *Stability of scrape-off layer plasma: a modified Rayleigh-Bénard problem*	Leeds
11:45	**Colin Hardy** *Quasi-analytic inviscid geodynamo solutions*	Leeds
12:00	**Yue-Kin Tsang** *Magnetic power spectrum in a dynamo model of Jupiter*	Leeds
12:15	**Alex Hindle** *Consequences of shallow water magnetohydrodynamic waves on hot Jupiters*	Newcastle

Session UKMHD 2

Monday 26th March 2018, 13:30-16:00

Room: Maths B

Chair: Dr D Pontin (Dundee)

TIME		
13:30	**David Hughes** *Vortex disruption by magnetohydrodynamic feedback*	Leeds
13:45	**Roger Scott** *Magnetic feature Detection through coronal volume segmentation along quasi-separatrix layers*	Dundee
14:00	**Peter Wyper** *A model for coronal hole bright points and jets due to moving magnetic features*	Durham
14:15	**Ross Pallister** *Test particle simulations at tearing of null-point current sheets*	Dundee
14:30	**Jonathan Thurgood** *Implosive collapse about 2D and 3D magnetic null points*	Dundee
14:45	**Dana-Camelia Talpeanu** *Numerical modelling of stealth solar eruptions; simulated and in-situ signatures at 1AU*	Leuven
15:00	**Jivraj Pipaliya** *Diffusive magnetic energy changes direction of the wave propagation with shock normal in earth magnetic ramp region co-related angle between shock normal and upstream magnetic field*	Sheffield
15:15	**Jin Matsumoto** *The stability of the gamma-ray burst jet propagating through a progenitor star*	Leeds
15:30	**Tim Whitbread** *The contribution of individual active regions to the Sun's axial dipole moment*	Exeter
15.45	**Raquel Vaz** *Unforced Navier-Stokes solutions with applications to magnetohydrodynamics*	Imperial

Session UKMHD 3

Tuesday 27th March 2018, 10:30-12:30

Room: Maths B

Chair: Prof D Hughes (Leeds)

TIME		
10:30	**Matthew Allcock** *Solar magneto-seismology with asymmetric waveguides*	Sheffield
10:45	**Mihai Barbulescu** *An analytical model of the Kelvin-Helmholtz Instability of transverse coronal loop oscillations*	Sheffield
11:00	**Timothy Duckenfield** *Detection of the second harmonic of decay-less kink oscillations*	Warwick
11:15	**Tom Howson** *Standing kink oscillations in coronal magnetic flux tubes*	St Andrews
11:30	**Konstantinos Karampelas** *Energy evolution in oscillating gravitationally stratified coronal loops*	Leuven
11:45	**Andrew Hillier** *The non-linear growth of the magnetic Rayleigh-Taylor instability*	Exeter
12:00	**Eun-jin Kim** *Effects of shear flows on the evolution of fluctuations in interchange turbulence*	Sheffield
12:15	**Thomas Williams** *Formation of a dense flux rope by a siphon flow*	Lancaster

Session UKMHD 4

Tuesday 28th March 2018, 13:30-16:00

Room: Maths B

Minisymposium: Helicity in Weakly Dissipative Systems

Organisers and Chairs: Dr A Russell (Dundee), Dr D Pontin (Dundee) & Dr A Yeates (Durham)

TIME		
13.30	**Mitchell Berger** *Absolute measures of helicity*	Exeter
14.00	**Chris Jones** *Kinetic and magnetic helicity in planetary dynamo models*	Leeds
14:30	**Gunnar Hornig** *States of maximum helicity*	Dundee
15.00	**Carlo Barenghi** *The topological of quantum turbulence*	Leeds
15.30	**David MacTaggart** *The emergence of braided magnetic fields*	Glasgow
15.45	**James Threlfall** *Flux rope formation due to shearing and zipper reconnection*	St Andrews

Session UKMHD 5

Wednesday 28th March 2018, 10:30-12:30

Room: Maths B

Chair: Prof A Hood (St Andrews)

TIME		
10:30	**Mat Hunt** *Rotating jets in magnetised fluids*	Brighton
10:45	**Erin Goldstraw** *Modelling coronal magnetic field evolution*	St Andrews
11:00	**Jack Reid** *Coronal energy release by MHD avalanches: continuous driving*	St Andrews
11:15	**Simon Candelaresi** *Estimating the rate of field line braiding in the solar corona by photospheric flows*	Dundee
11:30	**Anthony Yeates** *Consequences of delayed flux emergence in coronal magnetic models*	Durham
11:45	**Patrick Antolin** *Transverse MHD waves from colliding prominence flows*	St Andrews

Contributed Talks

Session CT 1

Monday 26th March 2018, 10:30-12:30

Room: Maths C

Chair: Professor C Budd (Bath)

TIME		
10:30	**Henry Allen** *Mathematical modelling and analysis of the interplay between auxin and brassinosteroid in plant tissues*	Dundee
10:50	**Anahita Bayani** *Modelling anti-inflammatory systems – spatial considerations in the resolution of inflammation*	Nottingham Trent
11:10	**Ruth Bowness** *Exploring dormancy in Mycobacterium tuberculosis using a hybrid discrete-continuum cellular automaton model*	St Andrews
11:30	**Adam Bridgewater** *Application of Lindstedt's method to a delayed model of glucose-insulin regulation*	Northumbria
11:50	**Tamsin Spelman** *Predicting patterns of retinal haemorrhage*	Glasgow
12:10	**Gergely Röst** *Waning and boosting of immunity – challenges in modeling, analysis and numerics*	Oxford

Session CT 2

Monday 26th March 2018, 10:30-12:30

Room: Maths D

Chair: Professor X Luo (Glasgow)

TIME		
10:30	**Nicholas Hill** *Gyrotactic suppression and emergence of chaotic trajectories of swimming particles in three-dimensional flows*	Glasgow
10:50	**Ahmed Ismaeel** *A mathematical model for photothermal ablation of spherical tumors*	Glasgow
11:10	**Peter Stewart** *Optic nerve sheath bleeding driven by rapid cerebrospinal fluid pressure amplification*	Glasgow
11:30	**Laura Sumner** *Steady streaming as a method of drug delivery to the inner ear*	Imperial
11:50	**Adam Yorkston** *The deformation and stability of elastic cells in an inviscid uniform flow*	East Anglia
12:10	**Shreya Seghal** *A bifurcation analysis of spiral waves using a FitzHugh-Nagumo model*	Liverpool Hope

Session CT 3

Monday 26th March 2018, 14:00-16:00

Room: Maths C

Chair: Prof D Dritschel (St Andrews)

TIME		
14:00	**Jonathan Healey** *Fractal sets of neutral curves in stably stratified plane Couette flow*	Keele
14:20	**Rishi Kumar** *Comparison of asymptotic and numerical approaches to plane Poiseuille-Couette flow stability*	Imperial
14:40	**Dan Lucas** *Layer formation and localisation in spanwise stratified plane Couette flow*	Keele
15:00	**Paul Mannix** *Mode interactions in spherical Rayleigh-Bénard convection*	Imperial
15:20	**Omar Al-Tameemi** *Accuracy and stability of virtual source method for numerical simulations of nonlinear water waves*	Plymouth
15:40	**Erietta Moulopoulou** *Numerical investigation of the hydrodynamics and tracking of the inflow and moving bed in a Hele-Shaw geometry*	Leeds

Session CT 4

Monday 26th March 2018, 14:00-16:00

Room: Maths D

Chair: Dr C Howls (Southampton)

TIME		
14:00	**Hamid Alemi Ardakani** *Variational principles for interactions between water-waves and a floating rigid-body with interior fluid motion*	Exeter
14:20	**Valon Blakaj** *Phase-space representation of the wave fields reflected by random rough surfaces*	Nottingham
14:40	**Mohammad Mahdi Jalali** *Shallow flow and Lagrangian particles tracking model for chaotic mixing processes*	St Andrews
15:00	**Sean Jamshidi** *Coastal outflows into a buoyant layer of arbitrary depth*	UCL
15:20	**Scott Richardson** *Bioconvection in a horizontally oriented cylinder rotating about its axis*	Glasgow
15:40	**Timothy Whiteley** *An integro-differential equation model for urban population dynamics predicting emergent pattern formation*	Nottingham

Session CT 5

Tuesday 27th March 2018, 10:30-12:30

Room: Maths C

Chair: Dr D Trucu (Dundee)

TIME		
10:30	**Simon Pearce** *How to bend a microtubule*	Manchester
10:50	**Noemi Picco** *Modelling transient properties of cortex formation highlights the importance of evolving cell division strategies*	Oxford
11:10	**Aleksandra Plochocka** *Robustness of the microtubule cytoskeleton self-organisation*	Heriot-Watt/Edinburgh
11:30	**Anne Skeldon** *Sleep/wake inspired insights on the creation of gaps and non-monotonicity in circle maps*	Surrey
11:50	**Gleb Zhelezov** *Coalescing particle systems in models of chemotaxis*	Edinburgh
12:10	**Mauro Mobilia** *Eco-evolutionary dynamics in a randomly switching environment: competition and cooperation*	Leeds

Session CT 6

Tuesday 27th March 2018, 14:00-16:00

Room: Maths C

Chair: Professor C Breward (Oxford)

TIME		
14:00	**Giorgio Carta** *Reduction of low-frequency vibrations in long structures by using gyro-elastic beams*	Liverpool John Moores
14:20	**Marta Garau** *Interfacial waves in chiral elastic structured media*	Keele
14:40	**Neekar Mohammed** *Tunnelling corrections to wave transmissions on shell structures*	Nottingham
15:00	**Michael Nieves** *Roto-flexural waves in elastic beams with gyro-hinges*	Keele & Cagliari
15:20	**Armin Krupp** *Inferring retention mechanisms from flux–throughput measurements is an ill-posed problem*	Oxford
15:40	**Hayley Wragg** *Propagation of signals from indoor small cells at ultra-high frequencies*	Bath

Session CT 7

Wednesday 28th March 2018, 10:30-12:30

Room: Physics C

Chair: Professor J Chapman (Oxford)

TIME		
10:30	**Chris Breward** *The kinetic barrier in surfactant adsorption and desorption*	Oxford
10:50	**Quenton Parsons** *Step formation in thin-film diblock-copolymers via a phase-field model with free surfaces*	Oxford
11:10	**Peter Yatsyshin** *Microscopic aspects of wetting using classical density-functional theory*	Imperial
11:30	**Maxim Zyskin** *Modeling actuators based on carbon nanotube yarn/nanoenergetic material composites*	Nottingham
11:50	**Attila Kovacs** *Mathematical modelling of alumina feeding*	Oxford
12:10	**Monisha Subbiah Renganathan** *Ionic diffusion model for the oxidation of uranium*	Manchester

Session CT 8

Wednesday 28th March 2018, 10:30-12:30

Room: Maths C

Chair: Professor N Mottram (Strathclyde)

TIME		
10:30	**Jonathan Deakin** *Optimal coordinate transformations for the perfectly matched layer method*	Manchester
10:50	**Csaba Katai** *Effects of streamwise elongated and spanwise periodic surface roughness elements on boundary layers*	Imperial
11:10	**Joseph Cousins** *Axisymmetric nematic liquid crystal squeeze film: low Ericksen number*	Strathclyde
11:30	**Taysir Dyhoum** *Detection of multiple inclusions and quantification of contact impedances from ERT data using the complete-electrode model Ahmed*	Leeds
11:50	**Mubarack Ahmed** *Transport of energy and information through networks of cables*	Nottingham
12:10	**Kate Powers** *Modelling surge in centrifugal compressors*	Bath
12:30	**Eun-jin Kim** *Effect of enhanced dissipation by shear flows on transient relaxation and probability density function*	Sheffield

Session CT 9

Wednesday 28th March 2018, 14:00-16:00

Room: Physics C

Chair: Professor N Hill (Glasgow)

TIME		
14:00	**Will Booker** *Internal wave attractors in stratified fluids*	Leeds
14:20	**Radu Cimpeanu** *Early-time jet formation in liquid-liquid impact problems*	Oxford
14:40	**James Munro** *The initial dynamics of drop coalescence*	Cambridge
15:00	**Thomas Goodfellow** *Instability of elevator modes in oscillatory double-diffusive convection*	Leeds
15:20	**Ying Huang** *Dynamically coupled shallow-water sloshing in two connected vessels*	Surrey
15:40	**Clint Wong** *Fluid flow through submerged vegetation*	Oxford

Session CT 10

Wednesday 28th March 2018, 14:00-16:00

Room: Maths B

Chair: Professor J Healey (Keele)

TIME		
14:00	**Avan Al-Saffar** *Population performance from the perspective of information theory*	Sheffield
14:20	**Peter Hicks** *Gas-cushioned droplet impacts in the compressible gas regime*	Aberdeen
14:40	**Godwin Madho** *Predicting chaotic behaviour using data assimilation*	Leeds
15:00	**Stanislav Mazurenko** *Acceleration and global convergence of a first-order primal–dual method for non-convex optimisation problems*	Liverpool
15:20	**Kgomotso Morupisi** *Non smooth model of Ice Ages*	Bath

Session CT 11

Wednesday 28th March 2018, 14:00-16:00

Room: Maths C

Chair: Prof J-M Vanden-Broeck (UCL)

TIME		
14:00	**Jack Atkinson** *Dynamics of a trapped vortex in rotating convection*	Cambridge
14:20	**Helen Burgess** *Vortex scaling ranges in two-dimensional turbulence*	St Andrews
14:40	**Xavier Carton** *Vortex merger near a continental shelf*	UBO
15:00	**David Dritschel** *Imperfect bifurcation for the quasi-geostrophic shallow-water equations*	St Andrews
15:20	**Jean Reinaud** *Binary quasi-geostrophic vortex interactions in a shear induced by a surface buoyancy filament*	St Andrews
15:40	**Younsi Abdelhafid** *A new criterion for uniqueness for weak solutions of the 3D Navier-Stokes equations*	Djelfa, Algeria

Session CT 12

Thursday 29th March 2018, 9:00-11:00

Room: Physics C

Chair: Dr J Reinaud (St Andrews)

TIME		
9:00	**Anna Kalogirou** *Interfacial stability of multilayer shear flows with surfactants*	East Anglia
9:20	**Daniel Peck** *Penny-shaped hydraulic fracture accounting for shear stress induced by the fluid*	Aberystwyth
9:40	**Ian Thorp** *Motion of non-axisymmetric particles in viscous shear flow*	Cambridge
10:00	**Robert Timms** *A boundary layer analysis of reactive shear bands*	East Anglia
10:20	**Norjan Jumaa** *Implementation of a lattice Boltzmann method for multiphase flows with high density and viscosity ratios*	Plymouth
10:40	**Seemaa Mohammed** *Using dipole wall collision to validate slip and no slip moment-based boundary conditions for the lattice Boltzmann equation*	Plymouth

Session CT 13

Thursday 29th March 2018, 9:00-11:00

Room: Maths B

Chair: Professor B Sleeman (Leeds)

TIME		
9:00	**Nurkanat Aimakov** *Wave scattering in coupled composite plates*	Nottingham
9:20	**Puneet Matharu** *The direct computation of time-periodic solutions of PDEs & applications to fluid dynamics*	Manchester
9:40	**Koji Okitani** *Applications of a Cole-Hopf transform to the 3D Navier-Stokes equations*	Sheffield
10:00	**Raphael Stuhlmeier** *Evolution of statistically inhomogeneous degenerate water wave quartets*	Plymouth
10:20	**Ruaa Wana** *Smoothed particle hydrodynamics (SPH) modelling of tsunami waves generated by a fault rupture*	Plymouth
10:40	**Oliver Dunbar** *First Traveltime Tomography*	Warwick

Session CT 14

Thursday 29th March 2018, 9:00-11:00

Room: Maths C

Chair: Dr C Venkataraman (St Andrews)

TIME		
9:00	**Long Chen** *Predicting self-organisation in conducting fluids*	Durham
9:20	**Alex Doak** *Steady travelling wave solutions on an axisymmetric ferrofluid jet*	UCL
9:40	**Dane Grundy** *The effect of surface stress on solitary waves*	East Anglia
10:00	**Paul Hammerton** *Effect of molecular relaxation of nonlinear evolution of N-waves*	East Anglia
10:20	**Kristian Kiradjiev** *Surface-tension-driven flow with liquid injection*	Oxford
10:40	**Feargus Schofield** *The influence of the thermal properties of the system on the evolution of an evaporating droplet*	Strathclyde

Abstracts of Plenary Talks

Relaxation, reconnection and avalanches in solar and laboratory plasma magnetic flux ropes

Philippa Browning - University of Manchester

Magnetic flux ropes - twisted bundles of magnetic field lines - are ubiquitous and fundamental structures in astrophysical and laboratory plasmas. Importantly, they are reservoirs of free magnetic energy, whose release may explain, for example, both large-scale solar flares and heating of the solar corona. This energy release may lead to plasma heating and the acceleration of non-thermal electrons and ions. I will give an overview of modelling of energy release in magnetic flux ropes through the process of magnetic reconnection, triggered by the onset of the ideal kink instability. Two complementary modelling approaches are used: relaxation theory, which assumes a relaxation to a state of minimum magnetic energy, and 3D magnetohydrodynamic simulations. The latter may be coupled with test-particle simulations in order also to model particle acceleration. First, I will consider an individual unstable flux rope, presenting models of flaring solar coronal loops. Then I will discuss interactions between multiple flux ropes, showing how one unstable flux rope within an array of stable neighbours may trigger a heating avalanche (as previously postulated by cellular automaton models). Some implications for the solar corona and for spherical tokamak fusion plasmas will be described.

Optimization with expensive and uncertain data - challenges and successes

Coralia Cartis - Mathematical Institute, University of Oxford
Lindon Roberts
Katya Scheinberg
Jan Fiala

Real-life applications often require the minimization/maximization of nonlinear functions with several unknowns or parameters - where the function is the result of highly expensive and complex model simulations involving noisy data (such as climate or financial models, chemical experiments), or the output of a black-box or legacy code, that prevent the numerical analyst from looking inside to find out or calculate problem information such as derivatives. Thus classical optimization algorithms, that use derivatives (steepest descent, Newton's methods) often fail or are entirely inapplicable in this context. Efficient derivative-free optimization algorithms have been developed in the last 15 years in response to these imperative practical requirements. As even approximate derivatives may be unavailable, these methods must explore the landscape differently and more creatively. In state of the art techniques, clouds of points are generated judiciously and sporadically updated to capture local geometries as inexpensively as possible; local function models around these points are built using techniques from approximation theory and carefully optimised over a local neighbourhood (a trust region) to give a better solution estimate.

In this talk, I will describe our improvements and implementations to state-of-the-art, model-based trust-region, methods. In the context of the ubiquitous data fitting/least-squares applications, we have developed an approach that uses highly flexible local models in terms of number of points (and hence function) evaluations needed to construct them; this allows us to make progress in the algorithm from very little problem information when the latter is preciously expensive, and also to incorporate additional sampling in the presence of noise. We show superior numerical performance of our techniques compared to state of the art methods. I will also prove robustness of these methods, even when the noise is biased or when sampling may not be sufficiently accurate, hence when we only have accurate local models occasionally. Despite derivative-free optimisation methods being able to only provably find local optima, we illustrate that, due to their construction and applicability, these methods can offer a practical alternative to global optimisation solvers, with improved scalability.

Clues to the identity of the dark matter in our local neighbourhood

Carlos Frenk - Institute for Computational Cosmology, University of Durham

One of the most impressive advances in Physics and Astronomy of the past thirty years is the development of the "standard model of cosmology" LCDM (where L stands for Einstein's cosmological constant and CDM for cold dark matter). Theoretical predictions formulated in the 1980s turned out to agree remarkably well with measurements, performed decades later, of the galaxy distribution and the temperature structure of the microwave background radiation. Yet, these successes do not inform us directly about the nature of the dark matter. Indeed, there are competing (and controversial) claims that the dark matter might have already been discovered, either through the annihilation of cold, or the decay of warm, dark matter particles. In an astrophysical context the identity of the dark matter manifests itself clearly in the properties of dwarf galaxies, such as the satellites of the Milky Way. I will discuss predictions from cosmological simulations assuming cold and warm (in the form of sterile neutrinos) dark matter and show how astronomical observations can, in principle, distinguish between these as well as other possibilities.

The mathematical theory of models for atmospheric flows

Beatrice Pelloni - Heriot-Watt University
Michael Cullen
Mark Wilkinson

The mathematical and numerical analysis of models for atmospheric flows is still a widely open problem. The semi-geostrophic equations are a particular system of partial differential equations, obtained as reduction of the full Euler system of equations, and used in the modelling of large-scale atmospheric flows. In this review, we first describe the history of the equations, and their validity as a geophysical model. We then review the mathematical ideas and results that have enabled spectacular progress in the past 25 years. This progress has seen rigorous proofs of several global and local existence results, in 2 and 3 dimensions, for various boundary value problems posed. However, in no case are we able to prove uniqueness of solutions. I will discuss why this is a significant and interesting problem from both a theoretical and computational point of view.

Multi-level mathematical models for cell migration in dense fibrous environments

Luigi Preziosi - Politecnico di Torino

Cell-extracellular matrix interaction and the mechanical properties of the cell nucleus have been demonstrated to play a fundamental role in cell movement across fibre networks and micro-channels and then in the spread of cancer metastases.

This talk will focus on several mathematical models dealing with such a problem, starting from modelling cell adhesion mechanics to the inclusion of the influence of nucleus stiffness in the motion of cells, through continuum mechanics, kinetic models and individual cell-based models.

Numerical models for the heart function

Alfio Quarteroni - Politecnico di Milano and EPFL, Lausanne

Mathematical models based on first principles allow the description of blood motion in the human circulatory system, as well as the interaction between electrical, mechanical and fluid-dynamical processes occurring in the heart. This is a classical environment where multi-physics processes have to be addressed. Appropriate numerical strategies can be devised to allow for an effective description of the fluid in large and medium size arteries, the analysis of physiological and pathological conditions, and the simulation, control and shape optimization of assisted devices or surgical prostheses. This presentation will address some of these issues and a few representative applications of clinical interest.

Geometry as a Key to the Virosphere: New Insights into Virus Structure, Assembly, Evolution & Therapy

Reidun Twarock - University of York

Viruses are remarkable examples of symmetry in biology. Many viral pathogens package their genetic material into protein shells that are organised with overall icosahedral symmetry. These containers, called capsids, act akin to Trojan horses, protecting the viral genomes between rounds of infection. Mathematical techniques from group, graph and tiling theory can be used to characterise capsid architecture, and better understand the mechanisms underpinning virus assembly. In this talk, I will demonstrate how mathematics has helped to uncover a mechanism underpinning the assembly of many important pathogens, including Hepatitis B and C virus and Picornaviruses, a family that includes the common cold. I will show that characteristic features of this mechanism are shared across different viral strain variants and therefore lend themselves as targets for novel forms of broad spectrum anti-viral therapy. Using mathematical models of viral evolution in the context of a viral infection and an implicit fitness landscape, I will moreover demonstrate that therapeutic interventions directed against these conserved viral features have advantages over conventional forms of therapy. I will also discuss how the new mathematical and mechanistic insights can be exploited in bio-nanotechnology for vaccination or gene delivery purposes.

Abstracts of Minisymposia Talks
and Contributed Talks

Parameter identification, uncertainty quantification, and design of experiments for soft tissues with exponential nonlinearity

Ankush Aggarwal - Swansea University
Yue Mei
Sanjay Pant

Many of the soft tissue constitutive relationships contain an exponential function that models the nonlinear behavior of fiber recruitment and rotation. This nonlinearity results in a highly nonlinear and ill-conditioned parameter estimation problem. In this talk, I will propose some simple modifications based on elementary algebra, which significantly improve the convergence of the inverse model to estimate parameters. I will present the basic case with two elastic parameters and a generic extension to multiple parameters.

I will also present an information theoretic approach to quantify the uncertainty in parameters. This is done by looking at decrease in the Shannon entropy of parameters, once the measurements are taken into account. Using the modifications proposed above, I will show that the information gain is improved, especially in the presence of noise.

Finally, based on the gain in information, I will present a framework for designing optimal set of experiments that can provide us with the maximum confidence in result parameters for a given soft tissue model. I will present results for biaxial testing of a tissue to determine the number protocols and for in-vivo imaging of heart valves to determine the number of frames required.

Transport of energy and information through networks of cables

Mubarack Ahmed - University of Nottingham
Gabriele Gradoni
Stephen Creagh
Gregor Tanner

High-frequency cables commonly connect modern devices and sensors. Interestingly, the proportion of electric components is rising fast in an attempt to achieve lighter and greener devices. Significant research has focused on this area to help replace the hydraulics in aeroplanes, for example (the “more electric aircraft”). Modelling the propagation of signals through these cable networks in the presence of parameter uncertainty, is a daunting task. In this work, we study the response of high-frequency cable networks using both Transmission Line and Quantum Graph (QG) theories. We have successfully compared the two theories in terms of reflection spectra using measurements on real, lossy cables. We have derived a generalisation of the vertex scattering matrix to include non-uniform networks – networks of cables with different characteristic impedances and propagation constants.

The QG model implicitly takes into account the pseudo-chaotic behaviour of the propagating electric signal. We have successfully compared the asymptotic growth of eigenvalues of the Laplacian with the predictions of the Weyl law. We investigate the nearest-neighbour level-spacing distribution of the resonances and compare our results with the predictions of Random Matrix Theory (RMT). To achieve this, we will compare our graphs with the generalisation of Wigner distribution recently proposed by Poli *et al*. for open systems.

The problem of scattering from networks of cables can also provide an analogue model for wireless communication in highly reverberant environments. In this context we provide a preliminary analysis of the statistics of communication capacity for communication across cable networks, whose eventual aim is to enable detailed laboratory testing of information transfer rates using software defined radio. We specialise this analysis in particular for the case of MIMO (Multiple-Input Multiple-Output) protocols.

Wave scattering in coupled composite plates

Nurkanat Aimakov - University of Nottingham
Gregor Tanner
Dimitrios Chronopoulos

Establishing the wave propagation characteristics at plate junctions is of great importance in the prediction of vibrational energy transmission across complex structures. A hybrid approach combining the finite element (FE) and the wave finite element (WFE) method at interconnects between flat, isotropic plates has recently been established for calculating reflection and transmission coefficients. The method is based on modelling joints such as an L-shaped joint with FE with boundary conditions given by the solutions of the WFE method for the infinite plate. We extend this method to study two dimensional anisotropic plates. Numerical results for the bending wave transmission across coplanar and L-shaped junctions are presented. Comparisons of numerically predicted scattering coefficients with analytical solutions for selected structures are used to validate the model. The results obtained are important for Statistical Energy Analysis (SEA) and Dynamical Energy Analysis (DEA) based calculations of wave energy distribution of the full structure.

A numerical study of initial flow past an oscillating circular cylinder

Qasem Al-Mdallal - UAEU

This presentation is a numerical study of the initial flow past a circular cylinder with combined stream-wise and transverse oscillations. The motion is governed by the two-dimensional unsteady Navier-Stokes equations in non-primitive variables. The method of solution is based on conjugating the perturbation theory with the collocation method. The development of the physical properties of the ow at early times is captured. Comparisons with existing results verify the accuracy of the present results.

Population performance from the perspective of information theory

Avan Al-saffar - Sheffield
Avan Al-saffar
Eun-jin Kim

The evolution of different dynamical systems has been investigated from the point of view of information theory. In particular, Fisher information has been employed as a measure of variability/sustainability. Our goal in this work is to investigate the system of three differential equations describing the interaction dynamics between cancer, immune and healthy cells from the point of view of Fisher information. Specifically, we examine in detail the system's evolution under different parameter values in the case of inclusion of perturbations in the form of immunotherapy. A further consideration in our model is that the immune population is promoted and cancer population is forced to vanish by analyzing a particular immunisation term. As a result, the population of cancer cells is negatively affected by a periodic therapy. In other words, prey density will break down at sufficiently specific doses of immunotherapy.

We relate our Fisher information results to the Probability Density Function (PDF) of the system at specific parameter values. In particular, one interesting feature of this work is the optimal value of amplitude (a_{23} =1.5) where the cancer cells started to vanish at this amplitude and we confirm that by exploring Fisher information dynamics with different values of amplitude, as a result, a Fisher information peak is observed at (a_{23} =1.5) which means less variability, with a shortest distance between the two PDF peaks at this amplitude value.

Accuracy and Stability of Virtual Source Method for Numerical Simulations of Nonlinear Water Waves

Omar Al-Tameemi - University of Plymouth
Omar Al-Tameemi
David Graham

The Virtual Source Method (VSM) is based upon the integral equations derived by using Green's identity with Laplace's equation for the velocity potential. The velocity potential within the fluid domain is completely determined by the potential on a virtual boundary located above the fluid, avoiding the need to evaluate the singular integrals normally associated with integral equation methods.

This talk will present numerical simulations of non-linear standing waves and sloshing problems using VSM. We will discuss stability and convergence of the method as well as looking at global and energy and volume conservation.

Arterial blood flow modelling using 1D/0D formulations

Jordi Alastruey - King's College London

Reduced-order models, generally consisting of one-dimensional (1-D) and/or lumped parameter (0-D) equations, offer an efficient way to simulate arterial blood flow in vascular networks and interactions between the heart and the vasculature. While the basic methodology is well established, the focus is now shifting to experimental and clinical validation of these models, to quantification of uncertainty, and to clinically relevant applications. The presentation will discuss some of these aspects, with a particular emphasis on challenges and translational opportunities of reduced-order models.

The presentation will start with an overview on different tests carried out to assess the accuracy of the 1-D/0-D formulation. These include comparisons of arterial pulse wave signals (blood pressure, blood flow, and luminal area waveforms) against *in vivo* data in humans, *in vitro* data in a 1:1 scale cardiovascular simulator rig of the aorta and its larger branches, and numerical data obtained by solving the full 3-D equations in compliant domains. The results of these comparisons show that the 1-D/0-D approach provides a good compromise between accuracy and computational cost for simulating arterial pulse waveforms.

Reduced-order models enable the assessment and development of methods to diagnose and monitor cardiovascular function based on the analysis of pulse wave signals. The presentation will show some clinically relevant applications involving 1-D/0-D models, including the estimation and analysis of central blood pressure and the quantification of aortic stiffness from non-invasive pulse-wave and imaging data. We will focus on blood pressure and arterial stiffness since both are important predictors of life-threatening cardiovascular events. The tools produced in both projects were initially developed and tested using a population of thousands of 'virtual' (computed) subjects, each with distinctive pulse waveforms generated using 1-D/0-D modelling (www.haemod.uk/virtual-database). Once optimised using synthetic data, the tools were then assessed *in vivo* and applied to study clinical cohorts.

Reduced-order modelling provides a good balance between accuracy and computational cost, making it appealing for clinical applications involving the analysis of pressure, flow and area waveforms.

Variational principles for interactions between water-waves and a floating rigid-body with interior fluid motion

Hamid Alemi Ardakani - University of Exeter

New variational principles are given for the two-dimensional interactions between gravity-driven water waves and a floating rectangular vessel dynamically coupled to its interior potential flow with uniform vorticity. The complete set of equations of motion for the exterior water waves, the exact nonlinear hydrodynamic equations of motion for the vessel in the roll/pitch, sway/surge and heave directions, and also the full set of equations of motion for the interior fluid of the vessel, relative to the body coordinate system attached to the rotating–translating vessel, are derived from two Lagrangian functionals.

The Effect of Time-Dependent γ-pumping on Buoyant Magnetic Structures

Abrar Ali - City, University of London
Lara Silvers

An investigation dedicated to greater understand the process of magnetic buoyancy, at the base of the solar convection zone, is presented. We explore the interactions of the net downward, time-dependent, γ-pumping of turbulent convective motions overlying an imposed layer of magnetic fluid, in a polytropic atmosphere. Results from our numerical calculations show that an equipartition of energy, between the magnetic and kinetic components, must be reached for buoyancy-driven magnetic structures to rise into the pumping region. However, structures do not rise unhindered, as in a previous investigation, and can be pushed back down. The evolution, and other features of the emerging magnetic flux tubes are significantly affected by the temporal variation of the γ-pumping. The rate of emergence, strength of magnetic concentrations and extent to how far magnetic field can travel were all found to depend on the time-scale of the γ-pumping.

Numerical Solution of the Ostrovsky equation with a Variable Topography

Azwani Alias - Universiti Malaysia Terengganu
Nik Nur Amiza Nik Ismail

In the real-world phenomena, internal nonlinear waves are often propagating through a non-uniform background medium known as variable topography. The effect of variable topography on the propagation of oceanic internal solitary waves in the absence of background rotation is relatively well studied and understood by variable-coefficient Korteweg-de Vries type equations. However, the works that collaborate with background rotation and various variable topography, known as variable-coefficient Ostrovsky equations is less understood. The aim of this work is to study the effects of various variable topography on the propagation of the internal nonlinear waves in the presence of rotation effect. Numerical simulation such as pseudo-spectra method will be used to analyze these effects. The intention is to get deeper understanding of the propagation of nonlinear wave packets with various variable topography.

Solar magneto-seismology with asymmetric waveguides

Matthew Allcock - University of Sheffield
Noemi Zsámberger
Róbert Erdélyi

Propagating MHD waves are guided along magnetic structures in the solar atmosphere. However, these waves are rarely observed as the symmetric kink and sausage modes that the community is familiar with. Rather, we see modulation and asymmetry. With a series of animations, we will demonstrate how asymmetry arises and how one can use solar magneto-seismology (SMS) to diagnose background parameters of the asymmetric waveguide that are difficult to measure using

traditional methods.

Two novel SMS tools are presented: the *amplitude ratio method* (ARM) and the *minimum perturbation shift method* (MPSM). These techniques use the observed asymmetry in MHD waves to diagnose, for example, the magnetic field strength in solar atmospheric structures such as elongated magnetic bright points, prominences, and sunspot light walls. We also present the surprising result that observationally symmetric MHD waves can propagate along asymmetric waveguides, which brings into question our present identification of MHD modes.

Mathematical modelling and analysis of the interplay between auxin and brassinosteroid in plant tissues

Henry Allen - University of Dundee
Mariya Ptashnyk

Plant hormone auxin has key roles in growth and development, many of which are defined by the heterogeneous distribution of auxin in tissues. It is thought that interactions between auxin signalling and auxin's efflux carrier protein PINFORMED (PIN), where PIN turnover within cells and PIN membrane localisation are governed by auxin and the auxin signalling pathway, are responsible for heterogeneous auxin patterning. It is observed that, in addition to auxin dynamics, a balance between auxin patterning and expression of brassinosteroid (BR), another key plant hormone, is required for optimal growth of plant tissues.

In this work, we derive a mathematical model that links the auxin signalling pathway with PIN-dependent auxin flux by establishing auxin signalling-mediated PIN turnover based upon experimental observations, as well as assuming that PIN membrane localisation is governed by the flux of auxin between neighbouring cells. A system of nonlinear ordinary differential equations is used to describe the dynamics of auxin, PIN, and the molecules involved in the auxin signalling pathway in a discrete representation of plant tissue. We show that our model can reproduce biologically observed patterns under certain conditions, and we also perform mathematical analysis to illustrate the conditions on the model parameters that are necessary to achieve various types of patterns in the distribution of auxin in tissue. When considering auxin flux through the apoplast we show that the dynamics of auxin's influx carrier protein AUXIN RESISTANT (AUX1) are essential for biologically realistic heterogeneous distributions of auxin. By including the crosstalk between the BR and auxin signalling pathways we analyse the impact of interactions between auxin and BR on auxin flux and distribution in a plant tissue.

Multiscale Modelling of Cancer Response to Viral Therapy

Talal Alzahrani - University of Dundee
Raluca Eftimie
Dumitru Trucu

Oncolytic viruses (OV) are modified engineered viruses with the ability of replication within cancer cells and that lead to cancer cells lysis. In this work, we present a novel multiscale moving boundary modelling for the tumour-OV interactions, which is based on coupled systems of partial differential equations both at macro-scale (tissue-scale) and at micro-scale (cell-scale) that are connected through a double feedback link. At the macro-scale, we account for the coupled dynamics of uninfected cancer cells, OV-infected cancer cells, extracellular matrix (ECM) and oncolytic viruses. At the same time, at the micro scale, we focus on essential dynamics of urokinase plasminogen activator (uPA) system which is one of the important proteolytic systems responsible for the degradation of the ECM, with notable influence in cancer invasion. While sourced by the cancer cells that arrive during their macro-dynamins within the outer proliferating rim of the tumour, the uPA micro-dynamics is crucial in determining the movement of the macro-scale tumour boundary (both in terms of direction and displacement magnitude).

In this investigation, we consider two scenarios for the macro-scale tumour-OV interactions. While assuming the usual context of reaction-diffusion-taxis coupled PDEs for the cancer cells (both uninfected and OV-infected) and ECM, in a first scenario for the OV-dynamics, we consider that per unit time, OV exercises a local diffusion in the presence of a natural replication source term and accounting at the same time for the OV-tumour infection rate. However, a second scenario for the OV-dynamics further considers that the OV-local movement is not only caused by local diffusion but it is also influenced by an arising ECM-OV-taxis process. Finally, we explore the special case of syncytia formation and we discuss its dynamics and influence in tumour progression within the context of the two OV-scenarios already considered. To explore these complex tumour-OV interactions numerically, we developed a new multiscale computational framework, and its results will be presented and discussed. While further investigation is needed to validate the findings of our modelling, for the parameter regimes that we considered, our numerical simulations indicate that the viral therapy leads to control and decrease of the overall cancer expansion and in certain cases this can result even in the elimination of the tumour.

A travelling wave analysis of a prolfierating cell population

Hanadi Alzubadi - University of Dundee
Hanadi Alzubadi
Philip Murray

Cell monolayers are a widely used tool in tumour and tissue repair studies. In many cell lines cell proliferation is contact-inhibited such that the cell population reaches confluence at long times. Given suitable initial conditions, moving fronts of proliferation and migration can be observed prior to confluence, with two key measurable quantities being the width and propagation speed of the proliferating region. In this study we consider the continuum limit of a model of an off-lattice, cell-based simulation of a contact inhibited cell monolayer. Numerical solutions of the continuum model, which can be formulated as a nonlinear diffusion free boundary (nonlinear Stefan) problem, indicate a travelling wave behaviour that is subsequently investigated using a travelling wave analysis. Considering first a simplified linear problem, we perform a travelling wave analysis that identifies a small parameter, the natural logarithm of the compression ratio of cells at which contact inhibition stops proliferation, from which asymptotic expressions for the wave speed and proliferating rim width are derived. Subsequently, we use pertur- bation theory to derive approximations for the wave speed and proliferating rim width in the case of a nonlinear diffusion coefficient that represents a linear force law.

Transverse MHD waves from colliding prominence flows

Patrick Antolin - University of St Andrews
Paolo Pagano
Ineke De Moortel

In this work, we analyse coordinated observations between Hinode/SOT and IRIS of a prominence/coronal rain complex at the limb of the Sun. Cool and dense downflows and upflows were observed along a loop-like structure stemming from a prominence core high above the solar surface. A collision between a downward and an upward flow is observed to generate MHD transverse and sausage modes of ~20 km/s amplitude, and a short-lived brightening event with the plasma temperature increasing to at least 10^5 K. We confirm this scenario through 2D MHD simulations of plasma flow collision. The density and velocity differences between the colliding clumps and the strength of the magnetic field are the major parameters defining the response to the collision. Asymmetries between the clumps (angle of impact surface and offset of flowing axis) define to a minor extent the nature of the resulting MHD waves. Using the observed values, we successfully reproduce the observed transverse perturbations and brightening. The numerical modelling indicates that the plasma beta in this loop-like structure is confined between 0.05 and 0.24. These results suggest that a source of transverse waves could be in these collisions from counter-streaming flows, expected from thermal instability at the origin of the condensations forming the prominence.

Systematic and Reliable Multiscale Modelling of Lithium Batteries

Selcuk Atalay - Heriot-Watt University
Markus Schmuck

Lithium-ion batteries play a significant role in daily life and industry, e.g. mobile devices, electrical tools, and electrical vehicles due to their beneficial features such as higher energy and power densities. Since understanding structural properties of batteries is of considerable significance to battery performance, we investigate two basic cells: (i) a single intercalation host and (ii) a periodic arrangement of intercalation hosts. We take systematically Li-ion transport and interfacial reactions into account and compute the associated characteristic voltage vs. state of charge curves under various geometric and design parameters. We expect this fundamental model formulation to be promising for future predictive battery studies and design optimization.

Dynamics of a Trapped Vortex in Rotating Convection

Jack Atkinson - Cambridge University
Peter Davidson
James Perry

The subject of eye formation in vortex flow is one that has received a good deal of attention from the meteorological and atmospheric fluid dynamics communities in recent years. This is due to it being a feature shared by a number of different environmental phenomena that fall in to the class known as atmospheric vortices, examples of which include tropical cyclones, tornadoes, dust devils, and waterspouts (Sinclair 1973, Golden 1974). Despite receiving this attention there is still no clear consensus as to how these flows develop the region of weak, reversed meridional flow at the axis which is known as an eye (Pearce 2005a,b; Smith 2005).

We present numerical simulations of an idealised atmospheric vortex; we consider a disk-like axisymmetric system containing a Boussinesq fluid with a heated no-slip lower surface and a cooled free-slip upper surface rotating at constant angular velocity. Under these conditions a swirling laminar flow develops as a result of convection to give a single meridional cell between the axis and the outer boundary. We observe that above a certain critical level of forcing this flow can develop a region of weak reversed flow at the axis which we call an eye. These results match those of Oruba et al. (2017) supporting their theory of eye formation in cyclonic vortices.

We extended our simulations to higher levels of forcing and observe that above a second critical level the flow bifurcates and the eye begins to display oscillatory behaviour moving away from, and then back towards the axis. We examine the nature of these oscillations and how the frequency and amplitude changes as forcing is increased, characterising the nature of the bifurcation. Finally, we seek to provide a physical explanation for the oscillations through the presence of a trapped inertial wave at the centre of the vortex. We examine the motion of the eye, and also the tilting of the angular momentum contours that can generate vorticity, comparing these to the modes of a trapped inertial wave in a cylinder or frustum (Beardsley 1970). We find that the oscillations fall within the expected frequency range for an inertial wave based upon the local angular velocity of the flow, and that the motions in the eye are similar to those of inertial waves in other systems supporting this as a theory for the mechanism driving the oscillations.

An Analytical Model of the Kelvin-Helmholtz Instability of Transverse Coronal Loop Oscillations

Mihai Barbulescu - University of Sheffield
Michael S. Ruderman
Tom Van Doorsselaere
Robert Erdelyi

Recent numerical simulations have demonstrated that transverse coronal loop oscillations are susceptible to the Kelvin-Helmholtz instability (KHI) due to the counter-streaming motions at the boundary of the loop. We present the first study of this mechanism using an analytical model. The region at the loop boundary where the shearing motions are the greatest is treated as a straight interface separating time-periodic counter-streaming flows. In order to consider a twisted tube, the magnetic field at one side of the interface is inclined. We find that the stability of the displacement at the boundary is governed by Mathieu's equation and we analyse its parametric stability. We prove that the inclination of the magnetic field stabilises the flows and reduces the instability increment, but that there always exist unstable perturbations. This result suggests that the KHI at the boundary of coronal loops is a widespread phenomenon. It also explains the numerically found fact that magnetic twist does not prevent the KHI at the boundary of an oscillating magnetic tube. Studying the stability of oscillating coronal loops is important since the turbulent behaviour caused by the KHI may lead to heating via ohmic dissipation.

The topological of quantum turbulence

Carlo Barenghi - Newcastle University
Carlo F. Barenghi

Superfluidity (the absence of viscous forces) is a well-known property of quantum fluids such as superfluid liquid helium and atomic Bose-Einstein condensates. Another property - the discrete nature of vorticity - is perhaps more important. In this talk I shall show how this property allows the introduction of new ways to characterize the topological complexity of the turbulence by associating to each vortex loop a knot invariant called the Alexander polynomial.

Strongly nonlinear effects on mode-2 ISW

Ricardo Barros - Loughborough University

Large amplitude internal waves in a three-layer flow confined between two rigid walls will be examined in this talk. The mathematical model under consideration arises as a particular case of the multi-layer model proposed by Choi (2000) and is an extension of the two-layer MCC (Miyata-Choi-Camassa) model. The model can be derived without imposing any smallness assumption on the wave amplitudes and is well-suited to describe internal waves within a strongly nonlinear regime. We will investigate its solitary-wave solutions and particular focus will be given to mode-2 ISW. Some strongly nonlinear effects on these waves will be illustrated by asymptotic and numerical solutions to the underlying dynamical system.

Discrete-to-continuum modelling of hyperelastic cells

Roxanna Barry - University of Glasgow
Peter Stewart
Nicholas Hill

Predictive modelling of the human cardiac cycle requires accurate tissue-scale models for the deformation of the heart wall in response to an electrical signal. There are many such macroscale mechanical models for the heart wall, typically constructed by fitting a given constitutive model for the tissue to experimental/clinical measurements of heart wall deformation. However, a significant challenge in multiscale modelling of the heart is to rationally derive macroscale models which incorporate microscale properties. Previous work has typically focussed on homogenisation techniques, which rely on underlying periodicity at the microscale. Here, we address the need for multiscale mechanical models for living tissues (the heart wall) which rationally encode the properties of the individual constituent cells at the microscale, namely the cardiac muscle cells (myocytes), extracellular matrix and collagen fibres.

In this work we derive new multiscale models for cardiac tissues which incorporate the mechanical properties of individual myocytes without assuming homogeneity, symmetry or periodicity at the individual cell level. In particular, we form a reduced mechanical model for the elastic deformation of individual cardiac cells, incorporating both an active response (contraction) to external electrical stimulus as well as a passive elastic response. We then couple these discrete units to form a large-scale network model to describe the deformation of the human left ventricle, where each constituent cell has independent mechanical and electrical properties. Furthermore, we utilise a discrete-to-continuum (asymptotic) approach to rationally upscale this microscale discrete network model to form novel (nonlinear) macroscale continuum PDE models for the overall mechanical deformation of the tissue. This new formulation allows for gradients in mechanical and electrical properties across the tissue and allows us to consider tissues where the mechanical properties of the cardiac cells are vastly different between different regions of the heart (for example in disease or following a myocardial infarction.

Predictions of stress and strain in the system are compared in two locations in the domain. We demonstrate that these predictions of these discrete and continuum models agree well. These predictions can be used to quantify macro scale tissue deformation in the left ventricle over a heartbeat. We further compare our new model to existing cardiac tissue models.

We use discrete-to-continuum upscaling with a mechanical microscale model of cardiac cells, to form new (continuum) PDE models for the mechanical deformation of the heart wall, rationally encoding the mechanical and electrical (active) properties of individual cardiac cells.

Modelling anti-inflammatory systems – spatial considerations in the resolution of inflammation

Anahita Bayani - Nottingham Trent University
Jonathan J Crofts
Joanne L Dunster
Martin R Nelson

There is growing interest in inflammation due to its involvement in myriad medical conditions. Recent investigations show that inflammation is actively controlled by anti-inflammatory processes that can be modulated therapeutically. Accordingly, the mechanisms that resolve inflammation are of great interest, the interactions between macrophages and neutrophil-mediated pro-inflammatory processes in particular.

Existing mathematical models describing macrophage-neutrophil interactions are limited by their design, which generally takes the approach of spatially averaging biological quantities across the affected tissue. Assuming spatial homogeneity becomes increasingly spurious as the spatial scale of the damage increases, with clusters of neutrophils causing significant local tissue damage, while inflammation may resolve elsewhere. Furthermore, recent evidence points to aspects of the inflammatory response being modified under aging and trauma, with changes in e.g. directed neutrophil motility and the macrophage functional response potentially influencing long-term outcomes.

We extend an existing ordinary differential equations (ODE) model to incorporate spatial information regarding inflammatory mediators and immune cells. We analyse the resulting partial differential equation model's solutions, illustrating that the model admits spatially inhomogeneous solutions (steady states and sustained oscillations) that the corresponding ODE model neglects. We examine the impact of changes to key biological mechanisms, in terms of permissible solutions, with a view to simulating new therapeutic interventions.

What do We Learn about embryogenesis with Nano-ablation: The Case of C. elegans Elongation?

Martine Ben Amar - LPS ENS

Morphogenesis of tissues, like the deformation of an object, results from the interplay between mechanical forces and their material properties. Whereas the importance of mechanical forces in influencing cell behaviour is widely recognized, the importance of tissue material properties has received much less attention. Using *C. elegans* as a model, we examine how both aspects contribute to embryonic elongation. Measuring the opening shape of the epidermal actin cortex after laser nano-ablation, the spatiotemporal changes of actomyosin-dependent force and stiffness along the antero-posterior and dorso-ventral axis have been considered. Experimental data and analytical modelling show that the force anisotropy in the lateral epidermis induces the morphological changes of the embryo. By contrast, the elongation of the fiber-reinforced dorso-ventral epidermis is mainly directed by stiffness anisotropy. These experimental results establish a quantitative link between cortical tension, material properties and morphogenesis of an entire embryo but ask the fundamental question of the biological origin of this tension.

Absolute Measures of Helicity

Mitchell Berger - University of Exeter

I will consider some new developments in absolute measures of helicity (as opposed to measures relative to a vacuum field). These measures are defined for arbitrary foliations of space by simply connected surfaces (e.g. start with a set of nested spheres with spherical coordinates, then deform the spheres). They are based on generalizations of Poloidal-Toroidal decompositions. One application lies in measuring the helicity contained within the Northern hemisphere interior of the sun (or Southern hemisphere). Helicity flow into a hemisphere during solar minimum is a good predictor of the strength of the next cycle, with some advantages over using polar flux alone.

Cardiac re-entry evolution in MRI-based models of the heart

Irina Biktasheva - University of Liverpool
Vadim Biktashev
Arun Holden
Sanjay Kharche
Eleftheria Pervolaraki
Girish Ramlugun

Despite over a century of study, mechanisms of cardiac arrhythmias are poorly understood. The recently discovered new phenomenon of dissipative vortices interaction with sharp variations of thickness in excitable layer suggests a cardiac re-entry interaction with fine anatomical features such as pectinate muscles and terminal crest in atria might cause considerable displacement of the re-entry's established localisation compared to where it was first initiated. With the recent advance in DT-MRI technology, more detail models of heart anatomy become available for inclusion into anatomically realistic computer simulations of cardiac arrhythmias, providing a fascinating in-silico test bed with unimpeded access to the whole heart, at a greater spatial and temporal resolution than in experiment, in order to test the effects of a patient anatomy on cardiac re-entry dynamics.

Phase-space representation of the wave fields reflected by random rough surfaces

Valon Blakaj - University of Nottingham
Stephen Creagh
Gabriele Gradoni
Gregor Tanner

In order to overcome the bottleneck of wireless bandwidth, it has been proposed that new wireless communication systems and 5G to operate at high frequencies, ranging from microwave (6 GHz) to mmWave (above 30 GHz). Wave propagation prediction is a challenging task at these frequencies and will need new modelling tools compared to those used for current wireless systems and also needs inclusion of the complexity of the environment at much finer scale. We introduce here a linear operator formulation of ray tracing, the so-called dynamical energy analysis method (DEA), which is a phase-space method for predicting the distribution of waves intensities in complex environments. However, the focus of this work is on integration of scattering from random rough surfaces into DEA type calculations. We present here a method for calculating reflecting wave correlation functions from random rough surfaces driven by statistical sources. This leads to the phase-space representation of wave fields in phase-space. By exploiting the Wigner distribution function (WDF) properties, we provide a model to compute the scattering correlation function from random rough surfaces. We present a scattering operator, which gives an explicit connection between the incident field and the reflected field. Furthermore, we present the integration of scattering operator into numerical phase-space methods, such as DEA. This method has applications in predicting wave field distribution in wireless communications, where one has to take into account effects of scattering. In addition to this, we present also an approach for computing the local means of the reflected correlation functions from random rough surfaces.

Internal wave attractors in stratified fluids

Will Booker - University of Leeds
Onno Bokhove
Mark Walkley

Confined internal gravity wave systems in asymmetric domains can lead to the evolution of a spatial singularity called a wave attractor. The existence of this singularity is the result of successive wave energy focusing, that is wave energy becomes localised on small spatial scales. It has been observed that ocean ridges, such as the Luzon strait, can support such energy localisation and a study of this mechanism will lead to a better understanding of ocean mixing in these areas.

We consider a computational model of stratified fluids using a non-dissipative Hamiltonian discontinuous finite element method. By conserving discrete energy our numerical method allows for accurate long time modelling of wave attractors. Three formulations of an inviscid, ideal fluid model are considered: compressible, incompressible and Euler-Boussinesq. In each case internal waves are generated by applying a body force to the fluid at rest in an asymmetric domain and the resulting system is monitored for the presence of wave attractors. Analytic techniques, such as ray tracing and stream function solutions, can be used to predict the geometry of a wave attractor in certain asymmetric domains. The comparison of these solutions to our numerical generated wave attractors allows us to verify the geometry of the singularities we generate, but can not confirm the dynamics of how the wave attractor is formed from rest. Hence to validate our model, we carried out experiments to generate wave attractors. A body force was applied to a wave tank containing stratified water, and the resulting wave patterns were captured with the synthetic schlieren technique. We observe good agreement between the inviscid numerical model and the viscous experimental model.

Exploring dormancy in Mycobacterium tuberculosis using a hybrid discrete-continuum cellular automaton model

Ruth Bowness - University of St Andrews
Mark Chaplain
Gibin Powathil
Stephen Gillespie

Tuberculosis (TB) is an infectious bacterial disease caused by *Mycobacterium tuberculosis*. Despite significant recent advances, TB is the biggest infectious killer globally with someone dying from the disease every 18 seconds. When *Mycobacterium tuberculosis* bacteria enter the lungs, a complex immune response ensues and results in the formation of granuloma structures. When these granulomas are unable to contain the bacteria, active disease develops. At different degrees of disease severity, patients seek medical assistance, after which antibiotics are prescribed. The degree of antibiotic penetration into and through the granuloma is uncertain. The outcome of treatment is complicated by dormancy when the bacteria become temporarily resistant to antibiotics. We have developed a hybrid discrete-continuum cellular automaton model to study disease progression and treatment in the lung. The model contains discrete agents, or individuals, which model the spatio-temporal interactions (migration, binding, killing etc.) of bacteria, macrophages and T cells. The spatial movement of cells is governed by biased random walks, while the various cell-cell and cell-bacteria interactions are governed by cellular automaton rules. Chemokine diffusion, oxygen diffusion and antibiotic diffusion are also incorporated in the model via the numerical solution of appropriate PDEs. Several definitions and theories regarding bacterial dormancy exist in the literature. In this work, we use our hybrid cellular automaton model to explore several concepts of dormancy and their effect on treatment outcome.

The kinetic barrier in surfactant adsorption and desorption

Chris Breward - University of Oxford

Surfactants are chemical species that are distinguished by two key properties. The first property is that they preferentially reside at interfaces, while the second is that, when adsorbed at an interface, they reduce the local surface energy. These two properties are related, with the adsorption driven by the reduction in surface energy. Often when modelling surfactant transport, the adsorbed surface surfactant is assumed to be in thermodynamic equilibrium with the surfactant just below the surface, and there is a local functional relationship between these two species. The surface energy (surface tension) is then determined using another equilibrium relationship. When the timescale for adsorption at the surface is important, the evolution of the surface concentration can be described by an ODE and such models and are commonly known as “mixed-kinetic” or “barrier-limited” models. However, the same equilibrium relationship between the surface tension and surface concentration is normally used. In this talk, we will describe and explore a theory in which the surfactant transport to and from an interface and the surface energy are related in a self-consistent way.

Application of Lindstedt’s method to a delayed model of glucose-insulin regulation

Adam Bridgewater - Northumbria University
Benoit Huard

Negative feedback loops are observed in many biological processes and are crucial in maintaining key elements within the human body at a dynamic equilibrium. This equilibrium may be either: (1) stable, leading to constant levels over time; or (2) unstable, leading to sustained oscillations.

In this contribution, we look to investigate the ultradian oscillations of both glucose and insulin that are known to occur in patients without diabetes but are lost in patients with type 2 diabetes.

More specifically, we focus on the effect of two diabetic parameters (representing insulin resistance and insulin secretion) on the period and amplitude of the oscillations. This is achieved by applying Lindstedt's perturbative method on a reduced one-delay model of the system used in to obtain approximate analytical expressions for the limit cycle. The model is then extended to include a commensurate delay. In both cases, the approximate expressions for the amplitude and period are a close match to the numerical solution, allowing the quantitative description of each model parameter's contribution to the oscillations.

What have mathematicians done for us?

Chris Budd - University of Bath

Let's face it, maths has generally got a pretty bad image, with the public, with the media, with policy makers and (sadly) many young people. Some of the reasons for this is that maths is thought of as useless, boring, and irrelevant to peoples' lives. Of course, this is completely opposite to the truth. In fact mathematics has been one of the driving forces in human civilisation from the very earliest times. The problem is, how can we convince everyone that this is true.

In this talk I will provide what I hope is useful ammunition to anyone working with any of the groups above, or indeed anyone at a party that confesses that they are a mathematician and is then asked to justify their existence. I hope that at the end of the talk that there will be no doubt about what mathematicians really have done for us.

Vortex Scaling Ranges in Two-Dimensional Turbulence

Helen Burgess - University of St Andrews
David Dritschel
Richard Scott

We review the role played by long-lived coherent vortices in freely evolving and forced two dimensional turbulence, and discuss a new way of conceptualising scaling theories for the distributions of vortex areas and intensities, and their time evolutions. Vortex populations in two-dimensional turbulence are many-body systems far from equilibrium, and such systems generically exhibit multiple regimes with distinct power law behaviour, each associated with transport of a conserved quantity across scales. Canonical examples in two-dimensional turbulence are the enstrophy cascade, in which classical theory predicts a k-independent enstrophy flux to small scales, and the inverse energy cascade, in which a k-independent energy flux to large scales is expected (Kraichnan 1967, Batchelor 1969). Here we extend these inertial range arguments to the vortex subfield in forced two-dimensional turbulence, with transport across scales in vortex area space mediated by vortex interactions taking the place of transport through wavenumber space. We construct a theoretical framework involving a three-part, time-evolving vortex number density distribution, $n(A) \sim t^{\alpha}{}_{i} A^{-r}{}_{i}$, $i \in 1,2,3$, conserving the first three moments of $\omega_v{}^2 n(A)$ in three distinct scaling ranges. Here $\omega_v{}^2$ is the `vortex intensity', or mean square vorticity evaluated over vortices, and n(A) is the number density distribution of vortices with area A, where areas are defined as intense regions of vorticity bounded by vorticity isolines. Conservation is enforced in `comoving intervals', whose endpoints evolve at the vortex growth rate; this amounts to assuming invariance under the dilatation of flow features associated with the inverse cascade, and in addition assuming that the growth in vortex area is the appropriate measure of dilatation in all scaling ranges. We discuss how the same conceptual framework can be applied to model vortices in decaying two-dimensional turbulence, signing the way toward a unified description of vortex populations in the two systems. The predictions are well-verified by high resolution numerical simulations, and insensitive to the vorticity threshold used to isolate the regions. We further discuss modifications to the theory taking into account time- and A-dependent vortex intensity $\omega_v{}^2$.

Estimating the rate of field line braiding in the solar corona by photospheric flows

Simon Candelaresi - University of Dundee
David Pontin
Anthony Yeates
Gunnar Hornig

We study the effect of photospheric motions on the braiding and tangling of coronal magnetic fields. For that we make use of horizontal velocity data that was extracted from magnetograms using the local correlation tracking technique. By tracing trajectories we are able to compute the injected field line winding and finite time topological entropy.
Both are quantitative measures of tangling that can be compared to the blinking vortex motion as a benchmark. Here we show that through photospheric motions there can indeed be substantial injection of winding into the system that can potentially be transfered into magnetic loops. This can potentially lead to thin magnetic structures that are potential sites of magnetic energy loss.

Reduction of low-frequency vibrations in long structures by using gyro-elastic beams

Giorgio Carta - Liverpool John Moores University
Ian Jones
Natasha Movchan
Alexander Movchan
Michael Nieves

Vibrations in engineering structures due to earthquakes are generally difficult to suppress since they occur at low frequencies and are characterised by a wide spectrum. It has been proposed to install multi-scale high-contrast resonators to reduce the vibrations generated by seismic waves in structural systems. The idea is that the energy coming from the external source is diverted from the main structure to the resonators, which act as “energy sinks”. Computations in the frequency and transient regimes demonstrate the effectiveness of the approach, whose drawback is that the resonators need to possess a large mass or to be connected by soft links.

An efficient alternative method is where the resonators consist of beams with distributed gyricity. These structural elements, named “gyro-elastic beams” or “gyrobeams”, exhibit many eigenfrequencies in the low-frequency range, hence they can be used as efficient resonators to mitigate low-frequency vibrations without requiring a high mass to stiffness ratio.

The notion of a gyrobeam was first formulated where the governing equations of a beam with a continuous distribution of stored angular momentum were derived. In this work it is shown that the number of eigenfrequencies in a finite low-frequency interval increases as the gyricity coefficient is increased. Furthermore, in a periodic structure made of repetitive frames containing gyro-elastic beams, large stop-bands are created in the low-frequency regime by the presence of gyrobeams. Numerical computations are presented that illustrate the effectiveness of a system of gyrobeams for the purpose of vibration isolation. In particular, they show that in a structure subjected to an external excitation the frequency intervals where waves propagate are reduced significantly if gyrobeams are attached to the structure.

Finally, we mention that gyro-elastic beams can be realised in practice by attaching gyroscopic spinners to Euler-Bernoulli beams.

Vortex merger near a continental shelf

Xavier Carton - Université de Bretagne Occidentale / IUEM / LOPS
Mathieu Morvan

The merger of two identical vortices near a localized bottom topographic slope is studied with one- and two-layer quasi-geostrophic models. Both models show that the sloping topography facilitates vortex merger but diminishes its efficiency. Indeed, the topographic wave formed under the influence of the primary (two) vortices, breaks into secondary vortices. An analytical solution for this topographic wave in the linear regime is provided. The secondary vortices interact with the primary, advect them towards the shelf (if the primary vortices are cyclones), or offshore (for an initial pair of anticyclones). They can also strongly shear out the primary vortices, into elongated filaments. The impact of such interactions on cross-slope tracer transport is assessed. In a stratified fluid, the interaction between the upper (primary) vortices and bottom topography, is mediated by the lower layer (secondary) vortices. Though a few nonlinear regimes differ between vortex interactions in homogeneous and stratified fluids, the main effects remain similar. Scientific perspectives on this subject and on its applicability to ocean dynamics are finally proposed.

Spatial and Spatial-Stochastic Modelling of Gene Regulatory Networks

Mark Chaplain - University of St Andrews
Cicely Macnamara
Elaine Mitchell
Mariya Ptashnyk
Marc Sturrock

Transcription factors are important molecules which control the levels of mRNA and proteins within cells by modulating the process of transcription (the mechanism by which mRNA is produced within cells) and hence translation (the mechanism by which proteins are produced within cells). Transcription factors are part of a wider family of molecular interaction networks known as gene regulatory networks (GRNs) which play an important role in key cellular processes such as cell differentiation, division and apoptosis (e.g. the Hes-1, p53-Mdm2, NF-KappaB pathways). Transcription factors exert control over molecular levels through feedback mechanisms, with proteins binding to gene sites in the nucleus and either up-regulating or down-regulating production of mRNA. In many GRNs, there is a negative feedback in the network and the transcription rate is reduced. Typically, this leads to the mRNA and protein levels oscillating over time and also spatially between the nucleus and cytoplasm. When experimental data for such systems is analysed, it is observed to be noisy and, in many cases, the actual numbers of molecules involved are quite low. In order to model such systems accurately and connect with the data in a quantitative way, it is therefore necessary to adopt a stochastic approach as well as take into account the spatial aspect of the problem. In this talk, we give an overview of recent work in the area by formulating and analysing spatio-temporal and stochastic spatio-temporal models of GRNs (Hes-1, p53-Mdm2, NF-KappaB), also including synthetic GRNs e.g. repressilators and activator-repressor systems.

A Mean-Field Approach to Evolving Spatial Networks, with an Application to Osteocyte Network Formation

Jon Chapman - University of Oxford
Jake Taylor-King
Mason Porter
David Basanta

We consider evolving networks in which each node can have various associated properties (a state) in addition to those that arise from network structure. For example, each node can have a spatial location and a velocity, or some more abstract internal property that describes something like social trait. Edges between nodes are created and destroyed, and new nodes enter the system. We introduce a "local state degree distribution" (LSDD) as the degree distribution at a particular point in state space. We then make a mean-field assumption to derive an integro-partial differential equation that is satisfied by the LSDD. We perform numerical experiments and find good agreement between solutions of the integro-differential equation and the LSDD from stochastic simulations of the full model. To illustrate our theory, we apply it to a simple continuum model for osteocyte network formation within bone, with a view to understanding changes that may take place during cancer.

Predicting self-organisation in conducting fluids

Long Chen - Durham University
Anthony Yeates

In conducting fluids, twisted magnetic field lines may relax to a simple equilibrium state as if they are self-organised. This relaxation process is turbulent, and often requires extensive computational resources to model. However, recent research has shown that certain quantities are quasi-conserved. The field line helicity, which measures the winding of magnetic flux, is such an example: it is approximately redistributed but not destroyed. If we consider these quasi-conserved quantities, the equilibrium state may be predictable through constrained optimization.

Here we construct a testing model where the field line helicity A(x) is mapped to a 2D periodic domain. Given an initial state, we hope to obtain an optimal distribution of A(x) that matches with the equilibrium state obtained through magnetic relaxation. Preliminary tests have shown tantalising results for simple cases. We plan to extend this approach to more complex and realistic cases.

Leaky GFD problems

Lyubov Chumakova - University of Edinburgh
Lyubov Chumakova
Rodolfo R. Rosales
Andrew Rzeznik
Esteban G. Tabak

Semi-open physical domains that support dispersive waves are ubiquitous in nature, with the troposphere being one example. Such geometry allows for the energy to be lost due to waves propagating out of the domain of interest and never returning. This energy dissipation mechanism is quite efficient and amounts to a significant physical effect. However, the mathematical complication is that decaying dispersive waves have exponential envelopes, thus the classic group velocity concept does not apply to them. We developed mathematical tools to solve this family of problems by working directly with exponential waves and extending the concept of group velocity to this case. In this talk we will demonstrate this approach on classic GFD problems.

Early-time jet formation in liquid-liquid impact problems

Radu Cimpeanu - University of Oxford
Matthew Moore

We consider the problem of high speed vertical impact of two droplets of the same fluid, investigating this type of challenging interfacial flows by extending the analytical framework of Wagner theory, coupled with validation by means of direct numerical simulation. In particular, we show that the theoretical predictions for the location, angle and velocity of the jet-root are excellent in the early stages of the impact, while also favourably comparing the jet velocity profile before the jet begins to bend. The mathematical model is derived for general droplet velocities and radii, which encompasses a wide range of impact scenarios from the symmetric impact of identical drops to liquid drops impacting a deep pool. Our numerical simulations are performed using the open-source volume-of-fluid-based package Gerris, which provides the level of adaptive grid refinement necessary for a problem with such a wide variety of lengthscales. The numerical simulations incorporate more of the physics of the problem, in particular the surrounding gas, the fluid viscosities, gravity and surface tension. In light of the successful comparisons, we discuss the tangible benefits of using Wagner theory to confidently track properties such as the jet-root location, thickness and velocity in future studies of splash jet evolution.

Head-To-Nerve Analysis of Electro-Mechanical Impairments of Diffuse Axonal Injury

Ilaria Cinelli - NUI Galway
Michel Destrade
Peter McHugh
Maeve Duffy

To investigate mechanical and functional failure of diffuse axonal injury (DAI) in nerve bundles following frontal head impacts, by finite element simulations.

Anatomical changes following traumatic brain injury are simulated at the macroscale by using a 3D head model. Frontal head impacts at speeds of induce mild to moderate DAI in the white matter of the brain. Investigation of the changes in induced electro-mechanical responses at the cellular level is carried out in two scaled nerve bundle models, one with myelinated nerve fibres, the other with unmyelinated nerve fibres. DAI occurrence is simulated by using a real-time fully coupled electro-mechanical framework, which combines a modulated threshold for spiking activation and independent alteration of the electrical properties for each 3-layer fibre in the nerve bundle models. The magnitudes of simulated strains in the white matter of the brain model are used to determine the displacement boundary conditions in elongation simulations using the 3D nerve bundle models.

At high impact speed, mechanical failure occurs at lower strain values in large unmyelinated bundles than in myelinated bundles or small unmyelinated bundles; signal propagation continues in large myelinated bundles during and after loading, although there is a large shift in baseline voltage during loading; a linear relationship is observed between the generated plastic strain in the nerve bundle models and the impact speed and nominal strains of the head model.

The myelin layer protects the fibre from mechanical damage, preserving its functionalities.

Scattering of linear waves by dispersive hydrodynamic states

Thibault Congy - Loughborough University
Gennady El
Mark Hoefer
Nicolas Pavloff

Description of the scattering of surface waves by external obstacles or due to the propagation past variable topography is a well-studied and fundamental problem of fluid mechanics. In this work, we investigate a different type of scattering where the scatterer is an evolving large-scale fluid state satisfying the same equation as the scattered wave. As an example, we consider dispersive hydrodynamic scattering of shallow water waves in the framework of the Korteweg-de Vries (KdV) equation. The KdV equation supports the propagation of two well-known dispersive hydrodynamic states: the smooth rarefaction waves (RWs) and the rapidly oscillating dispersive shock waves (DSWs), and we study the scattering of linear waves by these two fundamental nonlinear structures. We derive modulation equations describing the wavepacket-mean flow interaction and show that the linear wave is either transmitted or trapped inside the hydrodynamic state. We identify two adiabatic invariants of motion determining the transmission and trapping conditions and show that the wavepackets incident upon smooth expansion waves or compressive, rapidly oscillating DSWs exhibit the so-called hydrodynamic reciprocity recently described in the context of hydrodynamic soliton tunnelling.

Multiphase Flow in Porous Media

Laura Cooper - University of Warwick
Keith Daly
Nicolai Koebernick
James Sprittles
Timothy George
A. Glyn Bengough

The behaviour of two or more fluids flowing though porous media has important implications for our understanding of biological processes, such as water movement in soil, and civil engineering applications, such as enhanced oil recovery. By combining mathematical modelling with the latest high-resolution imaging techniques and high performance computing it is now possible to investigate multiphase fluid flow at the pore scale, which can be difficult to achieve experimentally. However, this detailed modelling is computationally expensive and is only applicable to small length scales.

By combining finite element image-based modelling with multiscale homogenisation theory, it is possible to parameterise the equations for macroscale fluid flow based on real world pore scale structures. In this work we demonstrate how this can be applied to model fluid flow in soil, where the macroscale equations are usually parameterised using indirect experimental measurements and empirical fitting. To model the flow of two fluids, air and water, we use a phase field approach by combining the Cahn-Hilliard equations with the equations for incompressible Stokes' Flow. These equations are applied to a pore scale geometry based on a 3D high resolution X-ray computed tomography image of a soil sample. We show how this work may be extended to include a solute modelled with an advection-diffusion equation using the example of plant root exudates and consider how it could be used for further applications.

Axisymmetric Nematic Liquid Crystal Squeeze Film: Low Ericksen Number

Joseph Cousins - University of Strathclyde
Joseph Cousins
Stephen Wilson
Nigel Mottram

Nematic liquid crystal (nematic) devices are ubiquitous in modern day life but faster and more accurate manufacture of these devices is required to meet increasing global demand. The optimisation of the manufacturing process generally involves attempting to reduce manufacturing time by increasing flow velocities and filling speeds. However, these changes are often implemented without a proper understanding of the possible consequences of flow-driven director misalignment and damage caused to the director anchoring at the substrates. An understanding of the flow of a nematic during manufacturing can therefore potentially lead to improvements in manufacturing efficiency and device quality. Some manufacturers fill their devices using the so-called One-DropFilling method. In this method, nematic is dispensed in droplets onto a substrate and then compressed by a top plate moving downwards with a fixed constant speed, to ensure complete filling of the device. This process of compressing a film of nematic is similar to the classical squeeze film problem in fluid dynamics. In the present work we undertake a full treatment of the nematic axisymmetric squeeze film problem using the Ericksen-Leslie equations with the goal of obtaining greater insight into the One-Drop-Filling method. Specifically, a thin layer of nematic with constant volume is confined between parallel plates and Dirichlet conditions on the director, known as infinite anchoring conditions, are imposed at both boundaries. The nematic is squeezed by the downward motion of the top plate, and the resulting flow velocity, pressure and director angle are investigated asymptotically in the limit of small reduced Reynolds number Re and Ericksen number Er. Leading order solutions show elastic effects dominating with flow-driven director alignment appearing at higher order. The cases of a fixed constant speed squeezing, which corresponds to OneDrop-Filling, and a fixed constant force squeezing are both considered. In the fixed constant speed case, solutions are derived up to first order for the velocity, pressure, force, torque and director angle. The effect of changing the infinite anchoring conditions is investigated by considering four different infinite anchoring cases, namely planar, homeotropic, HAN and Pi-cell. In the fixed constant force case, the height of the squeeze film must be solved for asymptotically, whereas, in the fixed constant speed case the height is prescribed. At leading order, the height is found to behave like the fixed constant force Newtonian squeeze-film problem with the effective viscosity being dependent on the infinite anchoring conditions used. Higher order nematic effects are found to oppose or aid the closure of the squeeze film depending on the infinite anchoring.

The hydrostatics of Robert Boyle and George Sinclair

Alex Craik - University of St Andrews

The hydrostatical writings of the little-known George Sinclair and the famous Robert Boyle reveal an equivocal attitude to the employment of mathematical theorems in physical science. Both rightly rejected the philosophical speculations of various contemporaries that were not founded on experimental observation (notably those of Thomas Hobbes and Franciscus Linus). Their own approach was that of "experimental philosophy": Sinclair's "Theoremes" are based on physical observations rather than on mathematics.

Later anonymous and harsh criticisms of Sinclair by the mathematicians James Gregory and William Sanders favoured the earlier mathematical approach of Archimedes and Stevin.

Combining homogenisation theory and image based modelling to predict the poro-elastic properties of multi-constituent soils

Keith Daly - University of Southampton
Tiina Roose

The combined mechanical and hydraulic properties of soils are often described using the poro-elastic theory of Biot, which describes the flow of water through a porous elastic matrix. Typically however, soil structure is more complex and can be better described as being composed of solid particles, air filled pore space, and a mixed phase composed of water and micron sized deformable clay particles. We discuss how pore scale imaging and multi-scale homogenisation can be used to link the pore scale geometry and physics to the observable macro-scale properties of the soils.

On the pore scale we model the mixed phase as a poro-elastic mixture composed of elastic colloids and pore water. The solid particles move through rigid body translations, subject to an integral constraint on the total stress applied to the particle. Finally, the air-filled pore-space acts as an incompressible viscous fluid. Through application of homogenisation we derive a set of four cell problems, which capture the Darcy flow of air, the Darcy flow of water, the effective stiffness, and the pressure induced stress in the mixed phase.

From these, we derive a set of effective poro-elastic equations with parameters, which depend on the cell problems. These describe the mobility of air and water, the soil stiffness, a set of pressure induced stress coefficients, and a series of parameters that determine the change in air and water volume due to pressure variations and strain. By paying special attention to the integral constraint governing the stress balance on the solid particles, and the macro-scale deformation of the soils, we show that the each of the effective parameters contains a linear and a non-linear part. The non-linear part of these equations vanishes if the soils behave as an isotropic poro-elastic mixture.

We validate the homogenisation procedure by comparing the predictions of the original equations to the predictions of the homogenised version. Finally, we apply our theory to derive the poro-elastic properties of soil structures based on three dimensional geometries obtained using X-ray Computed Tomography. We considered three different initial soil treatments, comprised of two different compaction levels and two different moisture contents. We found that the effective properties of the soils were unaffected by the compaction. However, changing the moisture content significantly altered the hydraulic and mechanical properties of the soils. A key strength of this method is that it enables the optimization, or even design of soils composed from different constituents, with specific mechanical and hydraulic properties.

Fast Parameter Inference in a Computational Model of the Left-Ventricle using Emulation

Vinny Davies - University of Leeds
Umberto Noe
Hao Gao
Benn Macdonald
Xiaoyu Luo
Dirk Husmeier

A central problem in biomechanical studies of personalized human left ventricular (LV) modelling is estimating the material properties from in-vivo clinical measurements in a time frame suitable for use within a clinic. Understanding these properties can provide insight into heart function or dysfunction and help inform personalised treatment. However, finding a solution to the differential equations which describe the myocardium through numerical integration can be computationally expensive. To circumvent this issue, we use the concept of emulation to infer the myocardium properties of a healthy volunteer in a viable clinical time frame using in-vivo LV data. Emulation methods avoid computationally expensive simulations from the LV model by replacing it with a surrogate model inferred from simulations generated before the arrival of a patient, vastly improving efficiency at the clinic. We compare and contrast two emulation strategies: (i) emulation of the computational model outputs and (ii) emulation of the loss between the observed patient data and the computational model outputs. These strategies are tested with two different interpolation methods, as well as two different loss functions. The best combination of methods is found by comparing the accuracy of parameter inference on simulated data for each combination. This combination, using emulation of the model outputs (i), with local Gaussian process interpolation and mean squared error loss function, provides accurate parameter inference in both simulated and clinical data, with a reduction in computational cost of about 3 orders of magnitude compared to numerical integration of the differential equations using finite element discretisation techniques.

Optimal coordinate transformations for the perfectly matched layer method

Jonathan Deakin - The University of Manchester
Andrew Hazel
Robert Harter
Matthias Heil

The widely used perfectly matched layer (PML) method employs a complex coordinate transformation to impose non-reflecting boundary conditions in the finite element solution of wave equations. The coordinate transformation is applied in a layer surrounding the region of interest (the bulk) and in its exact form completely absorbs waves leaving the bulk, eliminating spurious reflections from the outer boundary. However, when the problem is discretised, if the solution in the PML region is under-resolved, spurious reflections are created, limiting the achievable accuracy of the computed solution in the bulk. Conversely, if the solution in the PML region is over-resolved then computational effort is wasted. The optimal balance between bulk and PML refinement is problem dependent and difficult to find a priori.

To address this problem we propose a PML which is optimal in the sense that the coordinate transformation ensures the solution varies linearly through the PML, making it trivial to discretise. We present an algebraic method for finding this optimal transformation which utilises information about the solution. While this makes the problem non-linear, we show that we can converge to the exact solution by iterating, using information from the previous solution. This iteration is natural if we are already performing spatial mesh adaptation in the bulk. We show that with this optimal PML, the numerical error is completely controlled by the refinement in the bulk.

Steady travelling wave solutions on an axisymmetric ferrofluid jet

Alex Doak - UCL

It has been known since Rayleigh that a gravity free capillary jet is unstable to long wave perturbations. This instability can be stabilised on jets of ferromagnetic fluid using a magnetic field, generated by a current running through a wire which the ferrofluid coats. The effects of buoyancy are negated by surrounding the ferrofluid in a non-magnetisable fluid of equal density. This model has been the subject of recent research. Previous authors (Blyth & Parau, 2014) have computed numerically steady solitary waves solutions under the assumption that the density of the second fluid is negligible. We extend these results to allow for periodic and generalised solitary wave solutions. Furthermore, we shall present a numerical scheme capable of solving for steady solutions when the density of the exterior fluid is not assumed to be negligible, offering a model which is more representative of experimental setups.

Imperfect bifurcation for the quasi-geostrophic shallow-water equations

David Dritschel - University of St Andrews
Taoufik Hmidi
Coralie Renault

We study analytical and numerical aspects of the bifurcation diagram of simply-connected rotating vortex patch equilibria for the quasi-geostrophic shallow-water (QGSW) equations. The QGSW equations are a generalisation of the Euler equations and contain an additional parameter, the Rossby deformation length L, which enters in the relation between streamfunction and (potential) vorticity. The Euler equations are recovered in the limit 1/L -> 0. We prove, close to circular (Rankine) vortices, the persistence of the bifurcation diagram for arbitrary Rossby deformation length. However, we show that the two-fold branch, corresponding to Kirchhoff ellipses for the Euler equations, is never connected even for small values 1/L, and indeed is split into a countable set of disjoint connected branches. Accurate numerical calculations of the global structure of the bifurcation diagram and of the limiting equilibrium states are also presented to complement the mathematical analysis.

Detection of the Second Harmonic of Decay-less Kink Oscillations

Timothy Duckenfield - Centre for Fusion, Space and Astrophysics (CFSA)
Sergey Anfinogentov
David Pascoe
Valery Nakariakov

EUV observations of a multi-thermal coronal loop, taken by the Atmospheric Imaging Assembly (AIA) of the Solar Dynamics Observatory (SDO), which exhibits decay-less kink oscillations are presented. The datacube of the quiet Sun coronal loop was passed through a motion magnification algorithm to accentuate transverse oscillations. Time-distance maps were made from multiple slits evenly spaced along the loop axis and oriented orthogonal to the loop axis. Displacements of the intensity peak were tracked to generate time series of the loop displacement. Periodogram analysis on the time series showed the presence of two periods within the loop; $P_1 = 10.3 \pm 1.7$ minutes and $P_2 = 7.4 \pm 1.3$ minutes. The longer period component was greatest in amplitude at the apex and remained in phase throughout the loop length. The shorter period component was strongest further down from the apex on both legs and displayed an anti-phase behaviour between the two loop legs. These results are interpreted as the coexistence of the fundamental and second harmonics of the standing kink mode within the loop in the decay-less oscillation regime. An illustration of seismological application using the ratio $P_1/2P_2 \sim 0.7$ to estimate the density scale height is presented. The existence of multiple harmonics has implications for understanding the driving and damping mechanisms for decay-less oscillations and adds credence to their interpretation as standing kink mode oscillations.

Multiple Scale Asymptotic Homogenisation of Nutrient Movement and Crop Growth in Partially Saturated Soil

Simon Duncan - University of Southampton
Keith Daly
Daniel McKay Fletcher
Paul Sweeney
Tiina Roose

Understanding water and solute movement in soil is vital in determining sustainable crop production for long-term food security. However, when studying transport in porous media such as soil, complex geometries are required to capture the intrinsic details contained in the microscopic structure. This typically requires vast amounts of computation time and resources. Hence, it is often favourable to construct an averaged macroscopic geometry such that the macroscale transport properties can be attained directly. One technique that is frequently used to obtain macroscale movement of fluids and solutes in soil is multiple scale homogenization.

We use multiple scale homogenisation to derive a set of averaged macroscale equations that describe the movement of nutrients in partially saturated soil, where the soil is modelled as a poro-elastic material. Furthermore, root vegetable crops are contained within the soil, in which the growth of each crop is dependent on the uptake of nutrients via a sink term within the soil adjacent to each crop, thereby representing root uptake. Special attention is paid to the reduction in void space, change in local porosity and the impact on nutrient diffusion within the soil due to compaction as the crops increase in size. We solve a coupled system of equations using a combination of Biot poro-elastic theory, Stokes flow and the diffusion equation for solutes within porous media. To validate the multiple scale homogenisation procedure, we compare the system of averaged equations to the original set of equations and find that the solutions between the two models differ by. However, we find that the computation time between the two sets of equations differs by several orders of magnitude. We observe that the computation time for the averaged system of equation to be over 1000 times faster than the original set of equations. This is due to the combined effects of the complex three-dimensional geometry and the implementation of a moving boundary condition to capture crop growth.

Detection of Multiple Inclusions and Quantification of Contact Impedances from ERT Data using the Complete-electrode Model

Taysir Dyhoum - University of Leeds, School of Mathematics
Robert Aykroyd
Daniel Lesnic

This contributed talk considers inverse shape reconstruction of rigid inclusions from electrical resistance tomography (ERT) boundary data and presents a novel combination of forward solution using the Method of Fundamental Solutions (MFS) and Bayesian inverse reconstruction through a Markov chain Monte Carlo (MCMC) algorithm. The approach solves the complete-electrode model (CEM) of electrical resistance tomography for detecting rigid inclusions embedded in a background bounded medium. The MFS provides fast and highly accurate forward solutions with less implementation effort, since it is meshless, compared to the more standard boundary element and finite element methods. This is particularly important as MCMC inverse solution requires the forward solver to be called thousands of time in an iterative approach. However, using the Bayesian approach and the MCMC estimation technique has big advantages in terms of uncertainty and reliability assessment of the obtained reconstructions. The mathematical model is governed by the two-dimensional Laplace's equation in a multiply-connected domain subject to a homogeneous Dirichlet boundary condition on the unknown rigid inclusion and a piecewise Robin boundary condition on the outer boundary containing the electrodes. The measurements are the constant voltage values on the electrodes calculated from multiple current patterns and solving the inverse problem means detecting the location, shape and size of the inner object, as well as the contact impedances for each electrode (if unknown). A series of numerical examples using simulated data demonstrate the accuracy and stability of the proposed approach.

ABC for Surface PDEs

Charles Elliott - University of Warwick

PDE models on complex and evolving domains involving unknown interfaces and interactions with discrete structures commonly feature in mathematical models in cell biology.

In this talk we focus on Analysis Based Computation with respect to a number of surface PDE problems relating to biomembranes and cell motility.

Blood flow and solute transfer in feto-placental capillary networks

Alexander Erlich - University of Manchester
Philip Pearce
Romina Plitman Mayo
Oliver E. Jensen
Igor L. Chernyavsky

What governs the supply of a human fetus with oxygen from the mother? We address this question with a model of the human feto-placental microvasculature. The physical setup in the human placenta is unique: the maternal and fetal blood supplies are separated by a thin layer of villous tissue (the syncytiotrophoblast), through which oxygen is exchanged by diffusion. Compared to other vasculatures, the capillaries are unusually loopy and bulged. Oxygen transfer from mother to fetus can be estimated using 3D finite-element simulations on realistic geometries obtained from confocal microscopy. However, owing to the computational expense of such calculations, it is infeasible to perform them on entire feto-placental capillary networks. Instead, we introduce a reduced 1D network model to simulate blood flow and oxygen transfer in feto-placental capillary networks efficiently. The model is used to study how network topology and vessel geometry affect oxygen transfer to the fetus.

For model input and validation we use several three-dimensional feto-placental geometries, obtained by confocal microscopy. These geometries allow us to extract properties such as network connectivity, average vessel radius and length, and vessel distance from the villous membrane.

Our model uses the topology and spatial properties extracted from real 3D data to construct a 1D reduction which captures the flow and oxygen transfer of the full 3D geometry. We validate the reduced 1D model against full 3D computational fluid dynamics. In contrast to 3D finite element models, our 1D model reduces flow and transport to linear algebraic equations and has virtually no computational cost. This allows us to relate oxygen transport to different network topologies, different vessel resistances, and to assess the impact of blood rheology on oxygen transport. This reduced 1D flow and transport model of the feto-placental microvasculature may contribute to multiscale models of the placenta and other biological systems.

Growth dynamics of elastic networks

Alexander Erlich - University of Manchester
Derek E. Moulton
Alain Goriely

Describing growth as a continuum in a soft elastic tissue is notoriously difficult. While it is understood that biological tissues can grow in response to mechanical stimuli, it is still widely debated whether growth should be modeled as a response to stress or strain. Various macroscopic growth laws (sometimes referred to as constitutive laws for growth) have been proposed based on phenomenological observation, thermodynamic restrictions, and computational pragmatism, each of which has developed a following. Comparing these growth laws is particularly difficult when applied to biological tissues that undergo large deformation (leading to constitutive non-linearities). The result is that it has been difficult to, in general, understand growth in a soft elastic continuum.

To shed light on growth dynamics, we propose a simple network model that is inspired by the mechanics of cells connected in a network. The idea is to understand the above mentioned growth laws in a network setting. As a first step, we consider one-dimensional networks, which allows us to use results from electric circuits (such as Kirchhoff's laws as a tool to balance lengths and forces across cell walls) to reach definite answers on growth dynamics. Our network description allows for a clear separation between mechanics and geometry, the latter being encoded by planar directed acyclic graphs. This network model allows us to directly compare strain-driven and stress-driven growth laws in terms of their growth dynamical stability, and to understand the role of geometry, constitutive laws, and the choice of growth law.

Maxwell, Kelvin, and the inverse square law of electrostatics

Isobel Falconer - St Andrews

In 1877 James Clerk Maxwell and his student Donald McAlister refined Henry Cavendish's 1773 null experiment demonstrating the absence of electric charge inside a charged conductor. The inverse square law of electrostatics predicted this absence, and both Cavendish and Maxwell took the experiment as verifying the law.

However, Maxwell, and his friend William Thomson (later Lord Kelvin), had previously expressed absolute conviction in the law, based on results of Faraday's. So why did Maxwell bother to repeat Cavendish's experiment?

To answer this question, I will focus on the mathematical tradition within which Maxwell and Kelvin were working, and the logic of the experimental evidence for the law. I set these in the context of contemporary British attitudes to the inverse square law and conclude that the status of the law was less secure than Maxwell and Kelvin would have us believe. The presentation of the experiments was a rhetorical move in an attempt to generate an apparently rigorous mathematical foundation for electrical science, as well as a British genealogy for such an approach.

Net reproduction functions for nonlinear structured population models

Jozsef Farkas - University of Stirling

The net reproduction number, i.e. the average number of offspring produced by a newborn individual in her expected lifetime, is an important quantity associated with population dynamical models. In most linear models it determines whether the population grows or decays (exponentially) over time. A fairly recent spectral result allows defining rigorously the net reproduction number for large classes of linear models. In this talk we will show how to define analogously net reproduction functions (or functionals) for nonlinear models.

On the chordae structure and dynamic behaviour of the mitral valve

Liuyang Feng - University of Glasgow
Nan Qi
Hao Gao
Wei Sun
Boyce Griffith
Xiaoyu Luo

Mitral valve (MV) dysfunction, including mitral valve stenosis, prolapse, and regurgitation, is one of the most common valvular heart diseases and hence has attracted significant research interest. Computational modelling of human MV function can improve our understanding of MV biomechanics, which is important for improving surgical procedures and medical therapies. However, because of the challenges of modelling the highly complex MV structure, its deformation, and its interaction with the left ventricle, only limited progress in multi-physics modelling of the MV has been made to date.

We develop a mitral valve (MV) model that includes physiologically detailed descriptions of the leaflets and the chordae tendineae. Three different chordae models – complex, "pseudo-fibre", and simplified chordae – are compared to determine how different chordae representations affect the dynamics of the mitral valve. To quantify the highly complex system behaviour resulting from the fluid-structure interaction (FSI), an energy budget analysis of the coupled MV FSI model is performed.

Our MV model is based on physiologically-detailed MV geometry from multi-slice CT scans of a normal mitral valve at middle diastole, from a 61-year-old male patient, which incorporates detailed leaflet thickness and chordal information. Both leaflets and chordae are modelled as fibre-reinforced hyperelastic materials. We use an immersed boundary-finite element (IB/FE) method for our dynamic MV modelling. The simulation has been carefully verified against the commercial software ABAQUS under static loading conditions. Energy budget is conducted with introduction of different energy terms appearing in the system: change of the kinetic energy, kinetic energy flux, rate of work done by the applied pressure, the rate of energy dissipation and the rate of change of elastic strain energy in the immersed structure.

Our results show that the complex and pseudo-fibre chordae models yield good MV closure during systole, but that the simplified chordae model leads to poorer leaflet coaptation and an unrealistic bulge in anterior leaflet belly. Energy budget analysis shows that the MV models with complex and pseudo-fibre chordae have similar energy distribution patterns, but the MV model with the simplified chordae consumes more energy, especially during valve closing and opening. Interesting flow patterns and vortex formulation are seen in all three cases.

In general, we show that the complex chordae and pseudo-fibre chordae have similar impact on the overall MV function, but that the simplified chordae representation is less accurate. Because a pseudo-fibre chordal structure is much easier to construct and less computationally intensive, it may be a good candidate for modelling MV dynamics or interaction between the MV and the heart in patient-specific applications.

Mathematical modelling acute myocardial infarction based on magnetic resonance imaging

Hao Gao - University of Glasgow
Kenneth Mangion
Colin Berry
Xiaoyu Luo

Although death rates from myocardial infarction (MI) are falling, the incidence of heart failure after acute-MI remains persistently high. LV dimensions and pump function change dynamically in the first few weeks post-MI. Left ventricular (LV) dysfunction after MI portends an adverse prognosis, which is responsible for nearly 70% of heart failure cases. In clinical practice, therapeutic decisions are informed by an evidence-base relating to LV ejection fraction (LVEF). However, on an individual basis, risk prediction using LVEF is limited as majority of patients who die prematurely have a normal or mildly reduced LVEF.

Computational heart modelling has potential to close some of these gaps in risk prediction in individual patients, i.e. myocardial stiffness and contractility. In order to improve patient-specific modelling of myocardial mechanics following an acute-MI, we developed a finite element model of a human LV with MI morphologies derived directly from Late-Gadolinium enhanced (LGE) magnetic resonance (MR) images. The LV geometry is reconstructed from in vivo short-/long-axis cine images of a patient after acute-MI. A linear relationship between LGE intensity and myocardial passive stiffness and contractility is assumed. We assume that the passive myocardium obeys the Holzapfel-Ogden constitutive law with eight unknown material parameters. We approximate the LV end-diastolic pressure with a population-based value (15 mmHg) and assume that the LV peak systolic pressure is the same as the cuff-measured value. The full scar region (normalised LGE intensity > 0.95) is considered to be passive and 50 times stiffer compared to the remote functional myocardium. To calibrate the passive stiffness, we inversely determine the eight unknown material parameters by matching the strain measurements estimated from MR images and the LV end-diastolic volume using a multi-step optimisation approach. We further match the LV EDP-EDV curve published previously. The myocardial contractility is determined by matching the systolic function, including the systolic peak circumferential strain and LVEF. We infer the myocardial contractility of this patient is around 180.8 kPa from our modelling, which is in the range of previously published human values. Finally, patient-specific myofibre stress distributions at end-diastole and end-systole are discussed in relation to the LV adverse remodelling.

In summary, our newly developed LV model by integrating MI pathology information directly from LGE MR images for estimating myocardial passive stiffness and contractility will be helpful in management of patients with acute-MI.

Interfacial waves in chiral elastic structured media

Marta Garau - Keele University
Giorgio Carta
Michael Nieves
Ian Jones
Natasha Movchan
Alexander Movchan

Chirality is the property of an object whereby it cannot be superimposed onto its mirror image. This important feature is ubiquitous throughout nature and science. It is also beginning to play a significant role in the design of a novel class of waveguides within metamaterials research.

The mechanical model of an elastic triangular lattice, with gyroscopic spinners attached to the junctions. There it is shown that the spinners, which confer chirality on the system, are capable of suppressing pressure waves in the medium in the low frequency regime. In addition, this system has led to the design of a continuous chiral medium, capable of acting as an efficient cloaking and shielding device. A detailed dispersion analysis of the chiral triangular system, which includes wave polarisation and dynamic anisotropy, has been undertaken. Recently, a triangular lattice connected to a non-uniform distribution of spinners was shown to exhibit very surprising wave propagation phenomena. At high frequencies, it is possible to observe exponentially localised waveforms and their paths can be controlled by appropriately arranging the spinners in the lattice. This localisation phenomenon is referred to as the DASER (Dynamic Amplification by means of Spinners in an Elastic Reticulated system).

In this talk, we extend the a previous model to introduce a new design for a periodic chiral medium possessing non-trivial topological features. An elastic hexagonal system of non-inertial rods connecting masses that are attached to gyroscopic spinners is considered. We analyse the dispersive nature of this structure and show how this is altered by the spinners. Following this, we show how uni-directional interfacial waveforms, possessing preferential directionality, can be created in the hexagonal lattice connected to an inhomogeneous array of spinners. We demonstrate that the direction of propagation is sensitive to the arrangement of spinners and to the frequency of the external excitation. Moreover, by introducing additional soft internal links into the hexagonal structure, we show that interfacial waveforms can be also realised in a heterogeneous chiral triangular lattice.

A viscous-hyperelastic constitutive model for transverse isotropic soft tissues

Daniel Garcia-Gonzalez - University of Oxford
Antoine Jerusalem
Sara Garzon-Hernandez
Ramon Zaera
Angel Arias

Soft tissues in humans, animals or plants often combine viscous behaviours with transverse anisotropy. The mechanical behaviour exhibited by these tissues involves complex phenomena such as large deformation and non-linear response. In addition, these tissues present dependencies on strain rate and temperature. In this work, we present a continuum constitutive model for soft tissues that incorporate strain rate and temperature dependencies as well as the transverse isotropy arising from fibres embedded into a soft matrix. The formulation is based on hyperelasticity and decouples the Helmholtz free energy function into the contributions of a viscous-hyperelastic matrix and the fibres, introducing dispersion dependent transverse isotropy. The proposed model allows for the particularisation of the energy potentials and flow equations of each constitutive branch within a thermodynamically consistent framework for finite deformation kinematics. In this regard, the approach developed herein provides the basis on which specific constitutive models can be potentially formulated for a wide variety of soft tissues. The versatility of this model is illustrated by its particularisation and application to animal and human white matter and skin. In both cases, different energy functions are considered: Neo-Hookean, Gent and Ogden. Finally, the predictive capability of the approach for the two soft tissues is confirmed by comparison with experimental data from the literature. More generally, this constitutive framework can be used to model specific soft tissues with strain rate and temperature dependencies through the selection of the free energy functions and flow equations of each constitutive branch.

Modelling coronal magnetic field evolution

Erin E. Goldstraw - University of St Andrews
Alan W. Hood
Klaus Galsgaard

One class of coronal heating mechanism involves the slow storage of magnetic energy, due to photospheric footpoint motions, in the coronal magnetic field. This is subsequently released rapidly. A second class assumes that the energy required to heat the corona is transported by waves. Previous slow storage models have used the ideal MHD kink instability to initiate the energy release. Here, the triggering of the resistive tearing mode results in reconnection and the release of the stored magnetic energy. The aim of this work is to understand how this storage and release mechanism occurs and how much energy is released to heat the plasma.

Previous MHD studies suggest that the process of storing and releasing magnetic energy can occur in a cyclic manner, with the system evolving through sequences of equilibria. However, recent simulations, using approximate methods have shown that this process only happens once, after which the system remains in a statistically steady turbulent state and magnetic energy does not build up again. Instead the input energy is immediately released.

We use full, 3D MHD simulations to investigate the dynamics of the tearing mode instability in a model coronal field. It is found that, after the initial build up and release of magnetic energy, the system remains in a lower energy state. Hence, the input of Poynting flux from continued shearing is immediately dissipated as heat.

The Evolution of magnetic fields: Comparison of approximate methods

Erin E. Goldstraw - University of St Andrews
Alan, W. Hood
Philippa, K Browning
Peter, J Cargill

Modelling the long time evolution of coronal magnetic fields is important in understanding the structure of the Sun's corona. An insight into this evolution provides a better interpretation of observations and allows predictions of the field's behaviour to be made more confidently. Magnetic field evolution is followed by solving the full MHD equations. However, due to their complex nonlinear nature, the present computational resources mean we are restricted in how long the field can be modelled. Therefore, it is often necessary to apply some additional assumptions to simplify the equations but it is not always clear whether these assumptions are valid or not. A simple experiment has been done to highlight the importance of considering the validity of a method. Four common approximate methods of MHD are compared to full MHD in the context of a sheared coronal field.

Bayesian parameter estimation for macroscopic pedestrian dynamics models

Susana Gomes - Imperial College London
Andrew Stuart
Marie-Therese Wolfram

The fundamental diagram of pedestrian dynamics is a function that relates the experimentally observed density of pedestrians to their average velocity. Although there is a general agreement on the basic shape of this function, its parametrization depends strongly on the measurement and averaging techniques used as well as the experimental setup considered.

We aim at developing a systematic approach to identify parameters in nonlinear macroscopic crowd motion models using multiple microscopic trajectories. We assume that each trajectory is a realization of an Ito-McKean process, where the individual velocity depends on the probability density of the process. The probability density satisfies a nonlinear Fokker-Planck equation itself, leading to a coupling between the microscopic SDE and the macroscopic PDE.

Motivated by the fundamental diagram we assume that individuals move with a maximum velocity, which decreases linearly as the probability density approaches the maximum crowd density. We are interested in identifying the maximum velocity and the maximum crowd density using multiple trajectories. We discuss Bayesian as well as other derivative free optimization methods to estimate these parameters and present analytic as well as numerical results, which give important insights into

the dynamics and challenges of this highly nonlinear inverse problem.

Instability of elevator modes in oscillatory double-diffusive convection

Thomas Goodfellow - University of Leeds
David Hughes
Stephen Griffiths
Peter Jimack

Fluid dynamical systems consisting of two or more diffusive components exhibit rich and interesting behaviour. In the oceans, heat and salt gradients interact to fuel phenomena such as salt fingers, oscillatory instabilities and long-lived density staircases (or layers). The linear stability of double-diffusive systems to small perturbations is well-studied: a static basic state is unstable to `elevator modes' (i.e. modes that are invariant in the vertical direction), which, in turn, are fully nonlinear solutions to the governing equations. In this study, we go beyond the linear theory for a static basic state, and instead investigate the stability of an elevator basic state to linear perturbations.

We focus on the diffusive case in which salt – the slower diffusing component – provides a stabilising gradient, and in which the primary elevator modes are oscillatory (but not growing) in time. We use a double-Floquet expansion in time and space to determine the growth rates of secondary modes. The structure of the fastest-growing secondary mode is influenced by the amplitude of the primary elevator mode, leading to two distinct solutions: at moderate amplitudes, the dominant secondary mode is horizontally and vertically dependent with a horizontal wavelength larger than that of the basic state; at large amplitudes, the secondary mode is horizontally and vertically dependent, but it now sits within the basic state. The mechanisms of these instabilities are discussed, along with their connection to instabilities of oscillating flows in fluids of constant density.

The effect of surface stress on solitary waves

Dane Grundy - University of East Anglia
Paul Hammerton

The theory of solitary waves is well developed when normal stress is applied at a fluid surface, leading to the KdV equation. However, when a fluid of finite depth is subjected to an electric field, or a surfactant is present, then a tangential stress arises at the free surface. Taking a large Reynolds Number limit leads to a boundary layer at the free surface and imposing a no-slip condition on the solid bottom leads to a viscous boundary layer at the bottom of the fluid. In this talk I will introduce each scenario and discuss recent numerical and analytical results describing a solitary wave in the presence of an electric field, and a uniform surfactant at the free surface.

Long-time behaviour and phase transitions for the McKean–Vlasov equation on the torus

Rishabh Gvalani - Imperial College London

We study the McKean—Vlasov equation on the flat torus which is obtained as the mean field limit of a system of interacting diffusion processes enclosed in a periodic box. The system acts as a model for several real-world phenomena from statistical physics, opinion dynamics, collective behaviour, stellar dynamics etc.

After commenting on the well-posedness of the equation, we study its long time behaviour and convergence to equilibrium. We then focus our attention on the stationary problem - under certain assumptions on the interaction potential, we show that the system exhibits multiple equilibria which arise from the uniform state through continuous bifurcations. This relates closely with previous work on phase transitions for the Mckean-Vlasov equation (cf. Chayes and Panferov, J. Stat. Phys., 2010). Finally, we attempt to classify continuous and discontinuous transitions for this system and show how this work, in conjunction with previous studies of the system, can be used to recover classical results on phase transitions for the noisy Kuramoto model. This is joint work with José Carrillo, Greg Pavliotis, and André Schlichting.

Effect of Molecular Relaxation of Nonlinear Evolution of N-Waves

Paul Hammerton - University of East Anglia

Burgers equation describes the evolution of finite-amplitude acoustic waves -- highlighting the balance between cumulative nonlinearity and thermoviscous diffusion. In the small diffusion limit, wave steepening leads to the appearance of viscosity controlled shocks. However, for propagation of acoustic waves through air or sea-water, molecular relaxation associated with the internal vibration of polyatomic molecules also plays an important role in determining the shock structure. Experimental measurements reveal the appearance of a slowly decaying tail behind the shock. In this paper asymptotic analysis is presented explaining the appearance of the shock tail, supplemented by numerical results using a pseudospectral scheme.

Quasi-analytic inviscid geodynamo solutions.

Colin Hardy - University of Leeds
Phil Livermore

The geodynamo is driven by the motion of the Earth's fluid outer core. In this motion, which is governed by the MHD equations, rotational forces are dominant over inertial and viscous forces, which means the Ekman and Rossby numbers are very small. Numerical models are used to simulate the geodynamo and have had some success but are only possible in the wrong parameter regime of Ekman and Rossby numbers being far too large. There is an alternative approach proposed by Taylor in 1963, of an inertia-free and viscous-free model as the asymptotic limit of Earth's dynamo. In this theoretical limit of a magnetostrophic balance, then he derived Taylor's constraint, that the azimuthal component of the Lorentz force must have zero average over any fluid cylinder coaxial with the rotation axis. The geostrophic flow is the component of the flow that evolves in such a manner as to ensure this constraint on the magnetic field is continuously satisfied through time. We propose a method for determining the instantaneous geostrophic flow, properly taking into account the boundary conditions and report some examples of geostrophic flows for a variety of simple instantaneous magnetic fields.

Fractal sets of neutral curves in stably stratified plane Couette flow

Jonathan Healey - Keele University

Plane Couette flow is famous for being stable to small amplitude disturbances. We consider the linear stability of plane Couette flow between horizontal plates where the fluid density increases monotonically with depth, i.e. stably stratified flow. Perhaps surprisingly, the presence of stabilizing buoyancy forces can destabilize the flow. This instability mechanism can be understood by studying the Taylor-Goldstein equation, a linear second order ordinary differential equation. We consider some simple model density profiles. In fact, they are sufficiently simple that explicit solutions in terms of special functions can be written down. However, the properties of the roots of the dispersion relation are nontrivial.

The strength of the density gradient at a point in the flow compared to the strength of the shear is characterized by a dimensionless parameter, the local Richardson number, Ri. A theorem due to Howard and Miles proves that a necessary condition for instability is that $Ri < 1/4$ somewhere in the flow. We introduce a parameter plane for our density profile where one axis gives the location in the flow of the point where Ri takes its minimum value, and the other axis characterizes the total change in density across the flow (a bulk, or global, Richardson number).

We show that when the minimum value of Ri in the flow domain is 1/4, an infinite number of points appear in the parameter plane at which neutral waves exist. These points lie on a fractal set. As soon as the minimum Ri drops below 1/4, each neutral point opens up into a neutral curve, with each neutral curve forming a closed loop. The neutral loops expand and overlap one-another as Ri is reduced further, giving a fractal set of neutral curves. When the minimum Ri is reduced to zero, a fractal set of branch-points is encountered where neutral modes coalesce to form complex conjugate pairs.

Fractals are usually associated with nonlinear processes, but fractal behaviour has been found here in a linear stability equation that has been studied since the 1930s. Qualitatively similar behaviour can be expected for other density profiles.

Gas-cushioned droplet impacts in the compressible gas regime

Peter Hicks - University of Aberdeen

When a droplet approaches impact with a rigid impermeable surface, there is a build-up of pressure in the gas separating the droplet and the impact site, which decelerates and deforms the droplet free-surface. This deformation traps a small gas bubble at the point of impact. For slow impact speeds, the gas behaviour is incompressible to leading order and this gives rise to a scaling law for the initial radius of the trapped gas bubble, which involves the droplet radius, the impact velocity, the liquid density, and the gas viscosity. Excellent agreement with this scaling law will be illustrated for both solid body impacts with quiescent liquid and droplet impacts with rigid substrates.

At higher impact speeds, gas compressibility is significant. In this regime, we show that a leading-order contribution from the viscous dissipation term in the gas energy conservation equation implies that neither of the existing models, which assume either isothermal or isentropic compression, are appropriate. The solvability condition for an inhomogeneous Neumann problem gives rise to a further relationship coupling the gas temperature, pressure, and density. Within this framework new scaling law behaviours for the size of the trapped gas pocket are discussed.

Recent results, published in collaboration with Prof. Sigurdur Thoroddsen's group in KAUST, Saudi Arabia (PRL, 119, 214502), involving ultra-high-speed photography of droplet impacts in the compressible cushioning regime will be described. A previously unknown double touchdown regime is observed, in which an annulus of gas is entrained in addition to the central bubble. This double touchdown will be shown to coincide with the onset of significant rarefied gas effects. Extensions of the compressible gas model to incorporate rarefied gas will be presented alongside preliminary numerical results.

Gyrotactic suppression and emergence of chaotic trajectories of swimming particles in three-dimensional flows

Nicholas Hill - University of Glasgow
Andrew Baggaley
Scott Heath Richardson

We study the effects of imposed three-dimensional flows on the trajectories and mixing of gyrotactic swimming microorganisms and identify phenomena not seen in flows restricted to two dimensions. Numerical simulations of Taylor-Green and Arnold-Beltrami-Childress (ABC) flows are used to explore the role that the flow and the cell shape play in determining the long-term configuration of the cells' trajectories, which often take the form of multiple sinuous and helical “plumelike” structures, even in the chaotic ABC flow. This gyrotactic suppression of Lagrangian chaos persists even in the presence of random noise. Analytical solutions for a number of cases reveal the how plumes form and the nature of the competition between torques acting on individual cells. Furthermore, studies of Lyapunov exponents reveal that, as the ratio of cell swimming speed relative to the flow speed increases from zero, the initial chaotic trajectories are first suppressed and then give way to a second unexpected window of chaotic trajectories at speeds greater than unity, before suppression of chaos at high relative swimming speeds.

The Non-Linear Growth of the Magnetic Rayleigh-Taylor Instability

Andrew Hillier - University of Exeter
Jack Carlyle

The nonlinear dynamics of the magnetic Rayleigh--Taylor instability (mRTi) appear in astrophysical phenomena as diverse as solar prominences and Supernovae. To improve our understanding of this instability, we examine the effect of the embedded magnetic field strength on the non-linear development of the mRTi. Numerical experiments are conducted in a domain sufficiently large so as to allow the predicted critical modes to develop in a physically realistic manner. The ratio between gravity, which drives the instability in this case, and magnetic field strength is taken up to a ratio which accurately reflects that of observed astrophysical plasma, in order to allow comparison between the results of the simulations and the observational data which served as inspiration for this work. This study finds reduced non-linear growth of the rising bubbles of the mRTi for stronger magnetic fields, and that this is directly due to the change in magnetic field strength, rather than the indirect effect of altering characteristic length scales with respect to domain size. By examining the growth of the falling spikes, the growth rate appears to be enhanced for the strongest magnetic field strengths, suggesting that rather than affecting the development of the system as a whole, increased magnetic field strengths in fact introduces an asymmetry to the system. Further investigation of this effect also revealed that the greater this asymmetry, the less efficiently the gravitational energy is released.

Consequences of shallow water magnetohydrodynamic waves on hot Jupiters

Alex Hindle - Newcastle University
Tamara Rogers
Paul Bushby

We discuss how interactions between magnetically modified planetary-scale waves and mean zonal flows can generate large variations in the equatorial circulation patterns of hot Jupiters. We highlight how an azimuthal background magnetic field changes the nature of the modes present in the inviscid, incompressible shallow water magnetohydrodynamic (SWMHD) system and propose how the magnetic modification of the planetary-scale waves can disrupt the hydrodynamic mechanism believed to cause equatorial superrotating jets. We discuss the consequences of this proposed mechanism and its implications on providing a constraint for the magnetic field strength of hot Jupiters, such as HAT-P-7 b and CoRoT-2b, that exhibit westward peak brightness offset.

Spatio-Metabolic Modelling Elucidates Resistance and Re-Sensitisation to Treatment in Heterogeneous Melanoma

Arran Hodgkinson - Université de Montellier
Laurent Le Cam
Ovidiu Radulescu
Dumitru Trucu

The targeted inhibition of specific oncogenes is now the most prevalent strategy for treating aggressive or invasive melanomas and one combination of particular interest is that of BRAF inhibitor (BRAFi) and MEKi. One particularly prevalent problem with the use of targeted cancer ther- apies is that of acquired resistance to treatment and we also choose to look deeper at new evidence of re-sensitisation to treatments. Currently, discrete or non-spatial mathematical methods are normally employed to describe disparate cancer subspecies within the tumour.

We present a novel model for describing intra-tumour heterogeneity as a continuous distribution of metabolic states, using partial differential equation (PDE) techniques. Using this general model, and within a 4-dimensional context – 1 temporal t, 2 spatial x, and 1 metabolic y – we particularise the model such that our metabolic variable, y, represents an intra-cellular competition for resources between glycolytic and oxidative phosphorylative (OXPHOS) pathways. The BRAF and MEK associated pathways may then be specifically targeted within a subset of the total cellular population in y, with the metabolic behaviour that their transcription elicits.

Our models show that more adaptive mechanisms involved in cellular metabolism are likely responsible for for the response of aggressive melanoma species to targeted inhibitor therapies. Using this model, one can also make more general statements about the acquisition of resistance and the behaviour of cells once in this state.

Mean fields and fluctuations in a compressible, random medium from Gaussian smoothing

James Hollins - Newcastle University
Anvar Shukurov
Graeme Sarson
Andrew Fletcher
Frederick Gent

We apply Gaussian smoothing to obtain mean fields from simulations of the interstellar medium (ISM). We solve the three-dimensional magnetohydrodynamic (MHD) equations in a shearing, Cartesian box 1 × 1 × 2 kiloparsecs (kpc) in size. Mean fields obtained from Gaussian smoothing have the advantage of retaining the dimensional structure of the original fields.

Whilst Gaussian smoothing does not obey the Reynolds rules, we can define a mathematically robust set of central moments as shown in Germano (1992, JFM, 238, 325). Such a formulation allows for a separation into equations for the mean fields and fluctuations, similar to the separations for Reynolds averaging and horizontal averaging.

An important concern with the application of Gaussian smoothing is to find an appropriate choice of the smoothing length, l. From spectral analysis of magnetic field, density and velocity, we find a suitable smoothing length for all three fields: l = 75 parsecs (pc).

The increasing mean magnetic field, amplified by a mean-field dynamo, significantly alters the subdivision of kinetic energy at different scales. In particular, the magnitude of kinetic energy at intermediate scales is greatly reduced. Additionally, the maximum kinetic energy is shifted towards the midplane from heights of $|z| = 0.3$ kpc.

States of maximum helicity

Gunnar Hornig - University of Dundee

In 1974 V.I. Arnold showed in a classical paper that the total helicity of a given magnetic field B in a simply connected and magnetically closed domain is bounded by the energy in the form $|H(B)| \leq 2C\, E(B)$. This formula contains a constant C, which is the smallest possible eigenvalue of the curl operator in this domain. The corresponding eigenstate is therefore a state of maximum helicity for a given energy. Knowing these eigenstates is of interest for two reasons. First, whenever we measure the helicity H of a given field, we need some reference helicity to compare it with in order to decide whether this number is big or small. The helicity of the maximum helicity state is the natural choice for this reference. Secondly, in any turbulent evolution the magnetic helicity has a tendency to accumulate at the largest possible scales (inverse cascade). At the same time turbulence dissipates energy at a higher rate than helicity. Hence there is a tendency of a (freely decaying) turbulent plasma to maximise the ratio of helicity/energy and evolve towards such a maximum helicity state. However, the original formula by Arnold was derived for simply connected and magnetically closed domains. This restricts the range of applications drastically. We will explore how one can extend this result to other domains and boundary conditions and how the corresponding maximum helicity state look like.

Standing kink oscillations in coronal magnetic flux tubes

Thomas Howson - University of St Andrews
Ineke De Moortel
Patrick Antolin

Many recent studies have highlighted the apparent abundance of waves in the solar corona, however, the extent to which wave dissipation contributes to heating the corona remains unclear. Many observations have detected the rapid damping of MHD waves, expected to be the result of mode coupling (propagating waves) and resonant absorption (standing waves). In the presence of a non-uniform density profile, a resonance will efficiently transfer energy from large scale modes to more localised waves. Whilst this does not directly result in heating, energy is transferred to smaller length scales and hence, is more readily dissipated. Within a coronal loop, cross-sectional gradients in the Alfven speed can lead to phase mixing and may increase the efficiency of this energy dissipation. Despite this, unless dissipation coefficients are enhanced by many orders of magnitude above predicted values, the heating rate obtained in numerical simulations is typically too small to maintain the temperature of the corona. To investigate the propagation of MHD waves in loops with non-trivial magnetic topologies, we present the results of 3D numerical simulations in which a wave is introduced into a magnetic flux tube. We consider the subsequent evolution of the flux tube and investigate the effects of the Kelvin-Helmholtz instability.

Dynamically coupled shallow-water sloshing in two connected vessels

Ying Huang - University of Surrey
Matthew Turner
Tom Bridges

A Tuned Liquid Damper (TLD) is a damping device used to mitigate wind and earthquake induced vibrations in tall buildings and other large structures. It consists of a vessel containing a fluid which is attached to a solid wall through a spring. The natural frequency of the TLD is chosen so as to dampen out the motion of the building by dispersing its energy through the fluid. This type of system is one of the simplest dynamically coupled systems where the vessel motion and fluid motion are intrinsically coupled. Here we are interested in a system comprising of two TLDs which are joined together by a middle spring. Our interest lies in understanding whether or not this system is more or less effective than the one vessel counterpart.

In this talk we present numerical results for the system using a Lagrangian Particle Path (LPP) formulation which assumes that the fluids are in the shallow-water limit. The scheme can be time discretised such that it is symplectic which means it preserves the Hamiltonian structure of the system. In other words, the total energy is conserved in the absence of external forces, and the energy partition between the fluid and the vessel remains. Simulations of the system with linear and nonlinear springs are presented along with a discussion of the scheme's limitation and possible extension.

Vortex Disruption by Magnetohydrodynamic Feedback

David Hughes - University of Leeds
Julian Mak
Stephen Griffiths

In an electrically conducting fluid, vortices stretch out a weak, large-scale magnetic field to form strong current sheets on their edges. Associated with these current sheets are magnetic stresses, which are subsequently released through reconnection, leading to vortex disruption, and possibly even destruction. This disruption phenomenon is investigated here in the context of two-dimensional, homogeneous, incompressible magnetohydrodynamics. By using the classical ideas of flux expulsion, we derive a simple order of magnitude estimate for the magnetic stresses --- and hence the degree of disruption --- that depends on the strength of the background magnetic field (measured by the parameter M, a ratio between the Alfvén speed and a typical flow speed) and on the magnetic diffusivity (measured by the magnetic Reynolds number Rm). The resulting estimate suggests that significant disruption occurs when M^2 Rm=O(1). To test our prediction, we analyse direct numerical simulations of vortices generated by the breakup of unstable shear flows with an initially weak background magnetic field. Using the Okubo-Weiss vortex coherence criterion, we introduce a vortex disruption measure, and show that it is consistent with our predicted scaling, for vortices generated by instabilities of both a shear layer and a jet.

Rotating jets in magnetised fluids

Mat Hunt
Jey Sivaloganathan

In recent years the issue of jets in MHD have become important. This talk aims to present two simple models of a rotating jet in a magnetic field and how it can affect the dynamics in a drastic way. Issues of stability will be considered and shown under certain circumstances a stable jet is possible.

The analysis comes from a reformulation of the incompressible MHD equations which will be explained briefly at the beginning of the talk.

Control Strategies for Linear Systems with Cauchy Distributed Noises

Moshe Idan - Faculty of Aerospace Engineering
Jason L. Speyer

Many engineering problems are characterized by measurement and process noises that are better described by non-Gaussian, heavy tailed distributions. In this talk we will discuss control of linear systems with such noise, specifically additive Cauchy distributed noises that have an undefined first moment and infinite higher moments. Consequently, when addressing the related optimal control problem, one cannot use the commonly used second moment performance criteria. Two alternative control methods will be presented that account directly for the Cauchy distributed noises: the optimal predictive control and maximum conditional probability control schemes. Both methods rely on the exact analytical representation of the conditional distribution of the system state given the measurement history that were derived as part of the minimum conditional variance estimation solution for such systems. The two controllers will be presented and their performance will be compared and contrasted to the performance of a controller designed via Gaussian approximation of the noises. The advantage of the proposed controllers will be demonstrated and the difference between the two will be discussed.

A mathematical model for photothermal ablation of spherical tumours

Ahmed Ismaeel - University of Glasgow
Xiaoyu Luo
Peter Stewart

Photothermal ablation is a promising new technique for treatment of some cancers, where metal nanoparticles are introduced into the tumour and the system locally heated with a laser to destroy the malignant cells. The aim is to have nanoparticles accumulated within the tumour and not in the surrounding healthy tissues, so that the heat source leads to a differential increase in temperature in the cancer and hence cell death. In this study we consider a mathematical model for nanoparticle delivery to a vascular spherical tumour, examining the distribution of nanoparticles through the tumour and the surrounding tissue. In this model we consider nanoparticles are conjugated with ligands which are selectively bind to tumour cell surface receptors which leads to nanoparticle internalizing within the cell. We study how the mass of accumulated nanoparticles within the tumour (and the surrounding tissue) is influenced by the degree of tumour vascularity, ligand nanoparticle conjugation and tumour cell capacity for internalized nanoparticles; and suggest an optimal timescale to maximize the efficiency of ablation in each case.

Shallow Flow and Lagrangian Particles Tracking Model for Chaotic Mixing Processes

Mohammad Mahdi Jalali - University of St Andrews
Mohammad Reza Jalali

Mixing occurs due to advection and dispersion processes whereby species become distributed within a flow field. The present research aims to develop an idealized model of the hydrodynamics and the advection and mixing processes. First, a boundary-fitted grid generator is developed based on Poisson-type elliptic partial differential equations and multi-grids solver. The open channel hydrodynamics are modelled using the Reynolds-averaged Navier-Stokes equations derived by depth-averaging the continuity and Navier-Stokes momentum equations. The hydrodynamics equations are discretized spatially using finite differences and the solution marched forward in time using fourth-order Runge-Kutta scheme. Then a Lagrangian particle tracking model is applied to predict the trajectories of tracer particles. The model, which has been constructed as a numerical algorithm by integrating forward in time the advection equations. The combined shallow flow and Lagrangian particle tracking model is validated through simulating mixing of particles for wind-induced set-up in a circular basin, mixing in an open channel with a side wall expansion, particle advection in the flow past a side-wall cavity containing a groyne, mixing in a rectangular channel containing a pair of parallel groynes, particle advection in the vicinity of groynes in the river bends, chaotic advection due to propagation of sloshing wave in a square closed basin, and periodic and chaotic advection in the alternating flow field of a pair of blinking vortices.

Coastal outflows into a buoyant layer of arbitrary depth

Sean Jamshidi - University College London
Edward Johnson

The influx of river water into the oceans brings nutrients, sediments and pollutants, as well as driving coastal currents, and as such is an important area of study. Previous works have largely investigated the impact that density differences have on the dynamics of these systems, and it is well understood that these play an important role. However, large scale currents can also be driven by jumps in potential vorticity, and there are far fewer studies that explore the impact that this has on river outflows. The present work isolates the effect of potential vorticity, and develops a mathematical model based on a 1 1/2 layer Boussinesq system with a long-wave scaling (the semi-geostrophic equations). The model depends on two physical quantities: the volume flux of river water, and the depth of the buoyant oceanic layer. Although the system is fully nonlinear, it allows theoretical predictions to be made, which are then compared with numerical experiments.

Two mechanisms contribute: a nonlinear Kelvin wave, which travels at finite velocity ahead of the river water and disturbs the oceanic fluid, and a current driven by the jump in potential vorticity between the river and oceanic water. This latter can provide forcing in either direction, depending on the sign of the jump, and thus can allow for the transport of oceanic fluid from upstream of the river mouth to downstream. The method of characteristics is used to predict both the fluid and propagation speeds in the Kelvin wave and the river water. By considering the relative strengths of the two mechanisms, the qualitative behaviour of the solutions can also be categorised, with flows featuring coastal currents, anti-cyclonic gyres and plumes that expand offshore -- all of which have been observed in laboratory experiments and oceans.

Kinetic and magnetic helicity in planetary dynamo models

Chris Jones - University of Leeds

The parts played by kinetic and magnetic helicity in planetary dynamos have received less attention than the corresponding problem in stellar dynamos, perhaps because of the lower magnetic Reynolds numbers in planets. Kinetic helicity arises naturally from rapidly rotating convection, and it plays an important role in the dynamo generation mechanism. There is evidence that the Lorentz force can affect the production of kinetic helicity, and this may be how the large-scale pattern of nonlinear dynamos is determined. Boussinesq convection-driven dynamos which produce a large-scale field generally lead to axially dipolar magnetic fields, quadrupolar fields being quite rare. In anelastic dynamo models, used to investigate the magnetic fields of the giant planets, the Lorentz force can reverse the sign of the kinetic helicity compared to the Boussinesq case. This may be why quadrupolar fields are much more common in anelastic dynamo models. The development of the magnetic helicity is relatively unexplored in planetary convection-driven dynamo models, but we will present a few preliminary results.

Implementation of a Lattice Boltzmann Method for Multiphase flows with High Density and Viscosity ratios.

Norjan Jumaa - Plymouth University
David Graham

We present a Lattice Boltzmann Method (LBM) for multiphase flows with high viscosity and density ratios. Following Banari et al. (2014), the motion of the interface between fluids is modelled by solving the Cahn-Hilliard (CH) equation with LBM. Incompressibility of the velocity fields in each phase is imposed by using a pressure correction scheme. We use a unified LBM approach with separate formulations for the phase field, the pressure-less Navies-Stokes (NS) equations and the pressure Poisson equation required for correction of the velocity field. The implementation has been verified for various test cases. Here, we present results for some complex flow problems including two dimensional single and multiple mode Rayleigh-Taylor Instability (RTI). Also, we present the evolution of the height of a standing wave for both high and low viscosity and density ratios.

Interfacial stability of multilayer shear flows with surfactants

Anna Kalogirou - University of East Anglia
Mark Blyth

The flow of multiple superposed layers of viscous liquids, known as multilayer flow, is of central importance in the rapidly burgeoning field of microfluidics. The ability to manipulate flows of this type is fundamental; one possible approach is by using chemical additives known as surfactants, which can greatly influence such flows especially at small scales. This talk will present a theoretical study that utilises mathematical modelling and numerical computations to scrutinise the effect of (insoluble and soluble) surfactants on the stability of multilayer shear flows in channels. Understanding stability is essential for efficient flow control in applications where (stable) uniform films or (unstable) interfacial waves are desired.

The flow configuration comprises two superposed layers of viscous and immiscible fluids confined in a long horizontal channel. The two fluids can have in general different densities, viscosities and depths, but here we are interested in the case with one of the layers being very thin compared to the other. The surfactant can be insoluble, i.e. located at the interface between the two fluids only, or soluble in the thin film. We therefore derive an asymptotic model in the thin-layer approximation, consisting of a set of nonlinear PDEs to describe the evolution of the film, interfacial and bulk surfactant concentration. Interfacial instabilities are induced due to the acting forces of gravity and inertia, as well as the action of Marangoni forces generated as a result of the dependence of surface tension on the interfacial surfactant concentration. The complex flow dynamics will be discussed, as well as the underlying physical mechanisms responsible for the formation of interfacial waves.

Energy evolution in oscillating gravitationally stratified coronal loops

Konstantinos Karampelas - KU Leuven
Tom Van Doorsselaere

Coronal loops have been the focus of studies regarding the damping of different magnetohydrodynamic (MHD) surface waves over the recent years. A number of observational and numerical studies have tried to connect the theory of wave damping both with coronal seismology and wave heating. The use of dissipation parameters, the physical characteristics of the loops and the total energy available in these systems play an important role in the evolution of these phenomena. In the current work we focus on the energy dissipation from transverse waves in gravitationally stratified coronal loops. Using the MPI-AMRVAC code, we perform three dimensional MHD simulations of kink waves in a straight density enhanced coronal flux tube. The temporal evolution of the energy distribution is examined for different values of viscosity and resistivity. Expanding on our past studies, our simulations hint towards resistive heating as the main dissipation mechanism in the environment of a coronal loop. The spatial evolution of the energy distribution is studied both along and across the flux tube in conjunction with the development of the Kelvin Helmholtz instability, while different methods of energy input are employed. Finally, we compare the efficiency of wave heating in our models with the findings of our previous work, for straight flux tubes in the absence of gravity.

Effects of streamwise elongated and spanwise periodic surface roughness elements on boundary layers

Csaba Katai - Imperial College London
Xuesong Wu

The impact on the boundary layer of spanwise periodic and streamwise elongated surface roughness elements is investigated. These types of roughness elements have been found to be able to stabilise the flow and hence delay transition. Our interest here is in their effects on the instability characteristics of the so-called lower-branch modes, and so the spanwise spacing is taken to be comparable with the spanwise wavelength of the instability mode which is of $O(R^{-3/8} L)$, where L is the dimensional length from the leading edge of the flat plate to the surface roughness, and R is the Reynolds number defined as $R = U_{inf} L/\nu$ (ν is the kinematic viscosity of the fluid, and Uinf is the flow velocity far from the plate). The streamwise length is much longer, consistent with experimental set up. Starting from the generic triple-deck theory for three-dimensional roughness elements, with both the streamwise and spanwise length scales being of $O(R^{-3/8} L)$, we derived the relevant boundary layer equations by appropriate rescaling. The resulting equations are parabolic because the pressure gradient in the spanwise direction is completely determined by the roughness shape while that in the streamwise direction is negligible. For sufficiently small roughness height the equations can be linearised, but fully nonlinear for suitably large roughness of interest. Appropriate upstream as well as boundary and matching conditions are derived for the problem. Due to the parabolicity, the equations can be solved efficiently using a marching method. A stability analysis is conducted to investigate the effect of Tollmien-Schlichting waves in the boundary layer and how it compares with previous the experimental findings.

Scale selection in the stratified convection of the solar photosphere

Mouloud Kessar - University of Leeds
David Hughes
Evy Kersale
Krzysztof Mizerski
Steven Tobias

The solar photosphere is known for its wide range of observable scales, such as the granular or supergranular scales. Over the last decades, several studies have performed numerical simulations of incompressible and compressible convection. Most of those have been performed for small density contrast. In this work, we examine the role of a strong stratification in determining the scale for turbulent anelastic convection. We perform local numerical simulations of convection for a range of density contrasts in large domains. We analyse both the Eulerian and Lagrangian statistics of the convection and demonstrate that increasing the stratification shifts the scale of the most energetic structures in the flow to smaller scales; furthermore, the relative amplitude of vertical to horizontal flows in the convection decreases with increasing stratification.

Scattering of long weakly-nonlinear waves in bi-layers with delamination

Karima Khusnutdinova - Loughborough University
Matthew Tranter

We study the scattering of long longitudinal bulk strain solitary waves in delaminated elastic bars within the scope of single and coupled Boussinesq-type equations with piecewise-constant coefficients subject to natural continuity conditions across the jumps. We develop direct and semi-analytical approaches to the problem, using asymptotic multiple-scale expansions and averaging with respect to the fast variables. Both pure and radiating solitary waves are considered. We also obtain partial theoretical estimates using the Inverse Scattering Transform. The results indicate that solitary waves can be used to detect delamination.

Ubiquitous modulation instability: complex breathing scenarios in optics and hydrodynamics

Bertrand Kibler - Laboratoire ICB - CNRS / Université Bourgogne Franche-Comté
Amin Chabchoub

The modulation instability is a universal mechanism that is responsible for the disintegration of weakly nonlinear narrow-banded wave fields and the emergence of localized breather events in dispersive media. We consider the nonlinear stage of modulation instability in the case of periodic or localized weak perturbations, by using a unique mathematical formalism based on the nonlinear Schrödinger equation. We review the recent theoretical and experimental advances, more specifically, the distinct experimental configurations for the sake of observation of breather waves in optics and hydrodynamics. In conclusion, we provide an outlook on novel research directions.

Effect of enhanced dissipation by shear flows on transient relaxation and probability density function

Eun-jin Kim - University of Sheffield

Shear flows are one of the most ubiquitous structures that naturally occur in a variety of physical systems (e.g. oceans, atmosphere, stars) and play an essential role in determining mixing of chemical species and transport of various quantities (e.g. heat, momentum). For example, a stable shear flow can dramatically quench turbulence by enhanced dissipation, leading to a severe reduction in mixing and transport. One remarkable consequence of this turbulence quenching is the formation of transport barrier where the transport is dramatically reduced, which is now believed to be indispensable for a successful operation of magnetic fusion as an economic future energy. Understanding the effect of different types of shear flows is thus absolutely critical.

We report a non-perturbative study of the effects of shear flows on turbulence reduction in a decaying turbulence. By considering different initial power spectra and shear flows (zonal flows, combined zonal flows and streamers), we demonstrate how shear flows rapidly generate small scales, leading to a fast damping of turbulence amplitude. In particular, a double exponential decrease in turbulence amplitude is shown to occur due to an exponential increase in wavenumber. The scaling of the effective dissipation time scale τ_{e}, previously taken to be a hybrid time scale $\tau_{e} \propto \tau_{\Omega}^{{2/3}} \tau_{\eta}$, is shown to depend on types of shear flows as well as initial power spectra. Here, τ_{Ω} and τ_{η} are shearing and molecular diffusion times, respectively. Furthermore, we present time-dependent Probability Density Functions (PDFs) and discuss the effect of enhanced dissipation on PDFs and a dynamical time scale $\tau(t)$, which represents the time scale over which a system passes through statistically different states.

Effects of shear flows on the evolution of fluctuations in interchange turbulence

Eun-jin Kim - University of Sheffield

Shear flows are one of the most ubiquitous structures that naturally occur in a variety of physical systems (e.g. oceans, atmosphere, stars) and play an essential role in determining mixing of chemical species and transport of various quantities (e.g. heat, momentum). For example, a stable shear flow can dramatically quench turbulence by enhanced dissipation, leading to a severe reduction in mixing and transport. One remarkable consequence of this turbulence quenching is the formation of transport barrier where the transport is dramatically reduced, which is now believed to be indispensable for a successful operation of magnetic fusion as an economic future energy. Understanding the effect of different types of shear flows is thus absolutely critical.

We report a non-perturbative study of the effect of different shear flows on the evolution of vorticity and particle density fluctuations in the interchange turbulence, which is a prototypical turbulence model in magnetically confined plasmas. In the inviscid limit, vorticity (density) grows (decays) as a power law due to streamers or zonal flows. However, due to the anisotropic stretching of wave numbers, the transport of density is less reduced by streamer than by zonal flow, and zonal flows lead to oscillation death (reduced oscillation frequency), highlighting different effects of zonal flows and streamers on turbulence regulation. Furthermore, the combined zonal flow and streamer induce oscillations at one frequency with exponentially growing (decaying) amplitude of vorticity (density) or at multiple integer frequencies with constant amplitude, depending on the relative sign of shear.

Multiphase modelling of tissue growth

John King - University of Nottingham

Multiphase approaches to the growth and deformation of biological tissue will be described and some of the associated mathematical issues analysed.

Surface-Tension-Driven Flow with Liquid Injection

Kristian Kiradjiev - University of Oxford

Gore's GMCS filter removes sulphur dioxide from flue gases by passing the gas through a filter comprising a porous structure made of sorbent-polymer composite (SPC) material. The gas adsorbs onto microscopic sorbent pellets in the SPC and reacts producing liquid sulphuric acid, which drains away along a network of fibres. Understanding the drainage mechanism and its driving forces is key to designing an effective filter. In this talk we consider a simplified problem of the surface-tension-driven spreading of a viscous fluid on a flat surface from part of the surface through which fluid is injected. This set-up is designed to mimic the production and accumulation of liquid sulphuric acid on a pellet within the SPC material and the subsequent drainage along a fibre.

We use asymptotic techniques for a thin viscous layer to obtain power-law dependencies of the film thickness and the position of the apparent contact line on time, which agree with the numerical solution to the full problem. We also present a simple inverse problem for determining the time-dependent form of the injection rate by measuring the motion of the contact-line position. Close collaboration with our partners at Gore enables our work to be tailored to the design challenge of sulphur-dioxide removal from flue gases and to more general applications involving spreading processes with injection of liquid.

Mathematical modelling of alumina feeding

Attila Kovacs - University of Oxford
James Oliver
Chris Breward
Andreas Muench
Svenn Anton Halvorsen
Ellen Nordgard-Hansen

Aluminium is an important and widely used metal, so detailed understanding of its production using the Hall-Heroult process is of high importance. The main electrolytic process used in this process today was discovered and developed in the 19th century, but despite its age many of the aspects are still open to further research.

One such aspect is the feeding of raw materials (alumina) into the cell during operation. This feeding is achieved by periodically pouring the room temperature particulate material onto the surface of molten cryolite that is slightly above its melting point. This dose then can either break up and dissolve quickly leading to optimal operation, or aggregate which slows down the dissolution, increases the probability of sinking, causes harmful gases to be generated, and can contaminate the product.

In this talk we present a two-phase multiple-region moving-boundary model for a single particle in a bath. Our analysis reveals the different timescales in the problem and, with the help of asymptotic analysis, we derive the movement of the boundary. Furthermore, we will discuss some of the ongoing work and challenges related to modelling of the floating aggregate.

Emergent Dynamics due to Spatial Heterogeneity in Reaction-Diffusion Systems

Andrew Krause - University of Oxford
Vaclav Klika
Thomas Woolley
Eamonn Gaffney

Motivated by understanding hair follicle formation in the developing mouse, we explore the use of spatial heterogeneity as a form of developmental tuning of a Turing pattern to match experimental observations of size and wavelength modulation in embryonic hair placodes. We demonstrate novel effects due to heterogeneity in two-component reaction-diffusion systems and explore how this affects typical spatial and temporal patterning. We find a novel instability which gives rise to periodic creation, translation, and destruction of spikes in several classical reaction-diffusion systems and demonstrate that this periodic spatiotemporal behaviour appears robustly away from Hopf regimes or other oscillatory instabilities. We provide some evidence for the universal nature of this phenomenon and use it as an exemplar of the mostly unexplored territory of explicit heterogeneity in pattern formation.

Comparison of asymptotic and numerical approaches to plane Poiseuille-Couette flow stability.

Rishi Kumar - Imperial College London
Andrew Walton

Plane Poiseuille-Couette flow is produced in a channel by the application of a uniform pressure gradient and relative motion of the walls. This type of flow is the linear combination of Poiseuille flow (in which flow is driven by an applied pressure gradient parallel to the walls of the channel) with plane Couette flow (in which flow is produced in a channel when walls slide with same speed (say with V) but in opposite directions). The aim of the talk is to investigate the linear stability of plane Poiseuille-Couette flow both at finite Reynolds numbers and in the large Reynolds number limit. To examine the linear stability at finite Reynolds number the Orr-Sommerfeld equation which describes the linear three-dimensional modes of disturbance to this flow is solved numerically using a standard ‘Chebyshev-collocation method’. Although there is a unique neutral curve when the sliding speed is zero, it is found that for non-zero sliding speed, multiple neutral curves exist in the two-dimensional problem ($\beta = 0$) for Poiseuille-Couette flow at large Reynolds number consistent with the predictions of Cowley & Smith (1985) exclusively based on a high-Reynolds-number asymptotic approach to this flow. These curves retreat to infinity provided the non-dimensional wall speed V (corresponding to the dimensional speed $V^{*}= U_m V$ where U_m is the maximum speed of the Poiseuille component of the flow) is approximately 0.34 in line with Reynolds & Potter's (1967) conclusions. We will compare the large R asymptotes of these various neutral curves with asymptotic theory. The asymptotic analysis of the lower branch of the main neutral curve for $R \gg 1$ and $V = \varepsilon^2 V_0$ where $\varepsilon \ll 1$ will be discussed thoroughly. Finally, we consider a qualitative comparison of finite Reynolds number computations of the two-dimensional problem and the corresponding solutions to the lower branch eigenrelation for a specific sliding velocity.

Investigating left ventricular geometries from diseased patients and healthy volunteers

Alan Lazarus - University of Glasgow
Hao Gao
Dirk Husmeier
Xiaoyu Luo
Wenguang Li

Changes in material properties of the heart can be useful in identifying left ventricular dysfunction. If one considers the set of equations modelling the myocardium, expensive parameter inference methods can be applied that can estimate these material parameters, providing insight into the function or dysfunction of the left ventricles. Alternatively, one can construct an emulator that can be queried at arbitrary input points to assess the corresponding material properties. Taking a more pragmatic approach, we may consider the geometries over a sample of left ventricles, both healthy and diseased, to assess the changes in geometry with different underlying conditions. We consider each subject defined by 2896 nodes on both the epicardium and endocardium surfaces of the heart. If one wishes to adopt a Cartesian system, the space in which our subjects lie is 8688 dimensional. These points alone cannot provide much information on the structure of the different left ventricles. Instead, we may project these data points onto a lower dimensional latent subspace, describing most of the variation in the left ventricular geometries. This talk will discuss the outcome when projecting onto these lower dimensional spaces, providing the possibility to classify healthy and diseased patients based on the geometries of their hearts alone or to include these geometries in a Gaussian process emulator for the material properties of different left ventricles.

Linear Differential Game Capture Zones in the Case of Measurement Errors

Stephane Le Menec - MBDA FRANCE

The problem of intercepting a manoeuvrable target (Evader) by an interceptor (Pursuer) is considered using a linearized kinematical model with first order acceleration dynamics and bounded controls for both players.
Thanks to small angle assumptions, the original (non-linear) system is linearized and scalarized by using the guidance and control concept of zero-effort miss distance as a new scalar state variable. In the new model framework, the Pursuer and the Evader respectively minimizes and maximizes (perpendicular) terminal miss distance at prescribed terminal time. Differential game capture zones and robust capture zones are constructed in the scalarized system. Robust capture zones are capture zones when the Pursuer strategies are restricted to certain classes of pursuit strategies. Linear and saturated linear strategies are considered in the case of computing robust capture zones, meaning while the differential game analysis leads to bang-bang strategies. It is of first important to compare the bang-bang capture zones that are the maximum capture zones respect to the robust capture zones that are smaller but that consider more realistic pursuit strategies. Interval computation is a tool that allows to take into account imperfect state measurements in a robust sense.

The main tool to be used in interval analysis is based upon the very simple idea of enclosing real numbers in intervals and real vectors in boxes. Recent advances have been made to simulate uncertain systems modelled by Ordinary Differential Equations and interval state vectors. Therefore, Interval Analysis allows to take into account imperfect state measurements in a robust sense in a way to compute differential game capture zones and robust capture zones that are now also robust to measurement errors. Comparisons between noisy linear differential game capture zones and noisy robust capture zones will be presented.

New insights into optimal control in guidance applications

Chang-Hun Lee - Cranfield University
Hyo-Sang Shin
Antonios Tsourdos

This talk devotes to present new insights into optimal control in guidance applications. In this study, we investigate the physical meaning or the working principle of guidance laws based on the optimal control theory, which is unrevealed so far. Our finding is that guidance laws based on the optimal control theory can be newly interpreted as combination of a state predictor for estimating a final state to be constrained and a specific form of governing equation for reducing a tracking error optimally. In our finding, the physical meaning of the optimal guidance laws is clearly shown. In practice, this information is important to ensure confidence in the performance and reliability of guidance laws based on the optimal control theory when implementing the optimal guidance laws in a real system. Also, the results obtained can provide a link between existing guidance laws based on nonlinear control and guidance laws based on optimal control. Therefore, the advantages of both techniques can be fully exploited by using the results obtained: existing nonlinear guidance laws can be converted to their optimal forms and the physical meaning of them can then be easily explained. Four illustration examples are provided to show how to utilize the results obtained to guidance applications.

A consistency study of coarse-grained models for a large dynamical particle system

Ping Lin - University of Dundee

Many applications are involved or partially involved with the interaction of large number of particles, for example, multi-cellular systems or subcellular elements in biological processes and atomistic systems in material sciences. A major difficulty for such systems is their large size and thus computing their solution may be extremely expensive or even impossible. There are quite a number of coarse-grained models developed to make the computation possible or easier. For dynamical problems there is almost no analysis whether the coarse-grained solution would be a good approximation to the original fine-grained solution. In this talk as an early attempt we will focus on a one-dimensional dynamical particle chain with nearest neighbour interactions, which is a special case of a number of coarse-grained models. The corresponding PDE of the dynamical particle chain is a nonlinear wave equation of mixed type. We can possibly construct the solution of its Riemann problem under an generalised viscous admissible condition. We numerically study the approximation of the coarse-grained model and see whether it is consistent with the fine grained model in this viscous sense. The solution of the Riemann problem may be used to understand the numerical solution. We also numerically study the coarse-grained model in the sense of average. Our conclusion is that if the particle interaction goes to non-convex part of the phenomenological interacting potential such an approximation is not reliable.

Public Engagement as a career - from engaging professionally to an Engagement Profession

Alina Loth - University of St Andrews
Alina Loth

With a strong steer from funding bodies and policy makers, public engagement (PE) is becoming an important aspect of research practice, from the individual to the institutional level. Public Engagement training for researchers and the participation in delivery opportunities, offer a great potential for personal skill development. The thriving field of PE creates a multitude of new job opportunities for academics, which allows for a stronger focus on PE (e.g. through PE grants or PE postdoc positions). But more academics are also aiming for a full-time profession within the wider field of Public Engagement with Research. Job opportunities are diverse and offer specialisations for almost any skill-set ranging from research illustrators, science communicators to management focused positions.

Layer formation and localisation in spanwise stratified plane Couette flow.

Dan Lucas - Keele University
C. P. Caulfield
Rich Kerswell

Recent research has shed light on the role of coherent structures in forming layers when stably stratified turbulence is forced with horizontal shear (Lucas et. al. 2017 *J. Fluid. Mech.* 832). In the current work we investigate the role of stable stratification in modifying coherent structures in plane Couette flow when the mean shear is horizontal i.e. gravity points in the spanwise direction. Direct numerical simulations reveal near wall layering and associated modified mean flows in the form of flattened streamwise rolls. Stratification is also found to inhibit the vertical growth of localised structures, meaning that spanwise localisation in the form of deep mixed layers are found which fill the wall-normal and streamwise extents. We also use this geometry to investigate the influence of stratification on the growth and localisation of isolated turbulent spots using a recently developed adaptive control procedure (Taylor et. al. 2016 *J. Fluid Mech.* 808).

Three-dimensional flows in a hyperelastic tube under external pressure

Xiaoyu Luo - University of Glasgow
Sen Zhang
Zongxi Cai

We study the collapsible behaviour of a vessel conveying viscous flows subject to external pressure, a scenario that could occur in many physiological applications. The vessel is modelled as a three-dimensional cylindrical tube of nonlinear hyperelastic material. To solve the fully coupled fluid-structure interaction, we have developed a novel approach based on the Arbitrary Lagrangian-Eulerian (ALE) method and the frontal solver. The method of rotating spines is used to enable an automatic mesh adaptation. The numerical code is verified extensively with published results and those obtained using the commercial packages in simpler cases, e.g. ANSYS for the structure with the prescribed flow, and FLUENT for the fluid flow with prescribed structure deformation. We examine three different hyperelastic material models for the tube for the first time in this context and show that at the small strain, all three material models give similar results. However, for the large strain, results differ depending on the material model used. We further study the behaviour of the tube under a mode-3 buckling and reveal its complex flow patterns under various external pressures. To understand these flow patterns, we show how energy dissipation is associated with the boundary lay- ers created at the narrowest collapsed section of the tube, and how the transverse flow forms a virtual sink to feed a strong axial jet. We found that the energy dissipation associated with the recirculation does not coincide with the flow separation zone itself, but overlaps with the streamlines that divide the three recirculation zones. Finally, we exam the bifurcation diagrams for both mode-3 and mode-2 collapses and reveal that multiple solutions exist for a range of the Reynolds number. Our work is a step forward towards modelling more realistic physiological flows in arteries and veins.

Multiscale Soft Tissue Modelling: Parameter Inference using Gradient Matching.

Benn Macdonald - University of Glasgow
Dirk Husmeier

Parameter inference in biological mechanistic models is a challenging problem. Describing the system through the use of differential equations (DEs) is a powerful way of giving an observed process a mathematical description. The equations depend on parameters which typically cannot be measured and therefore the task becomes statistically inferring them using the observed trajectories. This can be done by integrating the DEs for a given parameter set (obtaining a predicted signal), then quantifying the difference to the observed signal (after assuming an appropriate distribution for the measurement noise). However, since closed-form solutions of the equations usually do not exist, every time the parameters are changed/re-sampled, the DEs need to be numerically integrated. The computational costs associated with repeatedly doing so (for instance during optimisation or MCMC) makes this approach impractical for many systems. This presentation will discuss how to circumvent this computational burden through the use of gradient matching. Rather than solving the DEs to obtain a signal, gradient matching involves first smoothing the data with an interpolation technique, then obtaining the slopes of tangents to the interpolants. Parameters are then optimised/sampled to minimise the mismatch between these slopes and the parameter-dependent gradients from the DEs.

Modelling the immune response to cancer: an individual-based approach accounting for the difference in movement between inactive and activated T cells

Fiona Macfarlane - University of St Andrews
Tommaso Lorenzi
Mark Chaplain

A growing body of experimental evidence indicates that immune cells move in an unrestricted search pattern if they are in the pre-activated state, while they tend to stay within a more restricted area upon activation induced by the presence of tumour antigens. This change in movement is not often considered in the existing mathematical models of the interactions between immune cells and cancer cells. With the aim to fill such a gap in the existing literature, in this work we present a spatially structured individual-based model of tumour-immune competition that takes explicitly into account the difference in movement between inactive and activated immune cells. In our model, a Lévy walk is used to capture the movement of inactive immune cells, whereas Brownian motion is used to describe the movement of antigen-activated immune cells. The effects of activation of immune cells, the proliferation of cancer cells and the immune destruction of cancer cells are also modelled. We illustrate the ability of our model to reproduce qualitatively the spatial trajectories of immune cells observed in experimental data of single cell tracking. Computational simulations of our model further clarify the conditions for the onset of a successful immune action against cancer cells and may suggest possible targets to improve the efficacy of cancer immunotherapy. Overall, our theoretical work highlights the importance of taking into account spatial interactions when modelling the immune response to cancer cells.

A mathematical model for blood flow in the coronary circulation

Jay A. Mackenzie - University of Glasgow
Nicholas A. Hill
Xiaoyu Luo

Full simulations of the beating heart require a computational model for blood flow and pressure within the coronary arteries and veins. The key challenge is that, while most of the large vessels lie on the surface of the heart, the arterial small arteries, capillaries, and veins lie within the myocardium itself, and are thus subject to a periodic, external pressure exerted by the contraction of the myocardium during systole.

By developing the model for the pulmonary circulation of Qureshi et al. (2014), we simulate the 1D cross-sectionally averaged flow and pressure in the large arteries and veins using a Lax-Wendroff scheme, with geometries and material parameters appropriate for the coronary vessels. The flow at the inlet is driven by a specified time-dependent pressure in the aortic root from which the left and right coronary arteries branch.

Each pair of large coronary arteries and veins is connected by a vascular bed of smaller vessels. We construct a mathematical model of each vascular bed using an asymmetric structured-tree approach with linearised governing equations. The boundary conditions connecting each pair of large vessels relate pressure and flow via admittance matrices. We derive a novel admittance matrix that allows us to include a periodic external pressure. We obtain results that are consistent with physiological measurements and investigate the effects of microvascular disease, and vasodilation tests that are used to evaluate the function of the coronary microcirculation post myocardial infarction.

A similar approach can be applied to simulate the effects of respiration on the pulmonary circulation, under the assumption that the period of a respiratory cycle is an integer multiple of the cardiac cycle. Here too, we validate our numerical results against data and see good agreement between them.

Models and Numerics: Tools for the Dissection of Cell Migration and Chemotaxis

John Mackenzie - Strathclyde University
Robert Insall
C.R. Rowlatt
M. Nolan

A mechanistic understanding of directed cell movement is essential to help answer a range of questions such as defective wound healing, incorrect embryonic development and cancer metastasis. The underlying biological process are complex; involving a myriad of biochemical pathways and their interaction with mechanical forces propelling cells forward.

To make progress with these issues it is now being accepted that a multidisciplinary approach is needed where mathematical and computational models play an important role to help unravel experimental data and the suggestion of additional experimental investigations. One of the main challenges is the interaction of cell with its microenvironment and the transduction of information through the cell membrane. These processes have recently been modelled using evolving bulk-surface PDEs. From a numerical viewpoint the efficient solution of these systems can be tackled by various techniques which depend on wether the cell membrane is treated as a sharp interface or is implicitly defined via an indicator function.

In this talk I will present some recent developments in numerical methods for geometric evolution laws which are normally used to model the movement of the cell membrane. I will indicate how to couple the resulting meshes to evolving bulk meshes. These will then be used with a conservative Arbitrary Lagrangian-Eulerian finite element method for reaction-diffusion systems on evolving domains. Examples will be given of the use of the computational tools to image segmentation, cell tracking and the simulation of self-generated cell chemotaxis.

The emergence of braided magnetic fields

David MacTaggart - University of Glasgow
Chris Prior

The "standard model" of flux emergence on the Sun involves twisted magnetic flux tubes that rise to the photosphere (solar surface) and then expand into the atmosphere to create active regions. Despite the many successes of this model, there are some dissenting voices. Simulations of the upper convection zone suggest that "complex bundles" of magnetic field can lead to active region formation. Observations of helicity in the atmosphere suggest that some active regions have "mixed chirality", differing from the "single chirality" twisted flux tube. To attempt to unify these differences to the standard model, we will consider the emergence of braided, rather than twisted flux tubes. Braided fields ("complex bundles") can be produced by convective motions and have complex helicity patterns similar to those that are observed. Braids can be classified by their helicity and so we can systematically study the emergence of flux tubes with different helicity structures (including the twisted tube). In this talk, we will look at the emergence of two braid models in particular - one braided with thick (sub-)flux tubes and the other braided with many smaller-scale flux tubes. We will display the resulting twist patterns that these models produce once they emerge into the atmosphere. We will discuss observational consequences and mention on-going work which uses different braid models to study complex helicity injection into the atmosphere.

Predicting chaotic behaviour using data assimilation

Godwin Madho - University of Leeds
Steven Tobias

All computer models are imperfect. This problem is intensified further when studying chaotic behaviour. Therefore, it is important to reconcile the differences in the results produced by modelling and observations. Many studies have used data assimilation to overcome this issue. In this study we aim to test the capabilities of the Ensemble Kalman Filter (EnKF) to track complex behaviour. We present benchmark results using the Lorenz model. We look at how factors such as ensemble size and inflation affect our ability to track the 'truth'. We also present results for parameter estimation in the Lorenz model. Finally, we present a few preliminary results for a thermally rotating annulus model.

A robust and efficient adaptive multigrid solver for the optimal control of phase field formulations of geometric evolution laws with applications to cell migration

Anotida Madzvamuse - Sussex

In this talk, I will present a novel solution strategy to efficiently and accurately compute approximate solutions to semilinear optimal control problems, focusing on the optimal control of phase field formulations of geometric evolution laws. The optimal control of geometric evolution laws arises in a number of applications in fields including material science, image processing, tumour growth and cell motility. Despite this, many open problems remain in the analysis and approximation of such problems. In the current work we focus on a phase field formulation of the optimal control problem, hence exploiting the well-developed mathematical theory for the optimal control of semilinear parabolic partial differential equations. Approximation of the resulting optimal control problem is computationally challenging, requiring massive amounts of computational time and memory storage. The main focus of this work is to propose, derive, implement and test an efficient solution method for such problems. The solver for the discretised partial differential equations is based upon a geometric multigrid method incorporating advanced techniques to deal with the nonlinearities in the problem and utilising adaptive mesh refinement. An in-house two-grid solution strategy for the forward and adjoint problems, that significantly reduces memory requirements and CPU time, is proposed and investigated computationally. Furthermore, parallelisation as well as an adaptive-step gradient update for the control are employed to further improve efficiency. Along with a detailed description of our proposed solution method together with its implementation we present a number of computational results that demonstrate and evaluate our algorithms with respect to accuracy and efficiency. A highlight of the present work is simulation results on the optimal control of phase field formulations of geometric evolution laws in 3-D which would be computationally infeasible without the solution strategies proposed in the present work.

Cross-diffusion-driven instability for reaction-diffusion systems on evolving domains and surfaces: models, analysis and simulations.

Anotida Madzvamuse - Sussex

In this talk, I will present cross-diffusion and domain-growth induced pattern formation for reaction-diffusion systems with linear cross-diffusion on evolving domains and surfaces. Our major contribution is that by selecting parameter values from spaces induced by cross-diffusion, domain and surface evolution, patterns emerge only when cross-diffusion and domain growth are present. Such patterns do not exist in the absence of cross-diffusion, domain and surface evolution. In order to compute these cross-diffusion domain-induced parameter spaces, linear stability theory is employed to establish the necessary conditions for domain-growth induced cross-diffusion-driven instability for reaction-diffusion systems with linear cross-diffusion. Model reaction-kinetic parameter values are then identified from parameter spaces induced by cross-diffusion and domain-growth only; these exist outside the classical standard Turing space on stationary domains and surfaces. To exhibit these patterns, we employ the surface finite element method for solving reaction-diffusion systems with cross-diffusion on continuously evolving domains and surfaces.

Coupled bulk-surface reaction-diffusion systems: modelling, analysis and simulations

Anotida Madzvamuse - Sussex

In this talk, I will formulate new models for coupled systems of bulk-surface reaction-diffusion equations on stationary and evolving volumes. The bulk reaction-diffusion equations are coupled to the surface reaction-diffusion equations through linear Robin-type boundary conditions. On stationary volumes, I will state and prove the necessary conditions for diffusion-driven instability for the coupled system. It turns out that using linear Robin-type boundary conditions enables us to decouple the stability analysis of the bulk and surface dynamics. Our most revealing result is that, under a suitable choice of model parameter values, the bulk reaction-diffusion system has the capability of inducing patterning everywhere on the surface independent of whether the surface reaction-diffusion system produces or not, patterning. On the other hand, the surface reaction-diffusion system is not capable of inducing patterning everywhere in the bulk in the absence of patterning from the bulk reaction-diffusion system. For this case, patterns can only be induced in regions close to the surface membrane. Various numerical experiments are presented to support our theoretical findings. I will then discuss extensions of the model framework to exponentially evolving volumes and present some preliminary analytical and numerical results with applications to pattern formation.

Mode Interactions in Spherical Rayleigh-Bénard Convection

Paul Mannix - Imperial College London
Jonathan Mestel

The critical Rayleigh number Ra_c for thermally convective instability depends on the wavelength of the disturbance. In an annular spherical domain with stress-free or rigid boundaries, there are special parameter values $(Ra_c,\; d _c)$ at which instability to two different sets of thermal-rolls occurs simultaneously. Weakly non-linear analysis of this multiple-bifurcation problem is used to derive a set of coupled amplitude equations; a proxy to facilitate understanding of the system geometry and Prandtl number σ dependence. The absence of mid-plane reflection symmetry is demonstrated to be of crucial significance in the resulting equations structure. Classifying solutions we identify mode transitions, temporal resonance behaviour and illicit parameter dependencies.

William John Macquorn Rankine (1820-1872) and the making of engineering science in nineteenth-century Glasgow.

Ben Marsden - University of Aberdeen

This talk explores the interface between mathematical and practical engineering in the third quarter of the nineteenth century, through the example of W. J. Macquorn Rankine, professor of civil engineering and mechanics at the University of Glasgow from 1855 to 1872 and author of an influential series of ‘manuals’ of engineering science. The paper sets Rankine's early life as a practical engineer, his forays into the new science of thermodynamics in the early 1850s, and his extensive publications in and beyond 'applied mechanics' against contemporary attempts to bring the training of British engineers into universities and colleges from the late 1830s onwards. Though Rankine insisted his mission was to build a bridge between 'theory' and 'practice', by developing the art of applying scientific principles to practice, working engineers often found him incomprehensible and, indeed, readily dispensable. Even committed advocates of mathematical engineering science found it necessary to digest Rankine's copious but dense writings for the common intellect.

The Direct Computation of Time-Periodic Solutions of PDEs & Applications to Fluid Dynamics

Puneet Matharu - University of Manchester
Andrew Hazel
Matthias Heil

Many PDEs have time-periodic solutions and it is often of interest to explore their dependence on problem parameters. The determination of such solutions by direct time-integration can be very inefficient as transients may take a long time to decay. Furthermore, this method cannot robustly find unstable solutions, which may be of interest.

In this talk we present a finite-element based space-time approach that allows the direct computation of time-periodic solutions. We demonstrate the application of the methodology for the forced unsteady heat equation and the diffusive Van der Pol equation. Finally, we illustrate its application to the study of flow past an oscillating cylinder.

The stability of the gamma-ray burst jet propagating through a progenitor star

Jin Matsumoto - University of Leeds

One of the most important and oldest problem in astrophysics is the evolution and stability of the relativistic jet which velocity is almost equal to the speed of light. Relativistic jets are collimated bipolar outflows and ubiquitous among astrophysical systems consisting of a compact object surrounded by an accretion disk, e.g., active galactic nuclei (AGNs), microquasars, and the central engine of gamma-ray bursts (GRBs). We addressed the stability of the jet interface in the context of the GRB through three-dimensional special relativistic magnetohydrodynamic (MHD) simulations.

Although the divergence-free property of the magnetic field is automatically satisfied in one-dimensional simulations, it is not easy to maintain the divergence-free condition in multi-dimensional MHD simulations. Numerical divergence of the magnetic field results in unphysical results and unexpected termination of the simulation in multi-dimensions. Therefore, the treatment of the divB=0 is crucial for the result of calculations and the most important issue in computational MHD. We have recently constructed a new relativistic MHD code based on upwind constrained transport methods (Londrillo & Del Zanna 2004; Minoshima et al. 2015). Numerical fluxes at cell surfaces are calculated using the relativistic HLLD scheme (Mignone et al. 2009) in order to update conservative variables. In addition, the electric field at cell edges for updating the magnetic field at cell surfaces is also estimated using the relativistic HLLD Riemann solver.

Using our new MHD code, the propagation of the GRB jet that was continuously injected from the boundary of the calculation domain into a progenitor star was solved. We found that the interface of the jet was deformed by the growth of the centrifugal instability (CFI) when the magnetic field of the jet was weak. The growth of the CFI at the interface of the relativistic jet is recently reported in the context of the AGN jet for the first time (Gourgouliatios & Komissarov 2017) and has a potential impact on the transition from the laminar to the turbulent flow at the reconfinement point of the jet. The evolution of the turbulence at the jet interface is important for the radiative outcome from the relativistic jet. In addition, we performed the parameter study with varying the strength of the magnetic field of the jet in order to investigate the dependence of the magnetic field on the growth of the CFI. Another intriguing finding in our study is that the strong toroidal magnetic field of the jet prevents the growth of the CFI due to the magnetic tension force.

Acceleration and global convergence of a first-order primal–dual method for non-convex optimisation problems

Stanislav Mazurenko - University of Liverpool
Christian Clason
Tuomo Valkonen

First-order primal–dual algorithms are of paramount significance in mathematical image processing and more general inverse problems due to the resulting trade-off between precision and computation time. Recent advances have extended their applicability to areas previously dominated by second-order algorithms, such as non-convex problems arising in optimal control. Nonetheless, their application to non-convex large-scale optimisation still requires investigation. In this study, we analyse an extension of the primal–dual hybrid gradient method (PDHGM, also known as the Chambolle–Pock method) designed to solve problems with a nonlinear operator in the saddle term. Based on the idea of testing, we derive new step length parameter conditions for the convergence in infinite-dimensional Hilbert spaces and provide acceleration rules for suitably locally monotone problems. Importantly, we demonstrate linear convergence rates and prove global convergence in certain cases. We also show the efficacy of these new step length rules on PDE-constrained optimisation problems.

'Graecum est legi non potest': James Thomson Snr and the teaching of arithmetic, trigonometry and calculus in early 19th century Belfast.

Mark McCartney - Ulster University

James Thomson Sr, father of Lord Kelvin, was both school teacher and college professor of Mathematics in Belfast from 1814/5-1832. During his time in the town he wrote a number of textbooks, and these, combined with correspondence and examination questions allow us to use them as a case study in the teaching of mathematics in the early 19th century.

A multi-image based approach for modelling plant-fertiliser interaction

Daniel McKay Fletcher - University of Southampton

The transport of nutrients from a fertiliser, towards a root and subsequent absorption by the plant is an important process in agriculture to ensure the health of crops. Phosphorus (P) is one of the most important nutrients for plant growth and is often the limiting factor in obtaining high crop yields. The main source of P for fertiliser is obtained from mines. However, these reserves are dwindling, resulting in increasingly expensive P and subsequently more expensive food. Like the global dependence of energy on fossil fuels, our dependence on mined P is unsustainable. Struvite ($NH_4MgPO_4{\cdot}6H_2O$) is a renewable source of P (and nitrogen), and is a candidate for next-generation sustainable fertilisation. This study assessed mixed struvite-Monoammonium Phosphate (MAP) pellets as a method for supplying P to plants. Root System Architecture (RSA) proliferation in nutrient rich regions may be important for P uptake from these fertiliser pellets.

Soil-root-fertiliser, or indeed any soil system, cannot be fully examined through just one experimental method. Structure, chemistry, physics and biology all play significant roles in many important functional properties of the soil systems, such as root nutrient uptake, effective diffusion, hydraulic conductivity and microbial activity. Therefore, three methods were used to determine a root structure's response to a P fertiliser pellet, the transport of P in soil from the fertiliser towards roots and the subsequent root uptake.

X-ray Computed Tomography (XCT) was used to visualise the undisturbed 3D structure of root-soil-fertiliser systems and produce computational meshes for numerical modelling. Root length density was seen to increase closer to the fertiliser pellet. Secondly, Scanning Electron Microscopy with Energy Dispersive X-ray Spectroscopy (SEM-EDS) was used to obtain 2D P maps of planes within

the imaged regions. With the use of averaging, high-resolution P gradients originating from the fertiliser pellet were detected. These two imaging methods were aligned using an object detection and descriptor approach to verify that the process to obtain the P maps did not disturb the sample and to reveal the 3D context of the 2D P maps.

Models describing the processes were parametrised by the P maps and XCT data. An image-based model, where the domain is extracted from 3D images obtained with XCT was used to describe the transport of P from the fertiliser pellet to the roots. Parameters in this model such as the root uptake rate and the buffer power of the soil for P were calculated by fitting the model to the P gradients detected in SEM-EDS maps. Finally, the fully parameterised model was used to assess the effect of enhanced root proliferation in the nutrient rich soil region near the fertiliser pellet. Predicted plant P uptake in a soil column with a plant and a fertiliser pellet was compared to a column containing a plant with an artificial fertiliser pellet added to the mesh. Total P uptake per-root-surface-area was higher in the case where the roots had responded to the fertiliser pellet.

Modelling the effect of intercellular plasmodesmata on auxin dynamics at the Arabidopsis root tip

Nathan Mellor - University of Nottingham

The plant hormone auxin plays an integral role in many different processes during growth and development, and in the root is known to affect growth rate, lateral root branching, and gravitropism (the bending of roots towards a gravity stimulus during growth). Auxin dynamics within plant tissues is controlled by a set of efflux and influx carriers, whose distribution and polarity at cell membranes determines the overall pattern of auxin concentration and direction of auxin flux. In addition to polar transport, intercellular plasmodesmata, which are narrow pores directly linking the cytoplasm of adjacent cells, are also likely to impact on auxin dynamics, an aspect not previously considered by existing models of auxin transport. By adding plasmodesmata to a detailed multicellular model of auxin transport at the Arabidopsis root tip we investigate the role plasmodesmata may play in auxin patterning and flux through root tissues. The model is tested and refined using experimental data showing the spatial distribution of the auxin reporter gene DII-VENUS under various genetic variations and experimental treatments.

The Generalised Structure Tensor approach for the mixed invariant I_8 and its application to constitutive modelling of passive myocardium

Andrey V. Melnik - University of Glasgow
Xiaoyu Luo
Ray W. Ogden

Generalised Structure Tensors (GSTs) are used to formulate constitutive models for anisotropic fibre-reinforced materials in which fibres are dispersed. The GST approach has been so far successfully applied to models based on invariants I_4 and I_5 (I_6 and I_7). These anisotropic invariants capture the effect of deformation on each fibre family in isolation, unlike the invariant I_8, which couples two fibre families. We extend the GST approach to models based on the invariant I_8. We consider two different formulations and discuss arising issues. We derive expressions for the dispersed invariant I_8^*, stress tensors and elasticity tensors in the general case, in the cases of axisymmetric distributions, and in the case of coaxiality of families' structures. We demonstrate that when accounting for fibre dispersion in the I_8 term, the effect on the predicted material response can be significant and may also reduce material symmetry.

We apply the proposed formulation to the hyperelastic HO model for myocardium and obtain a modified HO model, in which fibre dispersion is consistently accounted for in every term of the strain energy function. For the purpose of fitting constitutive parameters to the mechanical tests data, we consider a homogenised GST, which is associated with the changing myofibre direction. Such variability of the tissue structure should be addressed, since tissue samples used for mechanical tests are noticeably inhomogeneous. The homogenised GST (defined as the average value over a tissue region) does not necessarily possess axial symmetry even if fibre dispersion is assumed to be locally axially symmetric. Different but related values of homogenised structure tensors are obtained for biaxial and shear tests, due to the difference in dimensions of tissue samples. Using our new model, we are able to produce an improved fit to available biaxial and shear data for ventricular myocardium.

Eco-Evolutionary Dynamics in a Randomly Switching Environment: Competition and Cooperation

Mauro Mobilia - University of Leeds
Karl Wienand
Erwin Frey

Environmental variability greatly influences how the size and the composition of a population evolve, i.e. its eco-evolutionary dynamics. In this talk, we consider a population of finite and fluctuating size whose growth is limited by a randomly switching carrying capacity. This models the environmental fluctuations between states of resources abundance and scarcity. We consider a population consisting of two strains, one slightly faster than the other, that compete under two scenarios: one in which competition is solely for resources, and one in which the slow ("cooperating") strain produces a public good. We investigate how the coupling of demographic and environmental (external) noise affects the population's eco-evolutionary dynamics.

By analytical and computational means, we determine the system's fixation properties (evolutionary dynamics) and show that coupled internal and external noise can significantly enhance the fixation probability of the slower-growing species. We also study cooperation dilemma associated with the public good production and arising from the correlations between the size and composition of the population. This results in an "eco-evolutionary game" in a fluctuating environment, and we determine the circumstances under which it is best to cooperate. Time permitting, we will also outline how the generalized linear noise approximation to populations of varying size allows us to analyse the coupled effects of demographic and environmental noise on the full population size distribution.

Tunnelling corrections to wave transmissions on shell structures

Neekar Mohammed - University of Nottingham
Stephen Creagh
Gregor Tanner
Sharad Jain

Modelling and predicting the vibrational properties of very large mechanical built-up structures of complex shape, such as cars, ships, and aircraft is a challenging task, especially in the mid-to-high frequency regime. In the low-frequency regime, standard modelling tools, such as finite and boundary element methods, are robust but not scalable to higher frequencies due to the prohibitive increase in model size. In the mid-to-high frequency regime, ray tracing is well suited for capturing such propagation. However, so far it is not used for predicting structure borne sound on complex, curved built-up structures.

We present a method here that captures wave effects in a ray-tracing treatment on curved plates. The equations of a ray dynamics for energy transport on arbitrarily curved and inhomogeneous smooth thin shells can be obtained via the Eikonal approximation from the underlying wave equations. We analyse mid-frequency effects below the ring frequency of curved plates for a cylindrical region smoothly connected to two flat plates using Donnell shell theory. We perform ray tracing based on Hamilton's equations derived in the short wavelength regime for bending, shear and pressure incident waves. Rays incident on the curved shell structure may be reflected or transmitted. "Simple" ray tracing gives either total reflection or total transmission; the full wave solution shows in contrast a smooth transition and exhibits resonant states in the waist of the cylindrical region. In one-dimension, both the smooth transition and the resonant states can be treated using Wentzel-Kramers-Brillouin (WKB) approximation extended to complex rays. We use graph models to account for resonant tunnelling in such curved plates. For classically-transmitted bending rays, we find complex rays which connect them to the resonant states that are formed in the curved region. Similarly, for classically reflected rays, we identify those complex rays that connect to the phase space of transmitting trajectories. We successfully find a theoretical expression for calculating the scattering matrix for bending rays which accounts for resonant tunnelling mediated by resonant states. Our model agrees well with the full wave solution.

Using dipole wall collision to validate slip and no slip moment-based boundary conditions for the lattice Boltzmann equation

Seemaa Mohammed - Plymouth university
David Graham
Tim Reis
Seemaa Mohammed

Normal and oblique two-dimensional dipole wall collisions for various collision angles are simulated by using the lattice Boltzmann equation method (LBE) with moment boundary conditions. A two relaxation time (TRT) model is used throughout. Both slip and no slip boundary conditions are applied by specifying the appropriate hydrodynamic moments of LBE instead of the standard 'bounceback'-type conditions. The dissipation rate of the energy rate is examined and contrasted with theoretical predictions. It is shown that the normal relationship between dissipation and enstrophy is modified in the presence of wall slip: whilst peak enstrophy levels are diminished when wall slip is allowed, extra dissipation is found due to the interaction between wall voracity and slip velocity.

Numerical Simulations of Action Potential Propagation in Cardiac Tissues with Myocardial Infarction Scars

Peter Mortensen - University of Glasgow
Radostin Simitev
Hao Gao
Godfrey Smith

The heart beats with a particular pattern. Starting at the Sino-Atrial node in the right atrium, an electrical wave passes across the atria, past the Atrioventricular node, down the septum and around the outside of the ventricles. This wave causes blood to be pumped through with maximum efficiency. However, when a myocardial infarction damages the muscle tissue, this pattern is interrupted, leading to heart failure.

We use finite element methods to simulate and explore the propagation of electrical excitation waves in cardiac tissues in 2D and 3D slabs as well as in more realistic geometries of the left ventricle. We include regions with reduced conductivity to model tissue that has been affected by myocardial infarction. The simulations are compared to experiments on rabbit hearts, performed on sections of the left ventricle and on whole hearts, by R. Myles et al. and A. Allan, respectively.

Currently, the single-cell electrophysiology models being used are for human ventricles (by TusccherPanfilov et al.) and for rabbit ventricles (by Mahajan et al.). Both models are upscaled to monodomain tissue models and the simulations produce contour plots for key properties such as activation times, calcium concentration and action potential duration. Future work will include using parameter estimation with data from rabbit left ventricles and coupling with models of muscle contraction.

Non smooth model of Ice Ages

Kgomotso Morupisi - University of Bath

It has been observed that the Earth's climate can be charaterised by alternating glacial and interglacial cycles which have been found to have experienced different frequencies such as 23000years, 40 000years and 100 000 years. Of these frequencies, the dominant 100 kiloyear period is the only frequency that could not be explained by using Milankovitch theory. This has led to development of different models of Ice Ages such as PP04 model, in order to study the factor that causes Ice Ages and physical mechanisms that governs their initiation and termination.

PP04 model is a non-smooth quasi-periodically forced dynamical system made up of a number of relaxation systems with discontinuous jumps due to modelling of sudden release of carbon dioxide at end of each glacial termination. A number of studies have been done on the model but all have analysed it using the techniques of smooth dynamical systems.

In this talk, I will introduce how to solve the non-smooth dynamical system model of glacial cycles using non-smooth dynamical systems theory.

Understanding the noise generated by a jet engine: a study of the 3D diffraction by a quarter-plane.

Marianthi Moschou - University of Manchester
Raphael Assier
William Parnell

Living or working near an airport can prove to be quite disrupting. One of the reasons is the noise generated by aeroengines. Hence, the understanding of noise generated by a jet engine, which is linked with the theory of diffraction of acoustic waves, is of great importance.

Canonical diffraction theory investigates the diffraction of an incoming wave by an obstacle with simple geometry, but with certain characteristics, such as edges or corners. Most of the analytical methods in aeroacoustics are based on blade-gust interactions, where the blade is often modelled as a half-plane. The corresponding canonical problem is the Sommerfeld problem. Our goal is to understand the effect played by blade tips and hence we first need to study the three-dimensional diffraction by a quarter-plane, one of the last unsolved canonical problems in diffraction theory.

A lot of progress has recently been made about the quarter-plane problem with simple boundary conditions, the widely known Dirichlet and Neumann boundary conditions, thanks to the method of embedding formulae. This is a new technique used in diffraction theory that enables us to fully describe the far-field in a scattering problem.

In this talk, we will focus on extending this method to more realistic boundary conditions of impedance type.

Numerical investigation of the hydrodynamics and tracking of the inflow and moving bed in a Hele-Shaw geometry

Erietta Moulopoulou - University of Leeds
Onno Bokhove

Beaches consist of eroded material transported and deposited by the sea. Gravel beaches in particular are a natural form of sea defence as they can absorb a lot of the wave energy. Consequently, it is important to understand how gravel beaches and waves interact. To this end, a vertical Hele-Shaw tank was introduced as a means to explore this interaction between breaking waves and a “slice” of beach. Consisting of two narrowly-placed glass plates, the tank is an ideal set-up to study wave-beach interactions. Among its advantages turbulence remains significantly reduced and visibility of the beach and wave dynamics is ensured.

In the full case of the Hele-Shaw experiment, particles are inserted in the cell and transported by the wave, thus gradually forming a “gravel” beach. As a first step, focus was put on the hydrodynamics; two extended mathematical models were used to describe the waves observed in the Hele-Shaw tank. The presence of the pump and of the narrow plates required extension of the shallow-water and potential-flow for shallow-water models so as to include the effects of influx and dissipation of momentum. The aforementioned extended models were then implemented using Finite Volume and the Finite Element methods, respectively. Furthermore, both of these models were validated against experimental data; the data was acquired by running the experiment and then were embedded within the numerical codes through the boundary conditions.

In the case of a narrow-width tank such as the one used during this project, it is difficult to measure the inflow efficiently and acquiring experimental data can become tenuous. Tracking the flow and the moving bed in the Hele-Shaw tank using image processing methods was a simple but effective solution to this problem. An image processing algorithm was created and used extensively to process thousands of snapshots originating from runs of the experiment, effectively tracking and extracting the water's free surface. The analysis of the processed snapshots allows us to acquire information about the depth and inflow of water as well as the depth and shape of the moving bed.

The initial dynamics of drop coalescence

James Munro - University of Cambridge
John Lister

When two drops touch, a fluid bridge is formed between the drops. Surface tension acting on this tightly-curved surface pulls the fluid bridge wider and the drops together. Previously, it was thought that the initial dynamics could be modelled with a Stokes flow, but recent experiments and simulations have shown that the flow is qualitatively different for any fluid with inertia as well as viscosity. In this talk, I'll present a new theoretical solution for the initial dynamics of drop coalescence with viscosity and inertia. This asymptotic solution identifies the key lengthscales of the flow, including a new lengthscale on which inertia and viscosity are in balance. We solve appropriate forms of the Navier-Stokes equations analytically on each lengthscale and match the solutions together for a complete picture of the initial stage of drop coalescence.

High Contrast Approximation to Penetrable Wedge Problems

Matthew Nethercote - University of Manchester
Raphael Assier
Ian David Abrahams

The problem of wave scattering by a penetrable wedge is one of the most important canonical problems in diffraction theory. Solutions to this problem will be useful for both acoustic and electromagnetic applications, including noise transmission, antennas and propagation, and crystal diffraction. To this day there is no clear analytical solution for penetrable wedge diffraction, however there have been numerous attempts at computational and asymptotic solutions by authors such as A. Rawlins, M. Lyalinov, V. Daniele and A. Shanin.

This talk will use the difference in the material parameters of the host and the scatterer in order to create a high contrast asymptotic approximation. What follows is an iterative scheme where the complicated penetrable wedge problem is split into an infinite amount of “easier” impenetrable wedge problems. The first of the “easy” problems is well known and can be solved explicitly. Each stage following this uses boundary data from the previous problem and is solved by the Wiener-Hopf technique.

Roto-flexural waves in elastic beams with gyro-hinges

Michael Nieves - Keele University (UK) & University of Cagliari, (Italy)
Giorgio Carta
Ian Jones
Natasha Movchan
Alexander Movchan

In this talk, we consider a chiral system represented by the interaction between Euler Bernoulli beams and gyroscopic spinners. For the beams, we derive a novel chiral boundary condition embedding this interaction, which is referred to as a *gyro-hinge*, under the assumption the nutation angles of the spinners are small. We show this condition leads to the coupling of flexural waves with rotational motion in the time-harmonic regime, where it is also demonstrated how the spinner properties dramatically influence the eigenfrequencies of the system.

The formulation is then extended to investigate an infinite system of beams connected by periodically placed gyro-hinges. The effect of the gyroscopic spinners on the wave dispersion properties of the system is discussed in detail. In particular, in the low-frequency regime, we show this structure can approximate the *gyrobeam*. The gyrobeam is a theoretical mechanical element possessing a continuous distribution of stored angular momentum. Recently, it has been used to design an efficient support system of resonators for frame-like civil engineering structures exposed to seismic vibrations. One drawback in the formulation of the gyrobeam is that the gyricity parameter appearing in the governing equations, which couples the transverse motions of the beam, is not defined in terms of physical quantities. Hence the construction of such an element is impossible in practice. We give an interpretation of this parameter in terms of the physical properties of the beams and gyroscopic spinners comprising the infinite periodic system.

A generic model of single-cell cardiac electrophysiology

Muhamad Hifzhudin Noor Aziz - University of Glasgow
Radostin Simitev

We propose a prototypical model of single-cell cardiac electrophysiology. The model has only three variables, it is simple enough to allow an explicit analytical solution and it is inexpensive for numerical simulation. We fit the parameters of the model to reproduce the action potential morphology of several detailed models of atrial and ventricular cells as well as to reproduce experimental data from isolated rabbit ventricular cells. As a first step, we carry out a sensitivity analysis by perturbing the parameter values by 30% to understand the effect parameter values have on the action potential morphology. For the parameter estimation, we then use the derivative-free simplex-based method of Nelder-Mead with multiple fitness functions. With the so-estimated parameter values, we compute APD restitution curves using the prototypical model and we find a good agreement with the APD restitution curves computed using the various detailed ionic models and experimental data.

Applications of a Cole-Hopf transform to the 3D Navier-Stokes equations

Koji Okitani - University of Sheffield
Riccardo Vanon

The Navier-Stokes equations written in the vector potential can be recast as the nonlinear Schroedinger equations at imaginary times, i.e. the heat equations with a potential term, using a Cole-Hopf transform [K.Ohkitani (2017), Journal of Turbulence 18, 465]. On this basis, we study two kinds of Navier-Stokes flows by means of direct numerical simulations. In an experiment on vortex reconnection, it is found that the potential term takes large negative values in regions where intensive reconnection is taking place, whereas the signature of the nonlinear term is more broadly spread. For decaying turbulence starting from a random initial condition, such a correspondence is also observed in the early stage when the flow is dominated by vorticity layers. At later times, when the flow features several tubular vortices, this correspondence becomes weaker. Finally, a similar set of transformations is presented for the magneto-hydrodynamic equations, which reduces them to a set of heat equations with suitable potential terms, thereby obtaining new criteria for the regularity of their solutions.

On the origin of non gaussian statistics in equations of the Nonlinear Schrödinger type

Miguel Onorato - Dipartimento di Fisica Università di Torino

Here I study the formation of extreme events in incoherent systems described by the Nonlinear Schrödinger type of equations. I consider an exact identity that relates the evolution of the normalized fourth-order moment of the probability density function of the wave envelope to the rate of change of the width of the Fourier spectrum of the wave field. I show that, given an initial condition characterized by some distribution of the wave envelope, an increase of the spectral bandwidth in the focusing/defocusing regime leads to an increase/decrease of the probability of formation of rogue waves. Extensive numerical simulations in 1D+1 and 2D+1 are also performed to confirm the results.

Test Particle Simulations at Tearing of Null-Point Current Sheets

Ross Pallister - University of Dundee
David Pontin

Shearing of a magnetic null-point in the Solar corona can lead to the formation of a current sheet. The formation of flux ropes can destabilise the sheet, leading to tearing and quasi-turbulent dynamics. We have written a test-particle simulation code to model the full motions of protons in such a system, with electric and magnetic field values derived from a 3D MHD simulation of sheet tearing performed by Wyper and Pontin (2014). We present preliminary results of these test-particle simulations at different stages of sheet-tearing, giving an overview of how the acceleration profiles of the protons change over the course of the tearing. We also examine features in the acceleration profiles and trajectories that may warrant further investigation, as well as discuss further code development.

Fully dispersive equations for hydroelastic waves

Emilian Parau - UEA

A weakly nonlinear fully dispersive model equation is derived which describes the propagation of waves in a thin elastic body overlying an incompressible inviscid fluid. The equation is nonlocal in the linear part and is similar to the so-called Whitham equation which was proposed as a model for the description of wave motion at the free surface of an inviscid fluid. Steady solutions of the fully nonlinear hydro-elastic Euler equations are approximated numerically and compared to numerical approximations to steady solutions of the fully dispersive but weakly nonlinear model equation.'

This is joint work with Evgueni Dinvay and Henrik Kalisch, University of Bergen.

Step formation in thin-film diblock-copolymers via a phase-field model with free surfaces

Quentin Parsons - University of Oxford
Andreas Muench
David Kay

Phase separation in diblock-copolymers is an important process in nanotechnology that yields highly regular structures on very fine length scales.

In this talk, we discuss the evolution of two-phase, thin-film diblock-copolymers using a three-phase Ohta-Kawasaki (non-local Cahn-Hilliard) phase-field model, with an obstacle homogeneous energy potential. The free surface is represented as an interface with a third, homopolymer phase for the void.

We develop energy estimates for stationary solutions, based on an appropriate sharp interface limit. These suggest that films with steps have lower free energy than their flat counterparts. We present supporting results of numerical solutions of energy-decaying, time-dependent dynamics of the copolymer-void model, based on a Moreau-Yosida regularisation of the obstacle potential. These provide insight into the evolution of stepped films, and their associated domain patterns.

MCMC using Gaussian Processes for inference in a partial differential equations model of pulmonary circulation

Mihaela Paun - University of Glasgow
Dirk Husmeier
Nicholas Hill
Muhammad Qureshi
Mette Olufsen

This study performs parameter inference in a partial differential equations system of pulmonary circulation. We use a fluid dynamics network model that takes selected parameter values and mimics the behaviour of the pulmonary haemodynamics under normal physiological and pathological conditions. This is of medical interest as it enables tracking the progression of pulmonary hypertension. We use Monte Carlo Markov Chain (MCMC) methods to learn the parameters from observed data (blood flow and pressure) and to quantify the uncertainty. In order to reduce the computational cost associated with repeatedly evaluating the PDEs at every MCMC iteration, we use Gaussian Processes (GPs) to emulate the objective function (the residual-sum-of-squares, RSS), and run MCMC on the emulated landscape of RSS, which does not require the evaluation of PDEs. We compare several methods: History Matching using GPs coupled with Delayed Rejection Adaptive Metropolis algorithm, Adaptive Metropolis using GPs, Hamiltonian Monte Carlo using GPs, and delayed acceptance No U-turn Sampler using GPs. The last method represents methodological novelty, and we show its superior performance over the other methods (in terms of computational time and number of PDE evaluations). Formal convergence diagnostics (e.g. Gelman Rubin diagnostic) are also employed to check for convergence of the Markov chains.

Physical determinants of bacterial biofilm architectures

Philip Pearce - Massachusetts Institute of Technology

In many situations bacteria aggregate to form biofilms: dense, surface-associated, three-dimensional structures populated by cells embedded in matrix. Biofilm architectures are sculpted by mechanical processes including cell growth, cell-cell interactions and external forces. Using single-cell live imaging in combination with simulations we characterise the cell-cell interactions that generate Vibrio cholerae biofilm morphologies. Furthermore, we show how external fluid flow changes the intrinsic local order and the global morphology of growing biofilms. Our results demonstrate the importance of cell dynamics mediated by adhesion proteins and matrix generation in determining the global architecture of biofilm structures.

How to bend a microtubule

Simon Pearce - University of Manchester
Matthias Heil
Oliver Jensen
Gareth Jones
Andreas Prokop

Microtubules are one of the main components of cells and are essential for many biological functions. As the stiffest cytoskeletal polymer, they are generally seen to be very straight over cellular lengthscales. However, in areas of neurodegeneration highly curved microtubules are seen with radius of curvature of a micron. Similarly, curved rings are also sometimes seen in gliding assays, where microtubules are moved over a surface by the motor protein kinesin, amongst other microtubules translocating as rigid rods.

Recent evidence suggests that some microtubule-associated proteins such as kinesin are able to sense and alter microtubule curvature, and so we model microtubules as inextensible rods with a preferred curvature, which is controlled by the differential binding of the kinesin. We find that there exist parameter regimes wherein metastable rings can form, and hence offer this differential binding as an explanation for these highly curved microtubules seen both *in vitro* and *in vivo.*

For certain parameter regimes, the model predicts that both straight and curved microtubules can exist simultaneously as stable steady-states, as has been seen experimentally. Additionally, unsteady solutions are found, where a wave of differential binding propagates down the microtubule as it glides across the surface, which can lead to chaotic motion via a period doubling bifurcation.

I will also briefly mention the use of the compound matrix method (Evans function) for solving eigenvalue boundary-value problems and present a Mathematica package for calculating this.

Penny-shaped hydraulic fracture accounting for shear stress induced by the fluid

Daniel Peck - Aberystwyth University
Michal Wrobel
Gennady Mishuris

The classical 1D penny-shaped model of hydraulic fracturing (HF) is updated to incorporate the effects of shear stress on the crack faces, the importance of which was recently demonstrated for the KGD model. The primary effects of this modification are outlined, namely; the updated asymptotics for the viscosity dominated (zero toughness) regime and the need to implement a modified energy release rate fracture criterion (as opposed to the classical Irwin criterion).

The way in which incorporating the shear stress greatly improves the flexibility and efficiency of the final algorithm, in both the viscosity and toughness dominated regimes, is outlined. A highly efficient

numerical solver is demonstrated, and the level of computational accuracy is verified. The modified HF formulation is then compared with the classic radial model. Quantitative and qualitative analysis of the influence of hydraulic shear stress on the HF process is provided.

A Mathematical Model of Cerebral Vasospasm and Comparison between Treatment Strategies

Giulia Pederzani - University of Sheffield
Andrii Grytsan
Pervinder Bhogal
Anne Robertson
Paul Watton

Cerebral vasospasm (CVS) is an acute constriction of a cerebral artery the pathophysiology of which is still poorly understood. We extend a growth and remodelling framework to simulate the disease and the artery's response to mechanical treatment.

We model the arterial wall as a nonlinear elastic cylindrical membrane consisting of a constrained mixture of elastin, collagen and VSMCs. Constituents are configured within the tissue in a preferred stretch to achieve optimum mechanical response about the homeostatic configuration. We hypothesize that CVS is driven by VSMCs' remodelling about the new configuration and stiffening. Next we extend the membrane model to a thick-walled finite element (FE) model. We start from a framework describing growth and remodeling of fibre-reinforced soft tissue with anisotropic volumetric growth and sophisticate the constitutive model of collagen to incorporate its experimentally observed undulation distribution. The new constitutive model is verified against the analytical solution for a simple geometry and extended to allow remodeling of the constituent.

The model describes the mechanical response of an artery in health, vasospasm and following damage of VSMCs. Based on experimental measurements of the force exerted by stents, we assess whether they provide sufficient force to treat vasospasm. More specifically, assuming that failure of VSMCs occurs at an 80% cell stretch, we compare four stent-retrievers and determine their efficacy in resolving CVS in arteries of different diameters.

The model presented tests a novel hypothesis on the pathophysiology of vasospasm and predicts what magnitude of pressure is sufficient to resolve it. The predictions are consistent with clinical observations and support the use of stent-retrievers in some cases instead of balloon angioplasty, with significant clinical benefits.
The FE anisotropic growth model has been sophisticated to incorporate a representation of collagen undulation. It will next be extended to include VSMCs with active response, which allows broader application to modelling other soft-tissue diseases; examples are discussed.

Homogenized modeling for vascularized poroelastic materials

Raimondo Penta - University of Glasgow
Jose Merodio

A new mathematical model for the macroscopic behavior of a material composed of a poroelastic solid embedding a Newtonian fluid network phase (also referred to as *vascularized poroelastic material*), with fluid transport between them, is derived via asymptotic homogenization. The typical distance between the vessels/channels (*microscale*) is much smaller than the average size of a whole domain (*macroscale*). The homogeneous and isotropic Biot's equation (in the quasi-static case and in absence of volume forces) for the poroelastic phase and the Stokes' problem for the fluid network are coupled through a fluid-structure interaction problem which accounts for fluid transport between the two phases; the latter is driven by the pressure difference between the two compartments. The averaging process results in a new system of partial differential equations that formally reads as a *double poroelastic*, globally mass conserving, model, together with a new constitutive relationship for the whole material which encodes the role of both pore and fluid network pressures. The mathematical model describes the mutual interplay among fluid filling the pores, flow in the network, transport between compartments, and linear elastic deformation of the (potentially compressible) elastic matrix comprising the poroelastic phase. Assuming periodicity at the microscale level, the model is computationally feasible, as it holds on the macroscale only (where the microstructure is smoothed out) and encodes geometrical information on the microvessels in its coefficients, which are to be computed solving classical periodic cell problems. Recently developed *double porosity models* are recovered when deformations of the elastic matrix are neglected. The new model is relevant to a wide range of applications, such as fluid in porous, fractured rocks, blood transport in vascularized, deformable tumours, and interactions across different hierarchical levels of porosity in the bone.

Well-balanced finite volume schemes for hydrodynamic equations with general free energy

Sergio P. Perez - Departments of Chemical Engineering and Mathematics, Imperial College London
José Antonio Carrillo
Serafim Kalliadasis
Chi-Wang Shu

The construction of robust well-balanced numerical methods able to discretely preserve steady states of balance laws has attracted considerable attention since the initial works of LeRoux and collaborators. The so-called fractional-step methods have been widely employed to simulate the temporal evolution of balance laws but fail to accurately resolve the steady states of the corresponding systems, in which the fluxes need to be exactly balanced with the source terms. To correct this deficiency, well-balanced schemes are designed to discretely satisfy the balance between fluxes and sources when a steady state is reached. The strategy to construct such schemes relies on the fact that, when the steady state is reached, the variation of the free energy with respect to the density reaches a constant minimum in the support of the density. This allows the resolution of the fluxes and sources in the same level, i.e., not considering them separately as the fractional-step methods do, but rather taking into account that their common origin is the variation of the free energy with respect to the density.

Several well-balanced schemes have been proposed for particular choices of the free energy, leading to applications such as shallow water equations or cell chemotaxis. Here we present a well-balanced finite volume scheme for a general choice of free energy, which could contain different dependencies with respect to the density and external or interaction potentials. Our first- and second-order schemes preserve the steady states and the nonnegativity of the density, are consistent with the original system and satisfy a cell entropy inequality.

Mesoscopic description of a morphogen-controlled bistable switch

Ruben Perez-Carrasco - UCL
Karen Page
Pilar Guerrero
James Briscoe

When there are many molecules in a confined space, models using differential equations to describe concentrations work well. When there are very few molecules, simulations of individual molecules and their interactions are necessary. Mathematically the probabilities of having certain numbers of molecules can be described by the chemical master equation. There is an intermediate regime. When the numbers of molecules are small enough that stochastic effects matter but large enough that the rates of reactions do not change much on the timescale of the reactions themselves, the numbers of molecules can be modelled by Langevin equations, which are stochastic differential equations. Here we investigate the differences between this mesoscopic description and the macroscopic (differential equation) description in a model of a morphogen-controlled bistable switch.

Modelling transient properties of cortex formation highlights the importance of evolving cell division strategies

Noemi Picco - University of Oxford

The brain is the most complicated organ of any animal, formed and sculpted over 500 million years of evolution. The cerebral cortex is the folded grey matter that forms the outside of the brain and is the seat of higher cognitive function.

Many factors influence how neurogenesis in the cortex differs between species, including the types of neurons and neural progenitor cells, the different ways in which they proliferate and differentiate, and the length of the process. Critically, to fully understand the development of the cortex we are faced with the challenge of understanding the temporal changes in cell division strategy. Combining mathematical modelling and experimental observations we incorporate these different factors to model development and evolution of the mammalian cortex.

A key determinant of the neuronal production is the modulation of proliferative (self-amplifying) and differentiative (neurogenic) divisions. We propose a new ordinary differential equation model that incorporates our hypothesised temporal changes in the propensity of different cell division types. By analysing this model, we identify a developmental programme that is consistent with the temporal pattern of neuronal output in the cortex of different species. Additionally, we highlight the current limitations in the interpretation of model predictions, due to the limited data currently available and identify a specific need for experimental quantifications.

This work was carried out in collaboration with neuroscientists (Dr. Fernando García-Moreno, Achucarro Basque Center for Neuroscience in Bilbao; Professor Zoltán Molnár, University of Oxford) and mathematicians (Prof. Philip Maini, University of Oxford; Dr Thomas Woolley, University of Cardiff), and it is funded by the St John's College Research Centre, Oxford.

Diffusive magnetic energy changes direction of the wave propagation with shock normal in earth magnetic ramp region co-related angle between shock normal and upstream magnetic field

Jivraj Pipaliya - Sheffield University

One of the key aspects of the in-situ measurements of collision less shocks is to detect magnetic energy variation at shock front earth magnetic ramp region at sunward earth radial distance 10.05RE. earth magnetopause Diffusive 3D magnetic energy changes direction of the wave propagation with shock normal in earth magnetic ramp transition region co-related to angle variation between shock normal and upstream magnetic field

Robustness of the microtubule cytoskeleton self-organisation

Aleksandra Plochocka - Heriot-Watt, Edinburgh University
Natalia Bulgakova
Lyubov Chumakova
Sandy Davie

You and I are but a collection of trillions of cells. Why are we not falling apart? Consider this question on an inter-cellular level: to hold the cells together, the stickiness protein (E-cadherin) has to be delivered to the cell boundaries, where it is biologically relevant. This delivery is done along a network of highways (the microtubule cytoskeleton) via stochastically moving molecular motors. The cytoskeleton network self-organises to adjust to the functions of the cell and its developmental stage. Recently it was discovered that in epithelial cells cytoskeleton self-organisation is governed primarily by cell geometry. In this talk, I will present our recent work. Using a probabilistic toy model and stochastic simulations, we show that cytoskeleton self-organisation is independent of most of the biological parameters. This means that the following result is robust: it is the cell geometry that determines our tissue properties.

A quantitative framework for understanding cancer cell invasion through in vitro scratch assays

Ana Victoria Ponce Bobadilla - Heidelberg University
Thomas Carraro
Tomás Alarcón
Helen Byrne
Philip Maini

Scratch assays are standard in vitro experimental methods for studying cell migration. In these experiments, a scratch is made on a cell monolayer and imaging of the recolonisation of the scratched region is performed to quantify cell migration rates. This experimental technique is commonly used in the pharmaceutical industry to identify new compounds that may promote cell migration in wound healing; and to evaluate the efficacy of potential drugs that inhibit cancer invasion. Given the key role this method plays in assessing the potential of new compounds for clinical use, it is important to develop robust quantification frameworks that accurately describe the movement of the front of migrating cells. This then allows validation of mathematical models that can be used to test hypotheses about the physical mechanisms that govern cell migration.

In this talk, I will introduce a new migration quantification method that fits experimental data more closely, provides a more accurate statistical classification of the migration rate between different assays, and is able to analyse experimental data of lower quality than existing classical quantification methods. The method's robustness is validated using in vitro and in silico data.

Modelling Surge in Centrifugal Compressors

Kate Powers - University of Bath

Nearly all modern car turbochargers are formed of a centrifugal compressor and turbine. Understanding how turbochargers act under various flow regimes is important for many reasons, including improving vehicle fuel efficiency.

Surge is an unstable phenomenon that manifests itself in regions of low mass flow but high-pressure ratio. It results in pressure and mass flow fluctuations that severely reduce the efficiency of, and often cause damage to, the compressor. Current compressor modelling is predominantly done via CFD, which is computationally expensive and can be inaccurate near surge.

In this talk I will describe an analytical method for modelling compressors. Using the Euler equations, I will show how to construct a pressure/mass flow characteristic for a compressor from first principles. From there, I will explain how to couple this to a throttle in order to simulate surge-like phenomena.

Convergent Chaos

Marc Pradas - The Open University
Michael Wilkinson
Alain Pumir
Greg Huber

Chaos is widely understood as being a consequence of sensitive dependence upon initial conditions. This is the result of an instability in phase space, which separates trajectories exponentially. Here, we investigate the dynamics of a chaotic one-dimensional model for inertial particles in a random velocity field and demonstrate that despite their overall intrinsic instability, trajectories may be very strongly convergent in phase space over extremely long periods. We establish that this strong convergence is a multi-facetted phenomenon, in which the clustering is intense, widespread and balanced by lacunarity of other regions. Power laws, indicative of scale-free features, characterize the distribution of particles in the system. We use large-deviation theory and extreme-value statistics to explain this effect, and in particular, we develop the large-deviation theory for fluctuations of the finite-time Lyapunov exponent of this system. We show that the determination of the entropy function for the distribution reduces to the analysis of a Schrödinger equation, which is tackled by semi-classical methods. The system has 'generic' instability properties, and we consider the broader implications of our observation of long-term stability in chaotic systems. Our results show that the interpretation of the 'butterfly effect' needs to be carefully qualified.

Multiscale modelling and analysis of auxin transport in plant tissues

Mariya Ptashnyk - Heriot-Watt University
Alessia Sala

Auxins belong to the most important plant hormones and play a central role in growth and development regulation. Auxin is mostly produced in the plant shoot and is transported from cell to cell through the shoot and stem towards the primary root. In this talk we will consider two continuous microscopic models on the level of a single cell for transport of auxin in a plant tissue, by taking into account microscopic properties of the tissue and the heterogeneity of the transport process. Formal asymptotic expansion and two-scale convergence method are applied to derive macroscopic equations. To determine the effective macroscopic diffusion coefficients and transport velocities the corresponding unit cell problems are computed. The calculated transport velocity is shown to be of the same order as measured values. Numerical solutions for the macroscopic models are considered to demonstrate the dynamics in the transport and distribution of auxin in a plant tissue.

Stochastic homogenization for a chemotaxis system

Mariya Ptashnyk - Heriot-Watt University
Anastasios Matzavinos

An interesting and important question is how the heterogeneity of the environment influence the movement of cells and organisms. In this talk we consider multiscale analysis of the Keller-Segel chemotaxis system defined in a random heterogeneous domain in one and two dimensions. To model the random heterogeneity of the environment we assume that the diffusion and the chemotactic sensitivity coefficients are given by stationary ergodic random fields. We apply stochastic two-scale convergence methods to derive the homogenized macroscopic equations. The derivation of a priori estimates, which differs from classical results due to highly-oscillating diffusion and chemotaxis coefficients, constitutes an important part of the analysis.

Coronal energy release by MHD avalanches: continuous driving

Jack Reid - University of St Andrews
Alan Hood
Clare Parnell
Philippa Browning
Peter Cargill

One possible mechanism for heating and maintaining the high temperature of the solar corona is through a large number of "nanoflares", small-scale releases of magnetic energy by reconnection. We discuss how a small number of steady vortical motions lead to a dynamic instability, releasing a large amount of energy, followed by a series of smaller energy releases. Using three-dimensional MHD simulations, we demonstrate the creation and stressing of three threads within a coronal loop. The instability of the central thread excites and enhances the subsequent disruption of others, in a manner similar to an avalanche, enabling strong and repeated energetic releases. Thus, we can generate a sporadic heating profile, as required by observations.

Binary quasi-geostrophic vortex interactions in a shear induced by a surface buoyancy filament

Jean Reinaud - University of St Andrews

We study the interaction between two co-rotating quasi-geostrophic uniform potential vorticity deep vortices in the vicinity of a surface buoyancy anomaly filament in a three dimensional, stratified and rotating fluid. The surface buoyancy distribution generates a shear flow by locally modifying the pressure field which affects the interaction of the vortex pair. We first shown numerically that a uniform horizontal cooperative shear favours the merger of the vortices while an adverse shear impedes it. High values of adverse shear may in fact separate the pair of vortices by pushing them away from each other. The trend is explained by a simple Hamiltonian point vortex model. We next focus on the binary vortex interaction close to a surface buoyancy filament explicitly. The filament not only generates a shear flow, but it also destabilises and forms buoyancy billows at the surface due to the forcing imposed by the deep vortex pair. These billows interact with the deep vortices. In particular, a surface billow may pair with one of the internal vortices as either a heton-like structure for a filament in adverse shear with the vortices or an aligned co-rotating pair (anti-heton) for a cooperative shear. This provides a second mechanism for the separation of the deep vortex pair.

Bioconvection in a horizontally oriented cylinder rotating about its axis

Scott Richardson - University of Glasgow
Nick Hill
Andrew Baggaley

Despite their diminutive size, micro-organisms constitute the majority of terrestrial life in terms of both numbers and biomass but generally only become visible to the naked eye after aggregating into large clusters. The relevance of these aggregations is fascinating, both as an exercise in understanding the unknown, and as key applications include recent studies considering the use of such cells as a key constituent in fuel generation within bioreactors. These offer an increasingly viable and potentially effective alternative to fossil fuels where costs soar and supplies continue to dwindle. One such example of a process capable of producing these aggregations is bioconvection.

This biotic phenomenon is driven by a combination of individual reactions to ecological factors and correlating hydrodynamic responses resulting in the spontaneous development of patterns. As gravity imparts a substantial influence on concentrated suspensions, this presents a key example of group dynamics engendering feedback which would be impossible to achieve were individual elements not to aggregate.

In this computational exploration we explore this process for cells in a horizontally oriented chamber that rotates about its axis generating further variations within individual trajectories and augmenting the distribution of cells as a whole. So as to not rely solely on visual observations, differences within the patterns formed are quantified through exposing the cells to an overhead external light source, and thereafter establishing the level of light penetration, related to any shading caused by cell agglomerations. A range of angular velocities are considered in combination with a set of well established parameters which describe cell behaviour.

This computationally challenging task employs a range of different, case specific, computational techniques including an adaptive mesh refinement in the radial component and an implicit enforcement of a no-flux boundary condition for cells.

After allowing a suitable transient time the external light source is introduced where it was found that different angular velocities engendered significant variations. Specifically, it was observed that a lengthy period began to generate some distinct behaviour, but in the main results corresponded to those observed in the stationary case, rotation rates at the opposite extreme brought about the Taylor--Proudmann effect, which has previously reported in relation to the velocity field. However, for a range of angular velocities between these extreme rates, a significantly more beneficial cell pattern resulted with up to 100% more cells receiving what was defined a notable level of exposure.

Optimal Control and Estimation for Long-range AUV missions

Eric Rogers - University of Southampton

Acoustic assistance under ice is limited by issues such aslimited vessel manoeuvrability or inability to enter regions of thick ice. In such cases, an AUV it totally dependent its sensors and estimation algorithms.

One way forward in unaided navigation with a bound on the position error growth is to use geophysical information, such as Bathymetric and magnetic field reference maps. This contribution will cover recent research on terrain-aided navigation (TAN) in this area combined with estimation and optimal control algorithms.

Waning and boosting of immunity - challenges in modeling, analysis and numerics.

Gergely Röst - University of Oxford, WCMB
Maria Barbarossa
Mónika van Leeuwen-Polner

A challenging problem in mathematical immuno-epidemiology is to incorporate the within host processes of waning of immunity and immune system boosting (upon repeated exposure to the same pathogen) into population level disease dynamics. Since the frequency of boosting events depends on the density of infected individuals in the population, there are reciprocal feedbacks between the individual and the population levels.

We construct a dynamical model expressed by a structured PDE and show that for particular boosting mechanisms it can be reduced to delay differential equations. In the DDE formulation, analytical and numerical stability analysis is possible, and we can identify stability switches of the endemic steady state due to boosting, generating a complicated bifurcation diagram.

We systematically investigate (both in PDE and DDE settings) the temporal evolution of immunity distributions in the population under different assumptions on the biology of the boosting mechanism, connecting these processes to the qualitative nature of periodic disease outbreaks.

The Approximation of Generalized Log-Aesthetic Curves with Cubic Trigonometric Bezier Functions

Gobithaasan Rudrusamy - School of Informatics & Applied Mathematics
Diya' J. Albayari
Kenjiro T Miura

Curves with monotonic curvature profiles are coined as fair curves and it's essential for aesthetic product design. However, the de facto flexible curves available in CAD systems are Bézier, B-Spline and NURBS which has complex curvature function, thus the practitioners need to go through fairing procedure during design process. In 2011, Gobithaasan & Miura introduced Generalized Log-Aesthetic curve (GLAC) and it comprises of a family of aesthetic curves which possesses monotonic curvature profile. Even though it can be directly used for design feats without fairing process, but it is in the form of transcendental thus numerical method is necessary for GLAC rendering. Recently, Trigonometric Bezier (T- Bézier) has been introduced by Han (2009). The advantage of T-Bézier over traditional Bézier curves are twofold; firstly, it can precisely represents circular arcs, cylinders, cones, tori etc., thus suitable for CAD implementation. Secondly, T-Bézier curves located closer to control polygon as compared to traditional Bézier curves which is of use during designing process. In this paper, we propose an algorithm on employing cubic T- Bézier curves with two shape parameters to approximate GLAC with G^2 continuity by means of curvature error measure. The approximation formula inherits the shape parameters of GLAC which can be used for freeform design, whereas T- Béziers' shape parameters are utilized to satisfy terminal G^2 constraints. The final approximated GLAC is represented in the form of trigonometric basis functions, thus enabling us to preserve essential freeform curve design entities, e.g., convex hull property, partition of unity, symmetric, non-negativity and monotonicity properties. Numerical results indicate that the proposed algorithm capable of approximating GLAC within given tolerance in two iterations, indicating straightforward implementation for CAD systems.

Computational challenges in fluctuating hydrodynamics

Antonio Russo - Imperial College London
Miguel A. Duran-Oivencia
Sergio P. Perez
Peter Yatsyshin
José A. Carrillo
Serafim Kalliadasis

The description of soft matter systems out of equilibrium requires the inclusion of fluctuations in the standard hydrodynamic equations for the average evolution of conserved quantities, such as density or linear momentum. The associated general framework was postulated phenomenologically by Landau et. al., yielding what is known as Landau-Lifshitz fluctuating hydrodynamics (FH). This formalism has attracted considerable attention from the theoretical front, where a rigorous proof of the FH equations from the Hamiltonian microscopic description has only been developed recently. However, the numerical applicability of the fluctuating hydrodynamics entails several challenges which still remain elusive. In particular, conservative fluctuations, i.e. stochastic fluxes under the gradient operator, need to be properly accounted for. Besides, even for the simplest limit of these equations which corresponds to the stochastic diffusion equation, the presence of a normally-distributed flux in the time-evolution equation for the density involves non-positive solutions, which are clearly unphysical. To address this issue, recent works have proposed empirical limiters which artificially modified the natural fluctuations of the system to preserve positivity. However, such limiters do not guarantee mass conservation. This leads to a spurious contribution that might be negligible in themodynamic-equilibrium conditions. Nevertheless, in more general scenarios, e.g. the evolution of a thermodynamically-metastable system, the spurious contribution coming from the artificial limiter will lead to deviations from the actual solution. Hence the need for a robust method capable of handling stochastic fluctuations properly. In this work we present a positive-preserving finite-volume numerical approximation, based on an upwind discretisation of the fluxes. This is a generalisation of a recently proposed finite-volume method for nonlinear nonlocal equations with a gradient flow structure. To show the robustness of our method, we apply it to stochastic gradient flows with nonlinear energy functionals in both 1D and 2D. We highlight the applicability and versatility of our method for the solution of FH by considering a wide spectrum of physical settings. For instance, isothermal systems in the overdamped limit, in which case FH reduce to the fluctuating dynamic density-functional theory. A key point here is to replace the nonlinear energy functionals with the Helmholtz free-energy functional from classical density-functional theory.

The Influence of the Thermal Properties of the System on the Evolution of an Evaporating Droplet

Feargus Schofield - University of Strathclyde
Feargus Schofield
Stephen Wilson
David Pritchard

The evaporation of a sessile liquid droplet on a solid substrate is a fundamental problem in fluid mechanics linked to many industrial processes, such as ink-jet printing, coating, and spray cooling. As such, in recent years there has been fast growing scientific interest in all aspects of droplet evaporation, including determining the exact lifetime of an evaporating droplet.

There are many factors that can contribute to the evolution of a droplet. A key factor is the mode of evaporation, i.e. the manner in which the geometry of the droplet changes during the evaporative process. There are a wide variety of modes of evaporation, including a constant contact radius (CR) mode (for which the contact radius of the droplet is fixed and the contact angle decreases in time), a constant contact angle (CA) mode (for which the contact angle of the droplet is fixed and the contact radius decreases in time), a stick-slide (SS) mode (for which the droplet initially evaporates in a CR mode for part of its life, and then switches to a CA mode for the remainder of its life), and a stick-jump (SJ) mode (for which the droplet evaporates in a series of CR phases, separated by jump phases in which the contact angle and contact radius jumps instantaneously). Other key factors in determining the evolution of a droplet are the thermal properties of the liquid droplet and of the solid substrate. In particular, the relationship between the thermal conductivities of the droplet and of the substrate can have a strong influence on the process of droplet evaporation.

In the present work we investigate the combined influences of the thermal properties of the system and the mode of evaporation on the droplet evaporation in order to predict the evolution, and hence the lifetime, of the droplet. In particular, we investigate a droplet evaporating on a substrate with a relatively low thermal conductivity compared to that of the fluid, such as water on plastic, in the CR, CA, SS and SJ modes of evaporation.

Magnetic Feature Detection through Coronal Volume Segmentation along Quasi-separatrix Layers

Roger Scott - University of Dundee
David Pontin

Interchange reconnection is thought to play an important role in driving the dynamics of the slow solar wind. In order to understand the details of this process, it is important to catalog the various magnetic structures that are present at the open-closed flux boundary. To this end we have developed a method for identifying discrete flux domains using watershed segmentation along quasi-separatrix layers. In this way we are able to identify structures within the corona that affect the character of the open-closed boundary. By analyzing an assortment of global field models we can then build up a catalogue of common configurations and create a hierarchy for evaluating each one's relevance to the process of interchange reconnection. This, in turn, informs our understanding of which configurations should be given priority for upcoming numerical experiments, and how the results of these experiments can be brought together to form a more complete picture of the global evolution of the slow solar wind.

A Bifurcation Analysis of Spiral Waves using a FitzHugh-Nagumo Model

Shreya Sehgal - Liverpool Hope University
Andrew Foulkes

Spiral waves are spatio-temporal solutions to reaction-diffusion system of equations which have been observed in a wide variety of biological, chemical and physical systems, both artificial and in nature. There are several types of motion of spiral waves. In the simplest case, a free spiral rotates rigidly while its tip describes a circular trajectory. It was also observed that under certain conditions a spiral tip meanders rather than following a periodic circular orbit. Meandering is often not a random motion, rather the spiral tip traces a path resembling an epicycloid, exhibiting flower-like patterns. It is a type of quasiperiodic motion. It has been shown numerically by other authors that a supercritical Hopf bifurcation is responsible for the transition from rigid rotation to meander. However, these studies were limited to small core spirals.

In this project, we calculate the dynamics of the spiral wave in a co-moving frame of reference where the core of the spiral never approaches the boundaries of the computation box and it allows the computations of large core spirals using small numerical grids. This leads to a system of reaction-diffusion-advection equations in which the tip of the spiral wave is fixed in position and orientation. We conducted the bifurcation analysis of spiral waves by studying the underlying limit cycles of meandering spiral waves using the FitzHugh-Nagumo system of equations. Results show that indeed a Hopf bifurcation is responsible for the transition from rigid rotation to meander.

Maths by the back door - engaging new audiences with cross-discipline activities

Madeleine Shepherd - International Centre for Mathematical Sciences

Mathematics is notorious for turning off the public. Even those who are eager participants in science-based public engagement can exhibit maths-phobic tendencies. By finding and making a connection between individual audience member's context-based expertise and the underlying mathematics we can create a conversation that reaches past this barrier. In this talk I will discuss some activities and approaches to beginning these dialogues.

For example, at the International Centre for Mathematical Science we have presented film festivals, a mathematical art exhibition and local history walks and participated in architectural open days. As a textile artist I have embedded mathematics in craft work, art installations and community participation projects. Further afield, projects such as MathsCraft events in Australia and New Zealand and the International Hyperbolic Crochet Coral Reef are testament to the success of such approaches.

Ignition of waves in excitable systems

Radostin Simitev - University of Glasgow
Burhan Bezekci
Vadim Biktashev
Ibrahim Idris

Excitable and bistable systems are abundant in nature and technology, including nerve and cardiac electrical excitability, combustion and chemical reaction waves, self-heating in metals and superconductors, phase transitions, domain wall movement in liquid crystals to name a few. It is important not only to know that a particular system can support a non-decaying propagating wave, but also to know the specific initial conditions that can trigger it. However, the threshold of excitation, so readily apparent in local excitable dynamics, becomes rather complicated to find in the spatially extended context. We propose a semianalytical approach to establishing criteria for ignition/initiation of travelling waves in mathematical models of excitable media (Bezekci, Idris, Simitev, Biktashev (2015) Phys. Rev. E, 92:042917). Our approach is based on the idea of approximating the boundary between the basin of attraction of propagating waves (excited state) and of the basin of attraction of the resting state (non-excited state) as the stable manifold of a certain critical solution. In this talk we illustrate the methodology on a relatively simple example - ignition of a travelling front governed by the Zeldovich-Frank-Kamenetsky equation.

Sleep/wake inspired insights on the creation of gaps and non-monotonicity in circle maps

Anne Skeldon - University of Surrey
Gianne Derks
Paul Glendinning

Early mathematical models for cardiac arrhythmias, sleep-wake regulation and integrate and fire neuronal models all have a “threshold” structure, where a variable of interest increases until it hits an upper threshold and then decreases or is reset to a lower threshold value. Often the thresholds are periodically modulated, leading to a description as a circle map, and typical solutions that result in resonant phase-locked `tongue’-shaped regions (Arnold tongues).

Inspired by models of sleep-wake regulation, we show that threshold models can result in circle maps that are continuous or discontinuous, monotonic or non-monotonic. Understanding the transitions between different types of map provides a neat link between results in the literature in different areas and suggests some interesting new codimension two bifurcations.

We explain how the transitions between monotonicity and non-monotonicity and between gaps and no gaps occur and give a flavour of the many interesting possibilities that result. For example, unlike much existing work on maps with gaps, the generic form for discontinuities is for one side of the gap to have a square root singularity. Analysing the transition from no gap to gap enables one to understand how the Arnold tongue picture of periodic solutions that are created/destroyed by saddle-node bifurcations, as in continuous circle maps, relates to the creation/destruction of periodic solutions via border collision bifurcations, as seen in gap maps. Non-monotonicity leads to the presence of multiple gaps and novel codimension two bifurcations from which, like the “big-bang” bifurcations seen in other contexts, an infinite number of bifurcation curves emerge.

In the specific context of sleep-wake regulation, the bifurcation structures of mathematical models will be related to observed behaviour such as “internal desynchrony” where the sleep wake cycle desynchronises from the circadian rhythm (body clock), polyphasic sleeping patterns (more than one sleep a day) and circabidian patterns (patterns that repeat every two days).

Predicting patterns of retinal haemorrhage

Tamsin Spelman - University of Glasgow
Peter Stewart

Retinal haemorrhage (bleeding of the blood vessels in the retina) is often observed following a traumatic brain injury. However, it can be difficult to correlate the severity of the trauma to the observed bleeding. The central retinal artery and corresponding vein supply the retinal circulation of the eye. They penetrate the eye via the optic nerve, which they enter a few centimetres back from the eyeball, immediately after passing through a region filled with cerebrospinal fluid (CSF). The CSF also surrounds the brain, so the CSF pressure will be directly influenced by pressure changes in the brain. Using a multi-compartment model, we examine how the sudden change in CSF pressure drives a pulse along the blood vessels towards the retinal circulation; in some cases, the accompanying pressure wave can steepen to form a shock. We demonstrate that the strong confinement by the optic nerve on the section of blood vessel contained within it, significantly reduces the amplitude of the pressure wave transmitted into the eye, effectively protecting the eye from damage.

The Many Faces of Mathematical Engagement

Katie Steckles - Freelance

Public engagement with mathematics can take many forms. In order to give an idea of what kinds of mathematical engagement are out there, I'll list some of the many projects I've worked on within maths engagement, describe how each form of engagement differs and discuss what use may be made of each in popularising mathematics to the public.

Optic nerve sheath bleeding driven by rapid cerebrospinal fluid pressure amplification

Peter Stewart - University of Glasgow
Oliver Jensen
Richard Bonshek

The optic nerve is a dense collection of dendrite fibres which connect the eye to the brain. This nerve is surrounded by a sheath which contains a thin layer of cerebrospinal fluid (CSF); this CSF also surrounds the brain. Hence, the CSF in the optic nerve sheath is directly influenced by pressure changes in the brain. Bleeding of the optic nerve sheath (subdural haemorrhage) is frequently observed in conjunction with traumatic brain injury. We propose a mathematical model to explain this correlation, examining the flow of CSF along the optic nerve sheath driven by a sudden large pressure increase in the brain. We predict the subsequent temporal and spatial evolution of the CSF flow and pressure, as well as the corresponding optic nerve sheath thickness, using a spatially one-dimensional model. The model predicts that the pressure perturbation triggers a propagating pressure wave along the nerve towards the eye; for sufficiently large perturbations this wave can steepen to form an elastic jump (shock wave). Since the optic nerve sheath is closed-ended where it meets the eye, we show that this pressure wave is then reflected back towards the brain. This wave reflection results in a significant amplification of the CSF pressure in a region close to the eye, in some cases almost ten times larger than the corresponding CSF pressure at the inlet to the nerve. We hypothesise that this dramatic increase in CSF pressure, and accompanying rapid expansion of the sheath, can lead to rupture of blood vessels spanning across the optic nerve and subsequent bleeding close to the eye.

Finite element approximation of geometric PDEs coupled with surface PDEs

Bjorn Stinner - University of Warwick

Several phenomena in biophysics such as phase separation on biomembranes or cell motility can be modelled by geometric equations coupled to surface fields that are subject to PDEs on the thus described moving surface. We report on finite element schemes of such coupled systems and present some recent analytical findings in the one-dimensional case of moving curves.

Evolution of statistically inhomogeneous degenerate water wave quartets

Raphael Stuhlmeier - University of Plymouth
Michael Stiassnie

Nonlinear interaction, along with wind input and dissipation, is one of the three mechanisms which drive wave evolution, and is included in every modern wave–forecast model. The mechanism behind the nonlinear interaction terms in such models is based on the kinetic equation for wave spectra derived by Hasselmann. This does not allow, for example, for statistically inhomogeneous wave fields, nor for the modulational instability which depends on such inhomogeneity, and which has been implicated in the appearance of exceptionally high rogue waves.

Beginning with the basics of third-order wave theory, we sketch the derivation of a discretized equation for the evolution of random, inhomogeneous surface wave fields on deep water from Zakharov's equation, along lines first laid out by Crawford, Saffman, and Yuen. This allows for a general treatment of the stability and long-time behaviour of broad-banded sea states. It is investigated for the simple case of degenerate four-wave interaction, and the instability of statistically homogeneous states to small inhomogeneous disturbances is demonstrated. Furthermore, the long-time evolution is studied for several cases and shown to lead to a complex spatio-temporal energy distribution. The possible impact of this evolution on the statistics of rogue wave occurrence is explored within the framework of this simplified example.

Ionic Diffusion Model for the Oxidation of Uranium

Monisha Natchiar Subbiah Renganathan - The University of Manchester

Uranium reacts with oxygen in the air to form uranium oxide and/or uranium hydride, with the latter being formed only in the presence of moisture. The corrosion product uranium hydride, formed from the reaction of uranium with water vapour is undesirable as it is pyrophoric (i.e. self-ignites). The kinetics of uranium oxidation in dry air to form uranium oxide is dominated by the diffusion of oxygen anions, formed when the adsorbed oxygen atoms on the surface accept electrons from the underlying metal. There are empirical models available in the literature for the oxidation of uranium in dry and moist air, but very few mathematical models exist. A notable exception is that the diffusion of the oxygen anions through the uranium oxide layer has recently been studied by Gharagozloo et al., but only in the quasi-steady limit.

We formulate and solve a one-dimensional mathematical model, with diffusion as the rate-limiting step in the fast reaction limit, leading to a nonlinear unsteady moving boundary problem for the depth of the oxidised layer. A boundary-fixing transformation is used to solve the governing equation in a fixed domain, combined with Newton iteration and a second-order finite difference scheme in time and space. The overall process is one of combined temperature and time-dependent chemical and ionic diffusion within a growing substrate of constant composition (in the case of dry air) or variable composition (in the presence of moisture). The diffusion process also depends on the atmospheric concentration of the diffusing species, bulk temperature and the electric potential set up across the oxide layer. The resulting problem is of Stefan type, in which the density difference associated with the phase change remains important. We will briefly highlight the differences in the moist air problem, which results in an additional interface associated with hydride production.

Steady Streaming as a Method of Drug Delivery to the Inner Ear

Laura Sumner - Imperial College London
Tobias Reichenbach

The human ear converts pressure waves into electrical signals which are then relayed to the brain. This mechanotransduction occurs in the Organ of Corti situated on the basilar membrane inside the spiral-shaped temporal bone. The motion of the basilar membrane due to sound causes hair cells inside the Organ of Corti to open ion channel gates and trigger action potentials in attached auditory-nerve fibers. The hair cells can be damaged due to noise exposure or age, however. Mammalian hair cells cannot be regenerated, leading to sensorineural hearing loss. Sensorineural hearing loss can potentially be prevented or treated through drugs, but delivering the drugs to the hair cells remains one of the important problems. The inner ear is encased in the body's hardest bone, and drugs can only be injected through the round window or oval window at its base (Salt and Plontke 2009). Here we seek to investigate if steady streaming may be employed to distribute drugs from the base of the cochlea across its longitudinal extent. Steady streaming is indeed present in fluid systems with a fluctuating flow and results in a non-zero mean flow (Riley 2001). In the cochlea, the fluctuating flow results from the motion of the basilar membrane. This membrane has a spatially-varying impedance which allows to spatially segregate frequencies (Reichenbach and Hudspeth 2014). At a particular frequency the basilar membrane responds maximally at a frequency-specific location. Distributing drugs through steady streaming may thus employ a series of sounds of different frequencies to transport drugs from the high- to the low-frequency region.

Method: We model the basilar-membrane motion at a particular frequency through the WKB approximation. We then use this motion as input to a computational fluid-dynamics (CFD) simulation that we implement using OpenFoam. The simulation uses a dynamic mesh solver combined with a particle tracking solver to enable the coordinates of individual particles across the entire domain to be determined and post processed to find the steady streaming velocities. These results are compared to the acoustic streaming theory developed by Lighthill in order to validate his results in the context of the cochlea.

Results: Results for the longitudinal streaming between 5kHz and 20kHz show a reasonable agreement up to a scale factor with the theoretically proposed values. These values are also large enough such that therapies which utilise this effect are feasible (a time period of a few hours would be required). The boundary layer thickness is also an almost exact match to theory, meaning validation of the simulation.

Conclusions: We show that the simulation predicts considerable steady streaming over timescales reasonable for therapies, and plan to use it to investigate different types of sound stimulation for efficient drug transport.

Numerical Modelling of Stealth Solar Eruptions; Simulated and In-situ Signatures at 1AU

Dana-Camelia Talpeanu - KU Leuven, Royal Observatory of Belgium
Francesco P. Zuccarello
Emmanuel Chané
Stefaan Poedts
Elke D'Huys
Skralan Hosteaux

Coronal Mass Ejections (CMEs) are huge expulsions of magnetized plasma from the Sun into the interplanetary medium. A particular class of CMEs are the so-called stealth CMEs, i.e., solar eruptions that are clearly distinguished in coronagraph observations, but that are not associated with clear signatures close to the Sun, such as solar flares, coronal dimmings, EUV waves, or post-flare loop arcades.

Observational studies show that quite often (about 60%) stealth CMEs are preceded by another CME whose solar origin could be identified.

In order to determine the triggering mechanism for stealth CMEs we are using the MPI-AMRVAC code developed at KU Leuven. We simulate consecutive CMEs ejected from the southernmost part of an initial configuration constituted by three magnetic arcades embedded in a globally bipolar magnetic field. The first eruption is driven through shearing motions at the solar surface and the second is a stealth CME, both being expelled into a bimodal solar wind. We analyze the parameters that contribute to the occurrence of the second CME, which will lead to a better understanding of its triggering mechanism and improve the forecasting of the geomagnetic impact of stealth eruptions. We also compare the simulated signatures of the two CMEs with the in-situ data from ACE spacecraft at 1AU.

Connecting within-host and population-level models of HIV: untangling the mechanisms underlying transmission using data from both scales

Robin Thompson - University of Oxford
Chris Wymant
Rebecca Spriggs
Jayna Raghwani
Christophe Fraser
Katrina Lythgoe

Understanding HIV transmission between hosts is an ongoing challenge, with implications for vaccine design. A new HIV infection can either be initiated by a single viral variant or by multiple variants. Infections can also be founded with different numbers of viral particles, potentially including both cell-free virus and infected cells. However, it is unclear how the number of particles founding an infection relates to the number of variants. Naïvely it might be expected that more viral particles being transmitted is likely to lead to more variants being transmitted. However, this ignores the complex dynamics of HIV within infected hosts. In particular, donors in early infection typically have high viral loads made up of very limited viral diversity. In later infection, viral loads are lower, but the virus is composed of more variants. As a result, infections when donors are in early infection are often made up of large numbers of particles, yet very few variants. To demonstrate this effect, and uncover the mechanisms underlying HIV transmission, we characterised the spread of infection around an infected host using a simple model. This was then nested in a probabilistic population-level model describing the transmission of HIV between hosts. We parameterised our multi-scale within-host/between-host model using both within-host next generation sequence data and population-level data collected from a population of heterosexual individuals. Our model recapitulates features of transmission observed in real populations and allows us to test hypotheses about HIV transmission in different host populations.

Motion of non-axisymmetric particles in viscous shear flow.

Ian Thorp - University of Cambridge
John Lister

We investigate the dynamics of a non-axisymmetric particle immersed in an unbounded Stokes shear flow. Depending on the particle's initial orientation, the rotation is found to be (quasi)periodic or chaotic. We aim to elucidate the mathematical underpinnings of this motion, making use of the symmetries of the equations of motion and reversible KAM theory. Furthermore, particles with fewer than three planes of symmetry are sometimes observed to drift steadily across the streamlines in the direction of the gradient and/or vorticity of the shear flow. We will discuss the origins of this drift in terms of the symmetry groups of the orbit of the particle's orientation.

Flux Rope Formation Due to Shearing and Zipper Reconnection

James Threlfall - University of St Andrews
Alan Hood
Eric Priest

Zipper reconnection has been proposed as a mechanism for creating most of the twist in the flux tubes that are present in eruptive flares and coronal mass ejections. We have conducted a first numerical experiment on this new regime of reconnection, where two initially untwisted parallel flux tubes are sheared and reconnected to form a large flux rope. We describe the properties of this experiment, including the linkage of magnetic flux between concentrated flux sources at the base of the simulation, the twist of the newly formed flux rope and the conversion of mutual magnetic helicity in the sheared pre-reconnection state into the self-helicity of the newly formed flux rope.

Implosive Collapse about 2D and 3D Magnetic Null Points

Jonathan Thurgood - University of Dundee
David Pontin
James McLaughlin

Null collapse is an implosive process whereby MHD waves focus their energy in the vicinity of a null point, forming a current sheet and initiating magnetic reconnection. We consider, for the first time, the case of collapsing 3D magnetic null points in nonlinear, resistive MHD using numerical simulation, exploring key physical aspects of the system as well as performing a detailed parameter study. We find that within a particular plane containing the 3D null, the plasma and current density enhancements resulting from the collapse are quantitatively and qualitatively as per the 2D case in both the linear and nonlinear collapse regimes. However, the scaling with resistivity of the 3D reconnection rate - which is a global quantity - is found to be less favourable when the magnetic null point is more rotationally symmetric, due to the action of increased magnetic back-pressure. Furthermore, we find that with increasing ambient plasma pressure the collapse can be throttled, as is the case for 2D nulls. We discuss this pressure-limiting in the context of fast reconnection in the solar atmosphere and suggest mechanisms by which it may be overcome. We also discuss the implications of the results in the context of null collapse as a trigger mechanism of Oscillatory Reconnection, a time- dependent reconnection mechanism, and also within the wider subject of wave-null point interactions. We conclude that, in general, increasingly rotationally-asymmetric nulls will be more favourable in terms of magnetic energy release via null collapse than their more symmetric counterparts.

A Boundary Layer Analysis of Reactive Shear Bands

Robert Timms - University of East Anglia
Richard Purvis
John Curtis

The mechanisms responsible for ignition of explosive materials in response to low energy stimuli, known as "insults" in the literature, are still not well understood. It is in general believed that explosive ignition is of thermal origin, with mechanical energy being converted into heat energy in localised regions, forming so-called “hot spots”. Experimental evidence suggests that the phenomenon of shear banding, where plastic flow localises into thin planar regions, may be an important hot spot mechanism.

Here, a one-dimensional model for the initiation of shear bands in a reactive material is developed, with an Arrhenius source term to model the chemical reaction occurring in the band. An inhomogeneity in the heat flux is used as the stimulus for localised plastic deformation, and the problem is treated as a perturbation to the elastic solution. In the analysis, the thin zone of localisation is identified as a boundary layer. It is found that the behaviour of the perturbations to the temperature and stress in the band are governed by three dimensionless parameters which are known in terms of various material properties. The resulting equations are solved numerically and a criterion for the onset of shear banding is discussed.

Novel Multiscale Modelling Approaches in Cancer Invasion
Dumitru Trucu - University of Dundee

Recognized as one of the hallmarks of cancer, cancer cell invasion into tissue is a complex process that plays a key role in the growth and spread of cancer, culminating in metastatic spread (secondary cancers). One common aspect of all cancer progression is the secretion of matrix degrading enzymes (MDEs) by the cancer cells that modify or destroy the surrounding tissue or extracellular matrix (ECM) and support local cancer cell invasion. In conjunction with MDE activities, increased cancer cell motility due to changes in cell-adhesion properties further exacerbates the invasion. Transmembrane calcium-dependent adhesion molecules (cadherins) interact with intra-cellular proteins, such as β-catenin and give rise to adhesion junctions, resulting in cell-cell adhesion. In addition to cell-cell adhesion, the binding of various ECM ligands to cell-surface receptors (integrins) enables cell- matrix adhesion. Thus, processes occurring at a molecular (micro) scale give rise to processes occurring at the tissue (macro) scale, via processes taking place at the cellular (meso) scale. The interplay between micro-, meso- and macro-scale processes involved in cancer cell invasion are still not fully understood. This talk will address recent advancements in multiscale modelling of cell-cell adhesion inside the tumour in conjunction with the activity of various proteolytic processes occurring along the invasive edge of the tumour. Finally, we will present computational simulations of the resulting multiscale moving boundary model and discuss a number of important fundamental properties that follow.

Magnetic power spectrum in a dynamo model of Jupiter
Yue-Kin Tsang - University of Leeds
Chris Jones

The magnetic power spectrum, or the Lowes-Mauersberger spectrum, is the variance of the magnetic field on a planetary surface as a function of spherical harmonic degree. It has been used to infer the location of the Earth's core-mantle boundary which separate the conducting dynamo region and the non-conducting mantle. Here we investigate the form of the Lowes-Mauersberger spectrum in a dynamo model of Jupiter. The model consists of a set of anelastic, convection-driven magnetodynamic equations. A key feature is the electrical conductivity varies from high value in the interior metallic hydrogen layer to negligible value near the surface with a sharp but continuous transition. We discuss the relation between the Lowes-Mauersberger spectrum and the dynamo radius in our model and its dependence on model parameters. Comparing these results with observational data from the JUNO mission can constrain the location for the onset of the metallic hydrogen layer.

Tracking error and control effort trade-off in a robust trajectory tracking problem
Vladimir Turetsky - Ort Braude College of Engineering

Robust trajectory tracking problem by means of a feedback control in the presence of an unknown bounded disturbance is considered. The known solution is provided by an optimal strategy in an auxiliary linear-quadratic differential game with properly chosen penalty coefficients. In the case of a scalar system, explicit expressions are derived for the tracking error and the control effort as functions of the control penalty coefficient. This parameter varies in the open interval from zero to some critical value. In particular, it is shown that the tracking error tends to zero at the ends of this interval. Similar behaviour is demonstrated in a multi-dimensional case. Trade-off between the tracking error and the control effort is also investigated numerically for the case of a bounded control. Simulation examples are presented.

The wave-induced flow of internal gravity wavepackets with arbitrary aspect ratio

Ton van den Bremer - University of Oxford
Bruce Sutherland

We examine the wave-induced flow of small-amplitude, quasi-monochromatic, three-dimensional, Boussinesq internal gravity wavepackets in a uniformly stratified ambient. It has been known since Bretherton (J. Fluid Mech., vol. 36 (4), 1969, pp. 785–803) that one-, two- and three-dimensional wavepackets induce qualitatively different flows. Whereas the wave-induced mean flow for compact three-dimensional wavepackets consists of a purely horizontal localized circulation that translates with and around the wavepacket, known as the Bretherton flow, such a flow is prohibited for a two-dimensional wavepacket of infinite spanwise extent, which instead induces a non-local internal wave response that is long compared with the streamwise extent of the wavepacket. One-dimensional (horizontally periodic) wavepackets induce a horizontal, non-divergent unidirectional flow. Through perturbation theory for quasi-monochromatic wavepackets of arbitrary aspect ratio, we predict for which aspect ratios which type of induced mean flow dominates. We compose a regime diagram that delineates whether the induced flow is comparable to that of one-, two- or compact three-dimensional wavepackets. The predictions agree well with the results of fully nonlinear three-dimensional numerical simulations.

New family of gravity solitary of waves

Jean-Marc Vanden-Broeck - University College London

Nonlinear gravity waves propagating at the surface of a fluid are considered. The fluid is assumed to be incompressible and inviscid and the flow to be irrotational. Numerical evidence is presented for the existence of new families of non-periodic waves. Some of these new waves look like generalised solitary waves while others are similar to dark solitons. Results are presented in both finite and infinite depth.

Unforced Navier-Stokes solutions with applications to magnetohydrodynamics

Raquel Vaz - Imperial College London
Florencia Boshier
Jonathan Mestel

Steady fully developed flow in a cylindrical annulus with a similarity structure that imposes linear variation in the axial direction is shown to permit non-trivial unforced solutions. These constitute previously unreported solutions of the full Navier-Stokes equations in an annulus of arbitrary curvature. Dynamo action is investigated in these circumstances. Magnetic field instabilities with the same spatial structure are sought. The kinematic eigenvalue problem is found to have two growing modes for moderate values of the magnetic Reynolds number, R_m. As $R_m \to \infty$ it is shown that the modes are governed by layers on the outer wall. As the field grows, saturated solutions to the nonlinear dynamo problem are found in three distinct solution families. The complicated bifurcation structure is investigated.

Modelling receptor-ligand interactions

Chandrasekhar Venkataraman - University of St Andrews

We discuss the mathematical modelling of receptor-ligand interactions in cell biology via coupled bulk-surface PDEs and the computational approximation of the resultant model equations.

Ultrasound response to tumour induced angiogenesis model

Vasiliki Voulgaridou - Heriot Watt University
Steven McDougall

Ultrasound imaging is well established in daily clinical practice due to being safe, real-time in nature, portable, and inexpensive. However, it provides images with poorer quality compared to other medical imaging modalities. The use of microbubbles as contrast agents may improve the resolution and open the way for new possible applications. In this project, a tumour induced angiogenesis model in combination with a particle flow model are used to simulate the flow of microbubbles within the vasculature. The angiogenesis model describes vessel growth due to presence of a tumour. It is a model that couples growth due to metabolic stimuli and due to perfusion-related haemodynamic forces. The resulting vessel structure can be used as a base for the particle flow model. Then, the particle flow information is translated into microbubble flow information which, in turn, is used in an ultrasound imaging simulation. The result is a simulated ultrasound image of microbubbles flowing within the same vessel structure that was created by the presence of the tumour. The whole process allows for the study of ultrasound response to tumour induced angiogenesis and of the imaging parameters that can be altered in order to improve the outcome and aid diagnosis.

Lateral transport by vertical mode two internal waves in an enclosed basin

Danielle Wain - University of Bath

Boundary-interior transport is a key ecological process in lakes and on ocean shelves. In this talk, observations of transport away from the boundary by vertical mode two internal waves are presented. Tracer was released in a stratified lake where the thermocline intersects the shoreline. The lake is 25m deep and the early summer stratification (at the time of the tracer release) is composed of three layers of approximately equal depth; the surface and bottom layer are well mixed and separated by a stratified middle layer. The internal wave field is dominated by basin scale seiches, particularly of vertical mode one and two. During the time of internal wave activity, there was net transport up to a kilometre away from the injection location. Much of the transport can be attributed to the oscillating jet from the mode two wave, although this mechanism was not able to fully describe the observed spreading of the tracer. We propose and show that oscillating strain in combination with shear dispersion from the internal wave field can potentially explain the observed tracer distribution.

Smoothed Particle Hydrodynamics (SPH) Modelling of Tsunami Waves Generated by a Fault Rupture

Ruaa Wana - University of Plymouth
Jason Hughes
David Graham
Natalia Perez del Postigo Prieto
Alison Raby

Smoothed Particle Hydrodynamics (SPH) is a meshfree, Lagrangian, particle method. It is particularly well suited to simulating flow problems that have large deformations or contain free surfaces. In this work we use a single phase weakly compressible SPH model to simulate the flow that occurs in experimental models of Tsunamis generated by a fault rupture. The experiments have been carried out at Plymouth University's COAST lab. Fortran code has been written to implement the SPH method. The SPH model consists of a horizontal water tank, with water depths up to 0.6m. The Tsunami wave is generated by the rapid uplift of a plate on the bed of the tank, where it moves up very rapidly and the displacement follows a half-sine function. The uplifted plate is 0.6m long, and is positioned at one end of the tank. In order to obtain a good SPH particle distribution in the vicinity of the moving plate, and accurate predictions, careful consideration of the SPH boundary conditions is required. Wave heights are predicted at positions close to the uplift plate, in order to compare with experimental wave gauge data. The SPH model gives generally good predictions of the free surface position.

The energetics of flow in a flexible channel

Danyang Wang - Universtiy of Glasgow
Xiaoyu Luo
Peter Stewart

There are many examples of fluid-conveying vessels in the human body. When these vessels are subject to a negative transmural pressure (internal minus external) while conveying a flow, self-excited oscillations can occur. We explore the generation of self-excited oscillations in a two-dimensional channel that consists of rigid and flexible segments conveying a laminar high-Reynolds number flow. In particular, we construct a general framework for analysing the energy budget of self-excited oscillations about a non-uniform basic state. We then apply this general framework to consider two particular models for the flexible wall, namely a simple membrane model with an external pressure gradient and fluid-beam model.

Lorentzian Symmetry Predicts Universality Beyond Power Laws

Stephen Watson - University of Glasgow

The statistical physics governing phase-ordering dynamics following a symmetry breaking first-order phase transition continues to intrigue, but unifying principles have remained largely elusive. The Coarsening/Ageing of the ensuing ensemble of phase domains, wherein irreversible annihilation or joining of domains yields a growing characteristic domain length, is an omniprescent feature whose universal characteristics one would wish to understand. Power-laws for the growth in time of the characteristic size of domains (e.g., lengths), and a concomitant scale-invariance of the associated length distributions, has so frequently been empirically observed that their presence has acquired the status of a principle; the so-called Dynamic-Scaling Hypothesis. But the dynamical symmetries of a given phase-ordering system – its Coarsening Group G – may include more than the global spatio-temporal scalings underlying the Dynamic Scaling Hypothesis.

In this talk, I will present a recently developed theoretical framework that shows how the symmetry group G of a Coarsening (ageing) Dynamical System (CDS) necessarily yields G-equivariance (covariance) of the CDS's universal statistical observables. We exhibit this theory for a variety of model systems, with symmetries that may also be emergent and/or hidden. We will close with a novel theoretical coarsening law for a model driven spin system that combines Lorentzian and parabolic symmetries!

The Minimize-Effort-Specify-Performance approach to the design of missile guidance algorithms

Martin Weiss - Technion - Israel Techonology Institute

Every attempt to design a guidance algorithm, and more general, a control algorithm, faces a basic trade-off between the performance to be achieved and the effort that has to be paid for this purpose. The classic optimal control-based guidance design philosophy is to propose a cost function that is a weighted means of the two: the performance and the control effort. The trade-off is resolved by the choice of the weight coefficients. Another typical approach used both in optimal control-based approaches, but also in differential game based approaches, is to assume a maximum bound on the control input and optimize the performance. Both approaches have been used in different versions both for intercept guidance, as for evasion guidance.

In this contribution, we introduce a new systematic approach that attempts to solve the aforementioned trade-off in a different way. As the name suggests, the Minimize-Effort-Specify-Performance (MESP) approach is aimed at finding the minimum effort that achieves a specified performance. We will show that this approach leads to relatively simple guidance algorithms, that it is suited both for intercept as for evasion, and that it can be used in many situations that are currently very difficult to tackle with other approaches.

The MESP approach will be applied in three different settings: the Target-Attacker-Defender scenario, the problem of intercept with obstacle avoidance, the problem of intercepting a target that can be poorly distinguished from a decoy. In each case, we will show how the MESP leads to relatively simple, easy to implement guidance algorithms.

The Influence of Turbulent Pumping and Turbulent Diffusion on Magnetic Buoyancy Instability

Daniela Weston - University of Leeds

Magnetic buoyancy has been suggested as a mechanism for the rise of flux tubes through the solar convection zone to emerge as the structures we observe at the surface. The large scale of these structures, however, implies that the instability to magnetic buoyancy interacts with the effects of the small-scale, turbulent convection in the region through which they pass in such a way as to preserve the large-scale variation. Thus, we consider the instability of a horizontal layer to magnetic buoyancy, with a region of turbulence above and a non-turbulent region below, as a model for the base of the solar convection zone. The effects of turbulent motion are captured by the approach provided by mean field dynamo theory, which results in an additional turbulent diffusion effect and a downwards turbulent pumping velocity, acting on the mean field. Applying these two effects, we solve for an equilibrium field and then introduce 3D linear perturbations. We solve numerically for their growth rate and vertical dependence, using a Chebyshev spectral method. For horizontally periodic perturbations, we consider the horizontal scales that give rise to the greatest degree of instability. We consider the dominance of 2D versus 3D modes as the most unstable given the additional effects of mean field turbulent motion, and find the effects of parameter variation on the instability.

The contribution of individual active regions to the Sun's axial dipole moment

Tim Whitbread - Durham University
Anthony Yeates
Andrés Muñoz-Jaramillo

We test recent claims that the polar field at the end of Cycle 23 was weakened by a small number of large, abnormally oriented regions, and investigate what this means for solar cycle prediction. We isolate the contribution of individual regions from magnetograms for Cycles 21, 22 and 23 using a 2D surface flux transport model, and examine the relationship between a region's axial dipole moment contribution and its emergence latitude, flux, and initial axial dipole moment. We find, in agreement with other authors, that emergence latitude primarily determines the contribution of a region to the end-of-cycle axial dipole moment, with flux and initial axial dipole moment playing secondary, but nevertheless important, roles. Although the top ~10% of contributors tend to define sudden large variations in the axial dipole moment, the cumulative contribution of many weaker regions can not be ignored. In order to recreate the axial dipole moment to a reasonable degree, many more regions are required in Cycle 23 than in Cycles 21 and 22 when ordered by contribution. We suggest that the negative contribution of the most significant regions of Cycle 23 could indeed be a cause of the weak polar field at the following cycle minimum and the low-amplitude Cycle 24.

An integro-differential equation model for urban population dynamics predicting emergent pattern formation

Timothy Whiteley - University of Nottingham
Markus Owen
Daniele Avitabile
Peer-Olaf Siebers
Darren Robinson

Urban population distributions in large cities can show structure such as patchy patterning that may relate to important properties such as journey times, quality of life and sustainability. We use integro-differential equations to model the spatio-temporal dynamics of urban populations and services, under the assumption that they benefit from proximity to one another, as captured via spatial weight kernels. The system may tend towards a homogeneous state or a spatial pattern. With Gaussian kernels, linear stability of the spatially homogeneous steady state depends on a key function in the model, the carrying capacity for services given a local population density. In particular, patterning occurs only where the carrying capacity is convex with respect to population density. Furthermore, this spatial instability can only occur for perturbations with a sufficiently long lengthscale. Numerical continuation shows how multiple steady states corresponding to different spatial wavelengths can coexist and state transitions may occur as carrying capacity grows. Lastly, in urban centres, competition for space may cause services and population to be out of phase with one other. To generate such patterning in our model requires kernels with Fourier transforms that are negative for some wavelengths. With box and off-centre kernels, such out of phase patterning can occur and we show that this patterning occurs at a higher density and of a shorter lengthscale than in phase patterning.

Stability of scrape-off layer plasma: a modified Rayleigh-Bénard problem

Fryderyk Wilczynski - University of Leeds
David Hughes
Sven Van Loo
Wayne Arter
Fulvio Militello

In magnetic confinement devices, the boundary turbulence is characterised by intermittent ejection of coherent filamentary structures. These filaments transport plasma from the well-confined core region, through the Scrape-Off Layer (SOL), towards the material surfaces. This results in increased plasma-wall interaction, which has the potential to damage plasma-facing components and shorten the lifetime of the device. It is therefore essential to develop full understanding of the mechanisms behind the transport in the edge of the plasma.

We consider an established model of two-dimensional SOL plasma, for an insight into the instabilities present at the edge of magnetic confinement devices. The model equations are based on the Braginskii fluid equations under the assumption of drift ordering and electrostatic plasma. The model also employs the common slab geometry approximation, whereby the magnetic field is assumed constant and straight and the effects of curvature are reintroduced as effective gravitational terms.

Of particular interest from a fluid dynamical perspective is that in this limit, the SOL plasma equations can be reduced to a thermal convection problem with additional effects. These new effects include a non-uniform basic state gradient, linear damping terms, and additional advective terms. We analyse the linear stability of the plasma problem and relate the results to the established theory of Rayleigh-Bénard convection. Furthermore, we assess the importance of each of the new terms in establishing the overall stability threshold on the system.

Formation of a Dense Flux Rope by a Siphon Flow

Thomas Williams - UCLAN
Youra Taroyan

The interaction of siphon flow with an initially linear Alfvén wave within an isolated chromospheric loop is investigated. The loop is modeled using 1.5D magnetohydrodynamics (MHD). The siphon flow undergoes a hydrodynamic (HD) shock, which allows the Alfvén instability to amplify the propagating waves as they interact with the shock and loop footpoints. The amplification leads to nonlinear processes strongly altering the loop equilibrium. Azimuthal twists of 50 km s^{-1} are generated and the loop becomes globally twisted with an azimuthal magnetic field of $B_\theta \approx 5\times B_z$. The flow is accelerated to ≈ 70 km s^{-1} due to the propagating shock waves that form. Near the end of the simulation, where the nonlinear processes are strongest, flow reversal is seen within the descending leg of the loop, generating upflows up to 28 km s^{-1}. This flow reversal leads to photospheric material being "pulled" into the loop and spreading along its entirety. Within about 2.5 hr, the density increases by a factor of about 30 its original value.

Spike clusters for the Gierer-Meinhardt system

Matthias Winter - Brunel University London
Juncheng Wei
Weiwei Ao
Wen Yang

We consider the 1D and 2D Gierer-Meinhardt system on a bounded, smooth domain with two small diffusivities. We will show the existence of spike clusters (i.e. a pattern with multiple spikes converging to the same point in the limit of the diffusivities tending to zero) in the following settings: (i) interior spike cluster for the Gierer-Meinhardt system with a precursor gradient in 1D and 2D, (ii) boundary spike cluster in 2D. We will also compute the asymptotic behaviour of the eigenvalues of the system linearised around a spike cluster and show that spike clusters can be stable. These spike clusters play an important role in biological modelling to account for the bridging of lengthscales, e.g. between genetic, nuclear, intra-cellular, cellular and tissue levels, or for the time-hierarchy of biological processes, e.g. a large scale structure, which appears first, induces patterns on successively smaller scales. This is joint work with Weiwei Ao, Wen Yang and Juncheng Wei.

Fluid flow through submerged vegetation

Clint Wong - University of Oxford
S. Jonathan Chapman
Philippe Trinh

The study of fluid flows interacting with vegetative structures presents a significant theoretical and numerical challenge on account of its inherently multi-scale nature. Consider, for example, the case of fluid passing over a submerged vegetative bed or layer. Even in the case of inelastic vegetation, the flow characteristics can be highly dependent on the arrangement and configuration of the bed; here, strong shear effects along the top of the vegetation can trigger vortices, analogous to the case of the Kelvin-Helmholtz instability. In this talk, we will discuss the mathematical modelling of this fluid-structure problem, which now incorporates elastic deformation of the vegetation. A stability analysis of this problem suggests new insights into how the deformation of vegetation can affect vortex generation and the stability threshold.

Pattern production through a chiral chasing mechanism

Thomas Woolley - Cardiff

Recent experiments on zebrafish pigmentation suggests that their typical black and white striped skin pattern is made up of a number of interacting chromatophore families. Specifically, two of these cell families have been shown to interact through a nonlocal chasing mechanism, which has previously been modeled using integro-differential equations. We extend this framework to include the experimentally observed fact that the cells often exhibit chiral movement, in that the cells chase, and run away, at angles different to the line connecting their centers. This framework is simplified through the use of multiple small limits leading to a coupled set of partial differential equations which are amenable to Fourier analysis. This analysis results in the production of dispersion relations and necessary conditions for a patterning instability to occur. Beyond the theoretical development and the production of new pattern planiforms we are able to corroborate the experimental hypothesis that the global pigmentation patterns can be dependent on the chirality of the chromatophores.

Propagation of signals from indoor small cells at ultra-high frequencies.

Hayley Wragg - University of Bath
Hayley Wragg

As public demand for wireless communication increases the technology has developed. New high frequency technology has resulted in the need for new propagation models. Many of the current models are environment specific or don't adapt well to the high frequencies.

The Helmholtz equation models electromagnetic propagation. However, at frequencies above 2.4 GHz it becomes very difficult to compute. Taking the high frequency limit to the Helmholtz equation gives the Eikonal equation which justifies a ray-tracing model.

When considering domestic environments there is an additional complication in the lack of knowledge about the environment. The physical characteristics of obstacles are uncertain and the location of the obstacles is subject to change.

The approach presented in this talk is a stochastic ray-tracing model which offers an overview of the propagation within an environment where not all characteristics are known.

A Model for Coronal Hole Bright Points and Jets Due to Moving Magnetic Features

Peter Wyper - Durham
C. Richard DeVore
Judy Karpen
Spiro Antiochos
Anthony Yeates

Coronal jets and bright points occur prolifically in the predominantly unipolar magnetic regions of coronal holes, where they appear above minority-polarity intrusions. The magnetic field of these intrusions possesses a spine-fan topology with a coronal null point. The movement of magnetic flux by surface convection adds free energy to this field, forming current sheets and inducing magnetic reconnection and plasma heating. New three-dimensional MHD simulations will be presented where minority polarity magnetic features are subject to large-scale surface shear motions. In general, we find that steady interchange reconnection is driven at the null point. However, depending upon the nature of the surface shear, this background steady component is modulated by periodic low-intensity reconnection bursts in some cases and homologous non-helical jets in others. I will discuss how our results explain several key aspects of coronal hole bright points and jets, and the relationship between them.

Microscopic aspects of wetting using classical density-functional theory

Peter Yatsyshin - Imperial College London
Miguel A. Durán-Olivencia
Serafim Kalliadasis

Wetting is nucleation of a phase (typically liquid) on the interface between two other phases (typically solid and gas). In many experimentally accessible cases of wetting, the interplay between the substrate structure, and the fluid-fluid and fluid-substrate intermolecular interactions brings about an entire "zoo" of possible fluid configurations, such as liquid films with a thickness of a few nanometers, liquid nanodrops and liquid bridges. These fluid configurations are often associated with phase transitions occurring at the solid-gas interface and at lengths of just several molecular diameters away from the substrate. In this talk, we demonstrate how a fully microscopic classical density-functional framework can be applied to the efficient, rational and systematic exploration of the rich phase space of wetting phenomena. We consider a number of model prototype systems such as wetting on a planar wall, a chemically patterned wall and a wedge. Through density-functional computations we demonstrate that for these simply structured substrates the behaviour of the solid-gas interface is already highly complex and non-trivial. Our numerical methodology is based on spectral collocation for spatial discretisatoin and arc-length continuation with adaptive step-size control. The latter allows us to trace solutions in the regions of thermodynamic metastability, where two or more fluid configurations can coexist.

Consequences of delayed flux emergence in coronal magnetic models

Anthony Yeates - Durham University
Duncan Mackay

Time-evolving models of the global magnetic field in the solar corona depend upon newly emerging active regions as the fundamental sources of new magnetic flux. However, because available magnetogram observations are limited to the visible hemisphere of the Sun, these active regions are typically inserted at the time when they are first (fully) observed, as opposed to the time when they first emerge. We use a series of controlled numerical experiments, with a magneto-frictional model, to consider the lasting impact of such a delayed emergence. We find that delaying the insertion of a bipolar region significantly reduces the free magnetic energy added to the simulation. It can not only reduce the magnitude of magnetic helicity but also change the sign, as manifested in the chirality (direction) of axial magnetic fields in filament channels.

The deformation and stability of elastic cells in an inviscid uniform flow

Adam Yorkston - University of East Anglia
Mark Blyth
Emilian Parau

The deformation of an elastic cell in a fluid flow is a problem of interest in a variety of applications, from blood cells flowing through veins in biomechanics, to the use of inflatable elastic wings in aeronautics. Here we calculate steady state solutions for a deformable two-dimensional elastic cell in a potential flow with a uniform stream at infinity using a conformal mapping technique. Asymptotic methods are used to obtain analytical solutions for small flow speeds, and numerical results are presented for larger flow speeds for which the deformation is fully nonlinear. It is well known that, in the absence of flow, buckled solutions with n-fold symmetry appear at a set of critical transmural pressures. We demonstrate that as the strength of the flow is increased from zero, we uncover a rich solution space of steady solutions. While intuitively we expect the shapes to have both fore-aft and up-down symmetry, we find that solutions with only one of these symmetries are possible. We then present a linear stability analysis and demonstrate that the choice of constitutive relation for the in-wall tension plays a vital role in the stability of the solution. Finally, we present some unsteady calculations. In particular we find a solution in which the cell flips up and down continuously in an almost periodic manner whilst being carried downstream with the flow.

A new criterion for uniqueness for weak Solutions of the 3D Navier-Stokes equations

Abdelhafid Younsi - University of Djelfa, Algeria.

In this paper, we consider the 3D incompressible Navier-Stokes equations. We show a new uniqueness result for Leray-Hopf weak solutions in term of the gradient of the velocity.

Coalescing particle systems in models of chemotaxis

Gleb Zhelezov - University of Edinburgh
Ibrahim Fatkullin

The Patlak-Keller-Segel PDEs describe the chemotaxis of slime mould, and exhibit finite-time blow-up when the system mass is above a certain threshold--a mathematical manifestation of the biological observation that mould aggregates when its quantity is sufficiently high. In this talk, we associate a colliding stochastic particle system with these PDEs. By relating the squared Bessel process to the evolution of localized clusters of particles, we develop a numerical method capable of detecting collisions of many point particles without the use of pairwise computations, or very refined adaptive timestepping; and apply this method to the numerical solution of the Patlak-Keller-Segel system and its variants. We show that the presented numerical method is well-suited for the simulation of the formation of finite-time singularities in the PKS, as well as PKS pre- and post-blow-up dynamics.

The constitutive response of bodies undergoing surface growth.

Giuseppe Zurlo - National University of Ireland Galway

Bodies that are created through the process of surface growth, that is by the continuous deposition of mass over their boundary surface, are characterised by a constitutive response that depends on the conditions exerted on the new matter during deposition. In this talk we will present a new theory that explains the connection between the mechanical state of upcoming mass and the ensuing state of residual stress in the accreted body. It is shown that inelastic effects taking place during deposition can be fine-tuned by suitable controls on the deposition process, targeting both desired distributions of residual stress of the final body and its final shape. The results of this theory are of interest of various frameworks, spanning from the study of self-propulsion of cells on rigid substrates and of embryonic development, to the design of 3D printing protocols for the manufacturing of arteries.

Modeling actuators based on carbon nanotube yarn/nanoenergetic material composites

Maxim Zyskin - University of Nottingham

In recent experiments, it is shown that carbon nanotube yarn contracts when nanoenergetic material placed inside the yarn ignites. Such yarn contraction has applications in actuators or artificial muscles. I will describe modeling results trying to understand those experiments.

Abstracts of Posters

Towards a mathematical model of *in vitro* vascular network formation

Georgina Al-Badri - University College London
James Phillips
Rebecca Shipley
Nick Ovenden

Engineered tissues require intervention to ensure swift vascularisation of the construct once implanted *in vivo*. An existing method involves pre-vascularising the construct by inclusion of endothelial cells (ECs) and culture under appropriate conditions. We aim to emulate this process *in silico* in order to optimise the culture conditions used and to predict the stability and functionality of the resulting network.

Several mathematical frameworks have been previously utilised to investigate and model the key mechanisms of vascular network formation by ECs, however little work has been done in comparison with experimental data or in consideration of 3D tissue engineering applications. We will combine existing literature alongside specifically designed experimental work to identify the key mechanisms of cell behaviour, which may depend on the materials used, and parametrise a mathematical model of vascular network formation in 3D hydrogels.

The model involves three coupled PDEs, which describe the time and spatial evolution of the cell density, matrix (hydrogel) density, and chemoattractant concentration, alongside an additional force-balance equation involving cellular and matrix stress tensors.

To parameterize the model, we have specifically designed cell migration experiments to identify the dominant migration cues, for instance chemotaxis, haptotaxis, and durotaxis, by setting up opposing gradients and assessing how strongly the cells respond to each. Further experimental work will be undertaken to verify the model, and it can be compared to existing qualitative and quantitative results produced by the Phillip’s lab where human umbilical vein endothelial cells (HUVECs) have been cultured in a collagen gel to form connected capillary-like networks.

Our work will make progress towards providing a better understanding of how to effectively pre-vascularise an engineered tissue construct, a current key question in tissue engineering. We have also shown how experimental and mathematical methods can be used in conjunction to accelerate work in this field, additionally reducing time and cost involved.

Stability and Instability in Cross-Diffusion Systems

Mohammed Aldandani - University of Dundee
Fordyce Davidson

In this work, we investigate the stability of a reaction diffusion system. The main purpose is to discuss the stability analysis for steady-state solutions of a cross diffusion system used to model cell movement within bacterial biofilms. We show that cross-diffusion can destabilize a uniform equilibrium which is stable for the kinetic and self-diffusion reaction systems. In our system there is no Turing pattern formation with self-diffusion. However, with cross-diffusion, the system can exhibit pattern. We demonstrate this using both mathematical analysis and numerical simulations.

Analysis of a Fractal Ultrasonic Transducer

Ebrahem Algehyne - University of Strathclyde
Anthony Mulholland

Geometrical structures are used in the design of piezoelectric ultrasonic transducers. These structures are normally regular and periodic, with one principal length scale which, due to the resonant nature of the devices, determines the central operating frequency. There is engineering interest in building wide bandwidth devices, and so it follows that a range of length scales should be used. We will prove some results about a mathematical model of a fractal ultrasound transducer whose piezoelectric components span a range of length scales. There have been many previous studies of different types of wave propagation in the Sierpinski gasket but this work is the first to study wave propagation in the Sieprinski Gasket complement. This is a critically important mathematical development as the complement is formed from a broad distribution of triangle sizes whereas the Sierpinski gasket is formed from triangles of equal size. We will prove that a set of basis functions can be developed that allow us to describe this wave propagation via a finite element methodology, and that a renormalisation approach can be used to produce closed form expressions for the transducer's operational characteristics. It transpires that the fractal device has a significantly higher reception sensitivity (18 dB) and a significantly wider bandwidth (3 MHz) than an equivalent Euclidean (standard) device.

Anti-plane dynamic shear of strongly inhomogeneous laminates.

Mohammed Alkinidri - University of Keele
Yagmur Ece Aydin
Baris Erbas
Ludmila Prikazchikova

Anti-plane dynamic shear of two-layered and three-layered laminates is analysed. Two types of high contrast are considered, including composite structure with at least one stiff (thick or thin) layer. It is shown that for both contrast setups the value of the cut-off frequency, corresponding to the lowest vibration mode for a two-layer laminate with one free and one fixed faces, as well as for the lowest antisymmetric mode for a three-layered laminate with traction free faces tends to zero. For these modes shortened equations of motion are derived. Some of them appear to be valid over the whole low-frequency range, while the range of validity of others is restricted to narrow vicinities of the cut-off frequencies. Numerical data illustrating comparisons of exact and asymptotic results are presented.

The effect of heterogeneity on growth-induced buckling in elastic rods

Axel Almet - University of Oxford
Helen Byrne
Philip Maini
Derek Moulton

Mechanically-induced buckling underlies the shape and function of a number of biological structures, including intestinal crypt replication, brain tissue folding, and seashell formation. Unlike many typical engineering systems in which buckling is induced by an external compressive load, mechanical instability in biology is driven often by internal growth. In these contexts, it is just as important to consider how the system evolves beyond the onset of instability, as it is to understand when the instability occurs initially. In this talk, we work within the classical set-up of a growing elastic rod supported by an elastic foundation. In particular, through a combination of weakly nonlinear analysis and numerical methods, we examine how spatial heterogeneity in different material properties impacts the instability and resultant post-buckled shape evolution.

A Model of Groundwater Uptake by Plant Roots

Zuhur Alqahtani - Princess Nourah bint Abdulrahman University
Nigel Mottram
Moustafa Elshahed

Growth of soil-rooted vegetation crucially depends on the availability of water within soil. This availability of water for the roots to utilise in turn depends on factors such as rainfall, surface-water infiltration rate, porosity and permeability of the soil and the depth of the water table. The growth of vegetation, and the associate changes in root structure, can affect properties of the soil structure and thus a feedback mechanism is developed –soil-water availability affecting growth, growth affecting root density, root density affecting soil structure and soil structure affecting soil-water availability. As a mathematical model, we consider part of this feedback mechanism, modelling the soil water through the matric potential and using the classic Richards equation introduced in 1931. In our model, an adaption of the model of Yuan and Lu, we consider various forms for the root-water uptake rate and study the effect on water availability. We also consider non-local-in-time effects derived from aspects of delay and memory in the vegetation through the use of fractional time derivatives.

Stationary Solutions in Yip's Formulation of the Regularized Ericksen's Bar Model

Abdulmohsen Daham Alruwaili - University of Strathclyde
Dr. Michael Grinfeld
Dr. Gabriel Barrenechea

We consider Yip's formulation of the Ericksen model for an elastic bar on an elastic foundation (N. K. Yip, 2006. Structure of stable solutions of a one-dimensional variational problem, Control Optim. Calc. Variations 12, 721-751) which leads to the Euler-Lagrange equation for the functional E(u)= \int_0^1 (\frac{\gamma}{2} u^2_{xx} + W(u_x) + \frac{\alpha}{2} u^2) dx ,\ x \in (0,1), with double Dirichlet boundary conditions. Here the potential W(p)=(|p|-1)^2), is not differentiable at p=0.

We define and prove existence and uniqueness of periodic solutions with any number $n \geq 0$ of internal zeroes for all $\alpha,\gamma > 0$ and discuss the existence of non-periodic solutions.

A flexural wave on a coated edge of a thin elastic plate.

Ahmed Alzaidi - Keele University
Julius Kaplunov
Ludmila Prikazchikova

The study is concerned with flexural edge elastic waves. The focus is on a time-harmonic travelling wave localised near the edge of a semi-infinite plate coated by a narrow layer. An asymptotic procedure, accounting for the influence of the coating, leads to novel effective boundary conditions at the edge of a homogeneous plate known previously only within the framework of linear elasticity. The associated approximate dispersion relation is then analysed. The obtained results are compared with the exact solution of the original problem. The possibility of a stronger localisation effect, caused by the presence of the coating, is discussed. Similarity with localised buckling of a coated plate edge is also addressed.

Formulation and Estimation of ARCH and GARCH Models with Application to the Analysis of the Volatility Series

Guerouah Amin - Badji Mokhtar University Annaba Algeria
Zeghdoudi Halim
Khochmane Houssem eddine

This paper investigates the empirical properties of oil price and Stock market return volatilities using a range of univariate and multivariate GARCH models and monthly data from the U.S. The study relates the period August 1987 to October 2016, a total of 351 observations given. The aim of this paper is to examine the relationship between stock and oil markets. In addition, we evaluate the performance of each model with a range of diagnostic and forecast performance tests using univariate GARCH(1,1) and bivariate BEKK GARCH(1,1) , DCC GARCH(1,1) models.

Effects of Anomalous Resistivity on Particle Acceleration due to Pitch Angle Scattering.

Alexei Borissov - School of Mathematics and Statistics, University of St Andrews
Eduard Kontar, Thomas Neukirch
James Threlfall
Julie Stevenson
Clare Parnell

The mechanisms for generation of non-thermal accelerated particles in flares is one of the outstanding problems in solar physics. The energy for powering solar flares fundamentally comes from the coronal magnetic field and its release involves magnetic reconnection. One direct way of accelerating charged particles is due to the parallel electric field generated during magnetic reconnection. To achieve a sufficiently rapid release of energy an anomalous resistivity, several orders of magnitude larger than the Spitzer resistivity in the corona, is often invoked, particularly when performing magnetohydrodynamic (MHD) simulations of solar flares. Since resistivity is fundamentally connected to particle scattering, an enhanced anomalous resistivity relative to the Spitzer resistivity should result in an enhanced scattering frequency relative to the Coulomb scattering rate. We present results of test particle simulations that attempt to account for these phenomena by introducing pitch angle scattering at a rate dependent on the ratio of the anomalous to Spitzer resistivity in the context of MHD simulations of magnetic reconnection. We find that test particle trajectories and durations are significantly modified by the presence of resistivity dependent pitch angle scattering, with particle energy spectra also being affected in some cases.

Implementation of a Lattice Boltzmann Method with Moment based boundary conditions in two and three Dimensions.

Zainab Bu Sinnah - Plymouth University
Zainab Bu Sinnah
David Graham
Tim Reis

The Lattice Boltzmann Method (LBM) has been developed and used to simulate both steady and unsteady fluid flow problems such as turbulent flows, multiphase flow and flows in the vascular system. An example is the study of blood flow which can give a greater understanding of atherosclerosis and the flow parameters which influence this phenomenon. Blood flow is a kind of pulsatile flow. Here we study a simplified version of pulsatile flow - namely the standard Poiseuille flow between parallel plates driven by a periodic forcing term. Many boundary conditions have been developed in the LBM. The moment boundary condition approach is one such example. Here, boundary conditions at solid walls are specified in terms of various moments of the particle distribution function. In this paper, moment boundary conditions are imposed with non-slip and Navier-slip at solid walls and are applied in two and three dimensions. We used a second-order single relaxation time model and investigated grid convergence.

On Graph Mean Curvature Flow and related flows

Jeremy Budd - Nottingham University

In the continuum, mean curvature flow is closely related to Ginzburg-Landau dynamics and to flow under the Merriman-Bence-Osher algorithm. We introduce a way to understand these flows within a discrete graph-based setting, and discuss whether these connections observed in the continuous context remain in the graph context.

Mathematical Modelling of bladder mechanobiology during development and aging.

Fangzhou Cheng - University of Pittsburgh
Paul Watton, Giulia Pederzani
Lori Birder
Florenta Kullman
Anne Robertson

Storing and expelling urine are the two main functions of bladder. The mechanical functions are dominated by the bladder compliance. However, the bladder compliance has been found to decrease with age and disease. The loss of compliance not only diminishes the bladder capacity but also raises the filling pressure, which can be transmitted to kidney and cause renal insufficiency. Structurally, the bladder wall is a layered composite with passive components (elastin and collagen fibers) as well as active components (smooth muscle cells). There remain numerous open questions regarding the relationship between bladder wall architecture and mechanical function, even for the healthy bladder. Structurally motivated computational models of the bladder wall can provide a framework for addressing these questions.

We developed a conceptual model of the healthy bladder which is calibrated with experimental data. In order to provide information about the bladder loading mechanism, a custom bi-axial system was developed that is compatible with multi-photon microscopy (MPM), enabling simultaneous mechanical testing and imaging of collagen fibers without traditional destructive fixation methods. The value of the system for probing the mechanics of the bladder wall is illustrated with 9 rat bladders (4 adult, 5 aged). The collagen fiber distribution and recruitment were monitored from both the lumenal and adventitial sides during bi-axial loading, chosen to approximate bladder filling. The bladder is modeled as a nonlinear elastic spherical membrane using a constrained mixture approach. Constituents (elastin, collagen, smooth muscle) are configured in the loaded configuration to optimize its mechanical function. We then explore mechanistic hypotheses to describe the remodeling of bladder tissue that occurs during development and aging.

Localised Convection at Low Prandtl Number

Robert Cooper - Newcastle University
Celine Guervilly
Paul Bushby

We consider the most basic setup where localised convective structures are observed and study the motion of a layer of fluid in a rotating box, heated from below. We substitute Fourier modes into the governing two-dimensional Boussinesq equations and solve the system pseudo-spectrally. Studying localised convective structures, we find that such structures are evident at higher Taylor numbers and lower Prandtl number than originally considered.

A Constrained Mixture Model of the Left Ventricle and its application to simulating Myocardial Infarction

Shaktidhar Dandapani - The University of Sheffield
Namrata Gundiah
Xiaoyu Luo
Paul Watton

Myocardial infarction results in a change to the mechanical environment of the left ventricle (LV). Consequently, following an MI, both the localised infarct region and the LV will remodel. Computational models may help us to predict whether the remodelling will have a deleterious impact on LV mechanical function.

We model the left venticle as a nonlinear elastic spherical membrane using a constrained mixture approach. Constituents (elastin, collagen, cardiac myocytes) are configured in the loaded configuration to optimise the mechanical function of the ventricle. First, we illustrate a conceptual model of the healthy ventricle and its adaption to altered mechanical loading. We then consider its application to simulate myocardial infarction: an immediate loss of myocytes is prescribed and we simulate the growth and remodelling of the infarct.

We sophisticate the tissue model to represent a distribution of collagen attachment stretches and simulate the remodelling of the attachment stretch distribution as the infarct evolves: this enables the collagen to bear more load for a given mass and yields results consistent with in vivo observations for evolving thickness of the infarct region.

First Traveltime Tomography

Oliver Dunbar - University of Warwick

Seismic tomography is a technique in subsurface imaging where one uses the paths of pressure waves in the earth to reveal the underlying structure. In this talk we discuss first traveltime tomography, where only the first hitting time of waves on detectors is used in this reconstruction. One may formulate this as an inverse problem, with associated forward problem modeled by the Eikonal equation. We will introduce the model and discuss well posedness, regularisation and implementation.

Integral equation models of late-life acting insecticides and their role in malaria control

Andreas Foiniotis - Surrey University
Dr Stephen Gourley

Understanding the dynamics if an infectious disease, such as malaria, help us reducing the number of deaths and have better control of it spread.This is due to the evolution of more appropriate strategies, which are results of mathematical models which involve malaria. This poster is concerned with the mathematical modelling of the evolution of resistance to insecticides in disease-carrying insects such as mosquitoes. Insecticide resistance is a significant problem and can happen over a short time scale of as little as a few years in regions of intensive use. Killing young adults is not necessarily the right approach , the aim should be to try to delay the onset of resistance without compromising disease control. We aim to formulate a model for an insecticide that works more efficiently against older mosquitoes than young, newly matured adults. We model the effect of such an insecticide by using a function $\boldsymbol{\delta(\alpha)}$ of the age $\boldsymbol{\alpha}$ of a mosquito, that represents per-capita insecticide-induced mortality. As a mosquito ages, its chances of being killed by the insecticide increase. Our present work allows for the possibility of a lot of different characteristics between resistant and non-resistant mosquito strains and also, very importantly , keeps the per-capita insecticide-induced mortality as a general function rather than just a constant value. Our approach involves the formulation of models consisting mainly of integral equations rather than differential equations. We demonstrate results on linear and global stability of zero equilibrium point as well as results on linear stability of positive equilibrium points. We then examine under which specific conditions a late-life acting insecticide will slow down the evolution of mosquitoes towards insecticide resistance. In the second mathematical model we examine a more complex system of equations , that extends the ideas of the previous section to incorporate human population and malaria disease dynamics. Results for the local stability of a malaria-free equilibrium are presented. Last we present sufficient conditions to ensure that malaria is eradicated, leaving open the question of how the variables for the susceptible population behave as time tends to infinity. Those values does not necessarily approach equilibrium values.

Formation and growth of microsilica particles

Raquel Gonzalez Farina - University of Oxford
Robert A. Van Gorder
James Oliver
Andreas Muench
Rolf Birkeland

Microsilica particles, mainly composed of silicon dioxide, are spherical particles smaller than a micrometer generated as the byproduct of silicon furnaces. These particles grow by various mechanisms which are highly dependent on temperature, chemical species concentrations, and pressure. Being able to control the size and quality of the particles is vital since this affects the performance of the material used for numerous applications. The optimal operation of a furnace can result in the creation of particles with desired properties and yield, and motivated by this we derive a mathematical model that relates the local conditions of the furnace to the formation and growth of the microsilica particles. In particular, our model predicts chemical species concentration and temperature changes due to chemical reactions, as well as the particle size distributions, where the growth of the particles is assumed to be purely by condensation. We find that all the chemical reactions happen very close to the bottom of the furnace, and that the particles grow as they move upwards from the reaction zone since there is more silicon dioxide available.

On the interaction between fronts and vortices.

Thibault Jougla - University of St Andrews
Jan-Bert Flor
Jo-Hendrick Thysen

We investigate the formation and evolution of fronts and vortices in a two-layer stably-stratified fluid experiment. In a rotating tank with a differentially rotating rigid lid, we create a vertical shear across a density interface resulting in a baroclinic front. Different instabilities, such as baroclinic, Kelvin-Helmoltz or Rossby-Kelvin instabilities (RK), appear depending on the regime in Froude, Rossby and dissipation numbers. In the mean time, vortices naturally form that interact with the front. Using space-time analyses and Fourier filter analyses on particle Image Velocimetry (PIV) measurements and dye observations (LIF) for the RK and baroclinic unstable regimes, we investigate the interaction between small-waves with the baroclinic life cycle and the formation of these vortices. We characterise their size, the lifetime and the behaviour of vortices depending on the initial flow parameters.

Modelling and analysis of paste flow in high performance Søderberg electrodes

Alissa Kamilova - University of Oxford
Ian Hewitt
Peter Howell
Rolf Birkeland
Bjørnar Larsen

The Søderberg electrode is the most commonly used electrode system, which provides the energy necessary for the production of ferroalloys and calcium carbide. The paste, a mixture of tar binder and calcined anthracite, is the raw material that makes the electrodes. It is added in the form of solid cylinders, briquettes or blocks, while the casing is lowered at a constant rate. The viscosity of the paste is highly sensitive to changes in temperature, causing it to soften, flow, and then bake as the temperature increases in the electrode due to the heat supplied by the fans blowing hot air along the walls and the heat induced by the current from the current clamps.

A coupled fluid flow and heat transfer mathematical model was derived and solved for numerically in Matlab and Comsol to investigate the dependence on the governing parameters for this process. An understanding of the ideal operating conditions under which the correct paste melting profile is produced would prevent fires, explosions, and breakages which have resulted in costs of up to 1 million GBP to the company.

Inferring retention mechanisms from flux--throughput measurements is an ill-posed problem

Armin Krupp - University of Oxford
Ian Griffiths
Colin Please

Consider a particle-laden fluid flowing through a thin porous medium due to an applied fixed pressure gradient. If the particles are retained by the porous medium, they will reduce its permeability and so the flux through the medium will decline as more and more fluid has passed through the porous medium. The decline in flux, dependent on the total fluid that has passed through (throughput), is easy to measure in experiments.

For a given retention mechanism or set of retention mechanisms, the corresponding flux-throughput relationship can be directly computed. Different retention mechanisms, such as internal adhesion or retention at the surface, lead to qualitatively different declines in flux. As the retention typically happens on a scale that is too small to be readily observable, the retention mechanisms are inferred from the measured flux-throughput relationship.

Based on our previous work, we show that, in the limit of infinitesimally small particles, the relationship between the retention mechanisms and the flux-throughput measurements can be described using the Fokker-Planck equation.

The problem of inferring the retention mechanism from flux--throughput measurements thus becomes a coefficient inverse problem, which we show to be ill-posed, rendering the current practice for inferring retention mechanisms as problematic.

Wiener-Hopf problems in fracture mechanics with process zone

Pavlos Livasov - Aberystwyth University
Gennady Mishuris

In many problems of mechanics, especially fracture mechanics, the presence of mixed boundary conditions allows one to apply integral transforms which lead to a Wiener-Hopf problem. This applies to both static problems within a continuous model and dynamic problems, especially when it comes to steady-state regime. In addition, this technique is also effective in the case of discrete problems, which concern both lattice structures composed of masses and connecting springs and structures made of masses and beams. The Wiener-Hopf technique allows us to determine the basic properties of the solution and to identify important physical applications relating to the nature of crack propagation or phase transition. On the other hand, in the case when there is a significant process zone in the vicinity of the crack tip, the application of this method is much more complicated. Sometimes it can be done by reducing to a matrix Wiener-Hopf problem. In the present paper some of these problems are considered. Some numerical examples are presented and discussed.

Degradation of Lithium-ion Batteries

Scott Marquis - University of Oxford

A thorough understanding of the degradation of lithium-ion batteries is essential for their efficient application in industry. Degradation occurs as a result of a large number of complex mechanisms, which are not yet fully understood. Mathematical modelling is an important tool to develop understanding and provide a quantitative description of degradation.

We focus our attention on the growth of the solid--electrolyte interphase (SEI) which is one of the key degradation mechanisms. The SEI is a layer formed on the surfaces of negative electrode particles from unintended side reactions. Lithium is consumed in its formation and therefore capacity fade is observed. Furthermore, the layer impedes the flux of lithium and therefore the internal resistance of the battery increases, leading to power fade.

We model transport within the SEI and the electrochemical reactions on its surfaces. We then assume that the fluxes of species with the SEI are in steady state and develop simple flux--voltage relations for the fluxes of key species within the SEI. Finally, we introduce these relations into a full model of a lithium-ion battery to predict capacity fade.

Is the transition from dipolar to multi-polar numerical dynamos constrained by inertial wave propagation?

Ben McDermott - University of Cambridge
Peter Davidson

Numerical simulations of MHD convection in spherical shells have been surprisingly successful, in the sense that many of the published simulations produce large scale magnetic fields which are predominantly dipolar. The main features of the geomagnetic field are the westward drift, quasi-stationary high latitude flux lobes and strong dipolarity (Jackson et al., 2000); however, over geological time there have been numerous polarity reversals (Glatzmaier and Coe, 2015). The simulations necessarily operate in parameter regimes which are far from reality, and reversals are often found by increasing the forcing, which increases the importance of inertia as measured by the local Rossby number Ro_l. The transition from stable dipolar dynamos to reversing multi-polar dynamos is observed at *local* Rossby number order 0.1 (Christensen and Aubert, 2006; Aurnou et al., 2015).

Research into rotating hydrodynamic turbulence has shown that, for freely decaying rotating turbulence (Baqui and Davidson, 2015; Davidson et al., 2006) and for a localised vortical eddy (Sreenivasan and Davidson, 2008), a relatively abrupt change in dynamics occurs at $Ro = U/2\Omega l_\perp \approx 0.4$. For $Ro < 0.4$ inertial waves dominate the dynamics forming columnar structures reminiscent of Taylor columns, whereas for $Ro > 0.4$ energy is no longer dispersed preferentially along the rotation axis. We hypothesise that quasi-geostrophic structures in dynamo simulations are due to inertial waves, or magnetically influenced inertial waves, which cease to propagate as Ro_l increases beyond $Ro_l \sim 0.4$. Thus a lack of `columnarity' leads to a collapse of the magnetic dipole.

We present direct numerical simulations of the evolution of the velocity field generated by a vertically localised random cloud of buoyancy for a range of Rossby numbers $Ro = U/2\Omega l$, under the influence of a transverse magnetic field. The fluid considered is Boussinesq and incompressible and the computational domain is extended in the axial direction to allow for columnar structure formation. To reduce the effects of viscosity and thermal diffusion the Ekman number $E = Ro/Re$ is kept small— $10^{-2} – 10^{-4}$ based on the transverse integral scale. The Elasser number $\Lambda = (\Omega\tau)^{-1}$ (where τ is the magnetic damping time) is varied to explore the effect of anisotropic dissipation on flow structures and helicity propagation/segregation. The range $Ro = 0–1$ receives particular attention for reasons above, with the length scale based on the dominant length in the source, a the buoyant cloud, and the length scale in the isotropic direction perpendicular to both rotation and the applied field.

Often in studies of spherical shell convection the local length scale used to determine Ro_l is based on the mean spherical harmonic degree. However, recent studies have shown that at more Earth-like parameters convection becomes sheet-like, elongated in the cylindrical radial direction, this suggests the mean spherical harmonic *order* (azimuthal length scale) would be better suited for estimating Ro_l, reflected in our choice of Ro_l.

An asymptotic model of surface elastic waves in an isotropic half-space coated by a thin orthotropic layer.

Ali Mohammed A Mubaraki - Keele University
Danila Prikazchikov

This study is focused on surface elastic wave propagating in an isotropic half-space coated by a thin orthotropic layer subject to prescribed surface stress. The effect of the coating is modelled by means of efficient boundary conditions. A hyperbolic–elliptic model for the Rayleigh wave field is then derived, extending the previous results. The decay away from the surface is described by an elliptic equation, whereas behaviour on the surface is governed by a wave equation, singularly perturbed by a pseudo-differential operator.

A mechanochemical model for cell motility

Laura Murphy - University of Sussex
Anotida Madzvamuse

The way that cells move is key to the creation and development of most organisms on earth. Consequently a deeper understanding of cell motility is likely to have significant applications to medicine.

The aim of this work is to implement an efficient numerical method to study cell deformation and movement. The model we employ considers the actin filament network as a viscoelastic and contractile gel. The mechanical properties are modeled with with a force balancing equation for displacement. This is coupled to two reaction-diffusion equations describing the actin and myosin biochemical dynamics. We carry out linear stability analysis to determine key bifurcation parameters and find analytical solutions close to bifurcation points. We then use a finite element scheme to produce numerical solutions in 2 and 3 dimensions.

We see that the solutions predicted from linear stability theory are replicated in the early stages of movement. Subsequently we see significant deformations, some of which are consistent with shapes found in experiments. The model we describe could be thought of as the initial deformation which leads to movement.

Systematic analysis of a bifurcating model of tumour-immune interactions

Ana Osojnik - University of Oxford
Helen Byrne
Eamonn Gaffney
James Yates
Michael Davies

Due to cancer heterogeneity, outcomes of immunotherapy trials often show a separation of tumour responses into two groups; these are uncontrolled tumour growth vs tumour shrinkage or eradication. Our aim is to determine whether a mechanistic model of tumour-immune interactions with bifurcating switches or bistability between different tumour growth regimes can explain variability in responses observed experimentally. We focus on a simple two-compartment model, describing interactions between tumour and activated immune cells, that was proposed by Kuznetsov et al. (1994), and is frequently encountered in modelling literature. As parameter values are varied the model exhibits tumour escape, dormancy, and eradication; in some parameter regions bistability between tumour escape and dormancy is observed.

This model is a quasi-steady-state approximation (QSSA) of a full five-compartment model that incorporates kinetics of inactivated immune cells, programmed-for-death tumour cells, and conjugates of activated immune and tumour cells, dissociation of which results either in tumour cell programmed death or immune cell inactivation. By reconsidering the model simplifying assumptions, we deduce existence of a small parameter in the full model, and observe a separation of timescales. Using asymptotic methods, we deduce a two-compartment long-timescale model, which is different from the QSSA model, but has equivalent steady states. Numerical solutions of this asymptotically reduced model correspond with those of the full model. Further simulations and linear stability analysis of this model and the QSSA model reveal large differences in typical solution trajectories and the bifurcation structure between the two.

Our analysis suggests that the QSSA does not necessarily preserve model behaviour, and careful application of asymptotic methods is crucial for valid model simplifications. Understanding the behaviour of the asymptotically reduced model, which preserves the modelled mechanistic properties of tumour-immune interactions, will allow us to explore whether this model can explain the dichotomy in tumour responses observed in immunotherapy drug trials.

Measuring magnetic reconnection in the UCLA FLUX rope experiments

Christopher Prior - Durham University

We have developed a new mathematical tool to measure the changing connectivity of magnetic flux ropes. This method is valid no matter how complex the QSL structure of the field. We apply this tool to an experiment of interacting magnetic flux ropes in a laboratory plasma performed at the UCLA basic plasma physics laboratory. In particular we focus on the so called two-moon experiment in which the flux ropes were placed significantly close and the ensuing reconnective activity causes a field to develop with highly complex small scale current structure. This small scale structure had previously hindered attempts to measure the reconnective activity of the system using techniques which require the identification of a clear QSL layer (as was the case in other similar experiments), our new method makes no such requirements. We find the two interacting flux ropes merge to form a single current structure with significant small scale sub-structure. A periodic oscillation in the field's helicity develops which, whilst driven by the ideal kink instability is shown to develop due to non-ideal reconnective activity. We analyse the spatial structure of this periodic reconnective activity to show that the field varies from a coherent flux rope structure to a more complex structure which has some indication of braided entanglement. The conclusion is that the ideal instability drives the loss of coherent structure, this is then is rectified by the ensuing reconnetive rearrangement.

A mathematical model of the chemotherapeutic drug paclitaxel

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Paclitaxel (also known as taxol) is a highly effective anti-mitotic agent. By stabilising the microtubules within the nucleus, it prevents the contraction of the microtubules that is necessary for the separation of the chromosomes. This causes the cell to go into mitotic arrest and potentially apoptosis, hampering the proliferation of a tumour. However, optimal dosage and exposure times for this drug that maximise its effectiveness whilst minimising its unpleasant side effects have yet to be fully established. Such chemotherapeutic drugs take several years and cost many millions of pounds to produce. Hence, a mathematical model that describes the mechanism of paclitaxel could prove very useful to the industry.

A computational model consisting of a system of ODES for the pharmacokinetics of paclitaxel has been devised that takes the following into account: saturable drug binding to extracellular proteins and the containers used in the experiments, the accumulation and efflux of the drug in cells by diffusion across the cell membrane, saturable and nonsaturable binding of the drug to various components inside the cell, and the drug-induced stabilisation and enhancement of polymerised tubulin (microtubules) at high concentrations.

Optimisation algorithms were used to fit the model parameters to experimental data obtained in collaboration with a research group in Dundee’s School of Life Sciences. The most notable results are that paclitaxel has a high binding affinity to microtubules, corresponding to a rapid initial diffusion of paclitaxel into the cell. It was also found that the drug binds noticeably to the containers and medium used in the experiment. Thus, a model has been found that describes the pharmacokinetics of the drug on the level of a single cell and of the evolution of a homogeneous cell population in a uniform drug concentration.

Statistical analysis of the results for the cell mitosis experiments has revealed a significant increase in the average mitosis time and the rate of mitotic arrests of cells at higher concentrations. The distribution of the results thus far indicate that the mitosis phase can be compartmentalised into 3 main phases, with varying concentrations of the drug affecting the transition rate between each phase and the possibility of full mitotic arrest. Also, results from the microfluidic chip experiments has allowed the coupling of a diffusion equation into the system, with the aim of predicting how far the drug can penetrate into a region of tissue.

Observation of phase shift in Soliton-Hump Interaction in the framework of focusing Nonlinear Schrodinger Equation

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The framework of integrable turbulence has been recently recognised as a novel theoretical paradigm of major importance for a broad range of physical application from photonics to oceanography. In particular, in the context of nonlinear optics, the dynamics can be mathematically described by the focusing Nonlinear Schrodinger equation (NLSE). This integrable equation is known to possess soliton solutions and a more peculiar family of breather solutions. Our work is focused on the study of the interaction between these structures. In particular, we investigate the interaction between a "trial" soliton and a broad hump that can eventually evolve into a soliton gas. The macroscopic dynamics of soliton gas is determined by phase shifts occurring in two-soliton interactions. Interaction of a NLSE soliton with radiative component of the solution also leads to a phase shift, which should play a role in the full turbulent NLSE dynamics. To understand the interplay between the two phase shifts, we systematically performed a series of numerical simulations to study the delay of the moving trial soliton due to the interaction with a large hump. We investigated how the phase shift of the soliton is affected by its velocity and the initial position on the one hand, and by the soliton/radiative spectral content of the hump on the other. The curve that describes the soliton delay as function of the spectral content of the hump shows a monotonous behaviour at high velocity when the soliton content of the hump plays the dominant role in the interaction. Instead, at low velocity, the contribution of the radiative content of the hump to the interaction increases and the curve features a non-monotone behaviour with local maxima corresponding to the presence of an integer number of solitons in the hump. We compare our numerical results with analytical predictions.

Non-linear inverse problem related to electrostatic source localization

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Chris Budd

Electric charges in a conducting domain generate field and potential, the potential could be measured by electrodes. In most medical imaging applications the measurement is nonintrusive and thus can only be done on the boundary of the domain, by placing electrodes on the skin of a subject, like for instance the EEG imaging technique that measures electric potential on the scalp using an electrodes hat. To image the source setting from these partial measurements is a nonlinear inverse problem.

In this poster I will establish an analytically tractable problem using the simple geometry of a disk to study the structure of such an inverse problem with the aim of gradually building up the complexity to approach the real-life application of EEG imaging. I will present numerical formulation of the forward map which is established by using Boundary Element Methods, and a classical inverse problem follows by the means of gradient-based minimization with Tikhonov regularization. A key result of the numerical trials is the observation of a limit on the number of poles we can reconstruct, after which the stability of the solution deteriorates. We will present an analytic justification of this result by establishing a closed-form analytical solution for the Neumann boundary value problem. This then enables us to apply Fourier analysis and singular value expansion to probe more accurately into the decaying nature of the solution and explain the observations from the numerical results. For a more general geometry of the domain, we are looking into conformal mapping to extend the analytical solution acquired before.

Baroclinically-driven flows and dynamo action in rotating spherical fluid shells

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The dynamics of stably stratified stellar radiative zones is of considerable interest due to the availability of increasingly detailed observations of Solar and stellar interiors. We reports non-axisymmetric and time-dependent simulations of flows of anelastic fluids driven by baroclinic torques in stably stratified rotating spherical shells - a system serving as an elemental model of a stellar radiative zone. With increasing baroclinicity a sequence of bifurcations from simpler to more complex flows is found in which some of the available symmetries of the problem are broken subsequently. The poloidal component of the flow grows relative to the dominant toroidal component with increasing baroclinicity. The possibility of magnetic field generation thus arises and this paper proceeds to provide some indications for self-sustained dynamo action in baroclinically-driven flows. We speculate that magnetic fields in stably stratified stellar interiors are thus not necessarily of fossil origin as it is often assumed.

Higher-order effective boundary conditions for a coated isotropic elastic half-space

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Higher-order effective boundary conditions for a coated half-space are derived. The approach relies on the method of direct asymptotic integration of equations in elasticity. A comparison with the long-wave asymptotic expansion of the exact solution of the plane time-harmonic problem demonstrates the validity of the proposed formulation. At the same time, the corrections to the simplest leading order effective conditions, obtained previously, are proven to be asymptotically inconsistent, while the other previous results are confirmed and further refined.