EDINBURGH MATHEMATICAL SOCIETY
POSTGRADUATE STUDENTS' MEETING

The Burn, Edzell

16th – 18th May 2006

Organising Committee

Martin Bees (Glasgow)
Christina Cobbold (Glasgow)
Sophie Huczynska (St. Andrews)
Adam McBride (Strathclyde)

Tuesday 16th May

1515	TEA
1600	**Emma Coutts** Mathematical modelling of signal propagation in spiny dendritic tissue
1625	**Miles Gould** Coherence for weakened algebraic theories
1650	**Gavin Dunn** On the evaporation of a liquid droplet *Session Chairs: David Devlin & Dimitra Kosta*
1715	COFFEE
1740	**Caleb O'Loan** An introduction to quantum statistics
1805	**Aminur Miah** Some commutator properties of the generalised wreath product
1830	**Graeme Chalmers** Strong convergence and asymptotic stability of the theta-method for stochastic differential equations with jumps *Session Chairs: Armen Goroyan & Martin Hamilton*
1900	DINNER
2015	**Adam McBride** "Icebreaker" (bring pen, paper & brain. . .)

Wednesday 17th May

0900	BREAKFAST
1000	**Fiona Stewart** The Helfrich-Hurault transition in liquid crystals
1025	**Philipp Reinhard** André-Quillen homology
1050	**Samir Kumar Bhowmik** Numerical analysis of an integro-differential equation
	Session Chairs: David Devlin & James Trimble
1115	COFFEE
1140	**Rosie Williams** Bioconvection in a suspension of green algae
1205	**Yann Peresse** Generating sequences of functions
1230	**Richard Rankin** Non-conforming quadratic finite elements
	Session Chairs: Georgios Papageorgiou & Martin Hamilton
1300	LUNCH
1900	DINNER
2000	**Erik Baurdoux** Optimal stopping games for spectrally negative Levy processes
2025	**Neal Bez** Lebesgue's differentiation theorem along a parabola
2050	**Alan Walker** Periodic disturbances in cylindrically layered SmA
	Session Chairs: Qiming Li & Georgios Papageorgiou

Thursday 18th May

0900	BREAKFAST
1000	**Nicola Armstrong** A continuum approach to modelling cell adhesion
1025	**Richard Vale** The module of parking functions
1050	**Jemma Shipton** Spherical shallow water turbulence *Session Chairs: Jiajia Cui & Ewan Russell*
1115	COFFEE
1140	**Alan Taylor** Generating random networks
1205	**Craig Sooman** An Elementary Introduction to the Direct Method in Soliton Theory
1230	**Fiona Brunk** Intersecting subsets of words *Session Chairs: Mark Sorrell & Dimitra Kosta*
1300	LUNCH

Equipment supervisors: Tuesday & Wednesday am: Jiajia Cui & James Trimble

Equipment supervisors, Wednesday pm & Thursday: Qiming Li & Ewan Russell

Abstracts

A continuum approach to modelling cell adhesion
Nicola Armstrong, Heriot-Watt

Cells adhere to each other through the binding of cell adhesion molecules at the cell surface. This process, known as cell-cell adhesion, is fundamental in many areas of biology, including early embryo development, tissue homoeostasis and tumour growth. In this talk I will discuss a new continuous mathematical model of this phenomena. The model is derived by considering the movement of cells in response to the adhesive forces generated through binding. I will demonstrate that our model predicts the aggregation behaviour of a disassociated adhesive cell population. Further, when the model is extended to represent the interactions between multiple populations, I will show that it is capable of replicating the different types of cell sorting behaviour observed experimentally. The resulting pattern formation is a direct consequence of the relative strengths of self-population and cross-population adhesive bonds in the model. While cell sorting behaviour has been captured previously with discrete approaches, it has not, until now, been observed with a fully continuous model.

Optimal stopping games for spectrally negative Levy processes
Erik Baurdoux, Heriot-Watt

For diffusions (such as Brownian motion) the solution to an optimal stopping game (OSG) can often be found by solving a differential equation. For processes with jumps finding an explicit solution to OSG's is in general more difficult. After discussing some properties of spectrally negative Levy processes we indicate how, for these processes, a solution to a specific OSG can be found.

Lebesgue's differentiation theorem along a parabola
Neal Bez, Edinburgh

If f is in L^p and $p \in (1,\infty]$ then it is known that $\lim_{h\to 0} h^{-1}\int_0^h f(x-(t,t^2))\,dt = f(x)$ for almost every $x \in \mathbb{R}^2$. We shall discuss whether this version of Lebesgue's differentiation theorem along a parabola holds when $p = 1$.

Intersecting subsets of words

Fiona Brunk, St. Andrews

Let $[n]^k$ denote the set of all words of length k over the alphabet $\{1, 2, \ldots, n\}$. Two words $w = w_1 w_2 \ldots w_k, v = v_1 v_2 \ldots v_k \in [n]^k$ are said to *t-intersect* if they agree in at least t positions, that is if there exists $I \subseteq \{1, 2, \ldots, k\}$ with $|I| \geq t$ such that $w_i = v_i$ for all $i \in I$. A subset $\mathcal{F} \subseteq [n]^k$ is said to be t-intersecting if any two words in $\mathcal{F}$ mutually t-intersect.

There has been considerable interest in intersection problems of various combinatorial structures such as words, graphs or sets. My project currently focuses on investigating the structure of t-intersecting subsets of words as described above, particularly when the intersecting sets are of maximal size. For instance, if $\mathcal{F} \subseteq [n]^k$ is 1-intersecting then $|\mathcal{F}| \leq n^{k-1}$, and for $n \geq 3$ the only subsets of maximal size are those where all elements have a fixed position in common. In the talk, I will prove this result and explain the structure of maximal t-intersecting subsets for general t.

Strong convergence and asymptotic stability of the theta-method for stochastic differential equations with jumps

Graeme Chalmers, Strathclyde

An extensive body of research exists pertaining to the convergence and stability of numerical methods in the simulation of ordinary differential equations. This motivates such analyses in the approximation of stochastic differential equations and, more generally, stochastic differential equations with jumps of the type that are now popular in financial modelling.

Traditional finite-time convergence of numerical methods has been studied in a stochastic setting and the use of implicit methods to obtain good qualitative behaviour is now an active topic.

The theta-method class is known to possess good properties in the deterministic case, and extensions to the stochastic case have recently been established.

We examine a general jump-diffusion model, specifically the case where jump magnitudes follow some underlying distribution and explore strong convergence and asymptotic stability issues arising from its numerical approximation.

Such jump-diffusion models are widely used in mathematical finance; as such strong, pathwise simulations are therefore of practical significance (see, for instance, Cont and Tankov [1]). The present work builds upon existing frameworks of Higham [2] and Higham and Kloeden [3].

We derive a strong finite-time convergence result for the theta-method applied to the model, with a convergence rate that depends upon the moment bound for the jump magnitude.

For asymptotic stability, we consider the scalar and multiplicatively linear case, with constant jump magnitude and determine parameterised asymptotic stability regions for the model.

References

[1] R. Cont and P. Tankov (2004), *Financial Modelling With Jump Processes*, Chapman & Hall

[2] D. Higham (2000), *Mean-square and asymptotic stability of the stochastic theta method*, SIAM Journal of Numerical Analysis, 38

[3] D. Higham and P. Kloeden (2006), *Convergence and stability of implicit methods for jump-diffusion systems*, International Journal of Numerical Analysis and Modelling, 3

Mathematical modelling of signal propagation in spiny dendritic tissue

Emma Coutts, Heriot-Watt

There are approximately 100 billion neurons in the human brain. One of these neurons can be connected to thousands of others, and the way in which information propagates through these neural networks is of fundamental importance to the way we experience the world around us. At the single neuron scale the dendritic tree receives signals from other neurons and transmits them to the cell body. Each of these dendrites is covered with small bulbous protrusions called spines which provide a surface area for the synapses of other neurons to connect to. Mathematical modelling can be used to understand the ways in which these spiny dendrites generate and support action potentials and the way in which the intrinsic properties of the dendrite affect signal propagation. A brief description will be given of some existing models. The Baer-Rinzel model is a system of coupled PDEs which describes a cable with a continuum of spines and the Spike-Diffuse-Spike (SDS) model describes a cable with spines attached at discrete points and uses Green's functions to provide an analytical solution for the voltage. Extensions to the SDS model will be discussed, including the effects of tapering in the cable and noise in the system.

On the evaporation of a liquid droplet
Gavin Dunn, Strathclyde

The evaporation of a thin droplet is of fundamental importance to industry with a vast number of applications including ink-jet printing, spray cooling and DNA mapping. A mathematical model for the diffusion-limited evaporation of a thin axisymmetric droplet of liquid whose contact line is pinned by surface roughness (or other) effects is developed and analysed. Our work generalises and extends the work of earlier authors to include additional physical effects, such as the thermal properties of the substrate, and we compare the predictions of our model with recently obtained experimental results.

Coherence for weakened algebraic theories
Miles Gould, Glasgow

Consider the natural numbers $0, 1, 2, \ldots$ and addition. For any p q and r, we have that $(p+q)+r = p+(q+r)$ (associativity) and $p+0 = 0+p = p$ (0 is a unit). Similarly, for $n \times n$ matrices A, B and C, we have that $(AB)C = A(BC)$ and $IA = AI = A$. We summarise these statements by saying that $(\mathbb{N}, +, 0)$ and $(M_n(\mathbb{R}), ., I)$ are *monoids*. The theory of monoids is one of a large number of such theories: other examples are the theories of groups, rings, vector spaces, Lie algebras, rigs, crossed monoids, semigroups ...

In order to save effort, and to bring some order to this chaos, the field of universal algebra was invented. Universal algebra attempts to work with arbitrary algebraic theories, and prove general results that hold in all such cases.

In the last forty years, it has become increasingly apparent that the traditional "one-dimensional" theories are only the tip of a much larger iceberg, which is called *higher-dimensional algebra*. Higher-dimensional analogues of monoids, groups, Lie algebras, etc, have all been defined, in which the defining equations only hold "up to isomorphism". For instance, consider three sets, A, B and C. The repeated Cartesian product $A \times (B \times C) = \{(a, (b, c)) : a \in A, b \in B, c \in C\}$ is not equal to $(A \times B) \times C = \{((a, b), c) : a \in A, b \in B, c \in C\}$, but they are isomorphic in an obvious way, and similarly $\{*\} \times A = \{(*, a) : a \in A\} \cong A$. We say that **Set** is a *weak monoidal category*, but we could say that it's a higher-dimensional sort of monoid.

Defining these higher-dimensional structures one at a time is far too much effort: what's needed is a higher-dimensional sort of universal algebra, that can take a one-dimensional theory and give us its higher-dimensional analogue. Unfortunately, this is less easy than it sounds. We can't just replace sets with categories, functions with functors and equations with natural isomorphisms: to get a useful theory, the isomorphisms themselves must satisfy equations. Finding these equations is an example of a coherence problem, and I am trying to solve the general coherence problem of this type.

Numerical analysis of an integro-differential equation
Samir Kumar Bhowmik, Heriot-Watt
Consider the integro-differential equation

$$u_t(x,t) = \epsilon \int_\Omega J(x-y)(u(y,t)-u(x,t))dy + f(u),$$

with initial condition $u(x,t_0) = u_0(x)$, $x \in \Omega$, $t > 0$, $\Omega \subseteq \mathbb{R}^d$ with $d = 1,2,3$, $\epsilon > 0$, $J(x) = J(-x)$ and $f(u)$ is a bistable function. This type of initial value problem (IVP) arises in the modelling of various physical and biological processes. We study the numerical analysis of this IVP and also investigate its long time dynamics.

Some commutator properties of the generalised wreath product
Aminur Miah, Edinburgh
The (permutational) wreath product of two permutation groups arises naturally in mathematics and other areas of science. The study of wreath products is pretty extensive. Many algebraists use the wreath product as a tool to construct groups with interesting properties. Dixon and Fournelle generalise the wreath product. I will present this generalisation and deduce some commutator properties of it.

An Introduction to Quantum Statistics
Caleb O'Loan, St. Andrews
Quantum Statistics is an interesting research field that has made rapid progress since 1990. It has deep roots in the foundation of Quantum Physics and Mathematical statistics.

Quantum Systems give rise to probability distributions and so statistical problems occur naturally in the quantum context. I will talk about the basic mathematical foundations of the subject and some key ideas such as quantum information.

Generating sequences of functions

Yann Peresse, St. Andrews

In 1935 W. Sierpinski showed that if X is an infinite set then every countable family of functions from X to itself can be obtained as the composition of two such functions. We will explore the principal ideas behind the Theorem and it's proof and consider equivalent results about continuous, increasing, differentiable and measurable functions.

Non-Conforming Quadratic Finite Elements

Richard Rankin, Strathclyde

We will give an overview of the use of non-conforming quadratic finite elements to approximate elliptic partial differential equations.

André-Quillen Homology

Philipp Reinhard, Glasgow

In my talk I would like to discuss generalised Eilenberg-Steenrod axioms to define homology theories in other contexts than topological spaces, where the notion of a model category will substitute the definition of homotopy for spaces. I'll present how to define the André-Quillen homology, which is such an example for the categories of commutative and non-commutative rings.

A novel method for modelling geophysical flows

Jemma Shipton, St. Andrews

Atmospheric motion can be broadly categorized into two types: slow 'balanced' motion and relatively fast 'imbalanced' wave motion. However, these types of motion are inextricably linked and this poses a serious problem for numerical models. The observed motion of the atmosphere suggests that it is in a state of near balance which means that inertia-gravity waves do not make a significant contribution to the overall pattern of the flow. The flow properties are instead largely governed by the potential vorticity field. It is clear therefore that an accurate representation of the potential vorticity field is a requirement for any atmospheric model. What is perhaps not so clear is that an accurate representation of the imbalanced motion is also essential as any numerical generation of imbalance can soon swamp the flow. One way to deal with this is to introduce a damping term into the equations. However, this has drastic repercussions on accuracy.

An important property of the potential vorticity (PV) field is that, in the absence of forcing, it is materially conserved; i.e. it is observed following the flow. This property is not exploited by traditional numerical methods. The PV field has a tendency to develop fine scales and sharp gradients which are difficult to capture in numerical simulations as it is hard to retain this information whilst filtering out numerical imbalance.

The Contour Advective Semi-Lagrangian (CASL) algorithm exploits the different nature of each variable field. PV conservation is ensured since the PV field is held as a set of contours which are merely advected by the flow. This configuration allows for high resolution of the filamentary PV field without the inconvenience of storing the velocity field, which is typically much smoother, at an unnecessarily high resolution. Recent developments have enabled further exploitation of flow properties. By appropriate choice of variables it is possible to separate, to leading order, the balanced, PV controlled motion from the residual 'imbalanced' motion.

This talk will present the setup of the CASL algorithm in the context of the shallow water equations on a sphere, along with some results rom recent turbulence simulations. This work has important implications for short and medium range forecasting and for the initialization of numerical models.

An Elementary Introduction to the Direct Method in Soliton Theory
Craig M. Sooman, Glasgow

There are many nonlinear evolution equations with soliton solutions. Since the discovery of the soliton in 1965, various methods have been developed to find soliton solutions of these equations. In 1971, the Japanese mathematician Ryogo Hirota proposed the direct method for finding exact solutions. This involves transforming the original equation into a bilinear equation by using a new differential operator, the D-operator. The prototype for this theory is the Korteweg-de Vries equation (KdV equation). In this talk, I would like to show how to find soliton solutions of the KdV equation using the direct method. In particular, I will show how to find the one- and two-soliton solutions of the KdV equation. Along the way, I will show some animations of these soliton solutions.

The Helfrich–Hurault transition in liquid crystals

Fiona Stewart, Strathclyde

In the classical Helfrich–Hurault transition for non-chiral SmA liquid crystals, the director **n** is always assumed to be parallel to the smectic unit layer normal **a**. We examine theoretically the onset of the Helfrich–Hurault transition in SmA using a recent model which allows **n** to weakly decouple from **a**. This model has been further developed to incorporate nonlinear dynamics using the theory developed by I. W. Stewart (to be published).

We consider a planar aligned sample of SmA liquid crystal under the application of an electric field **E**, applied in the direction parallel to the smectic layers. Beyond some critical threshold for the electric field the layers desire to distort and undulate, see Figure 1. There are two novel features to the model introduced here which allow a more refined dynamical investigation of such distortions: (i) the Oseen constraint ($\nabla \times \mathbf{a} = 0$) is not imposed, (ii) a coupling term $B_1(1 - (\mathbf{n} \cdot \mathbf{a})^2)$, $B_1 > 0$, has been incorporated to reflect the inclination of the sample to prefer the SmA phase.

We show that the classical critical threshold E_c is modified to a reduced value which converges to the classical value as $B_1 \to \infty$. Furthermore we identify a mechanism for measuring B_1 experimentally.

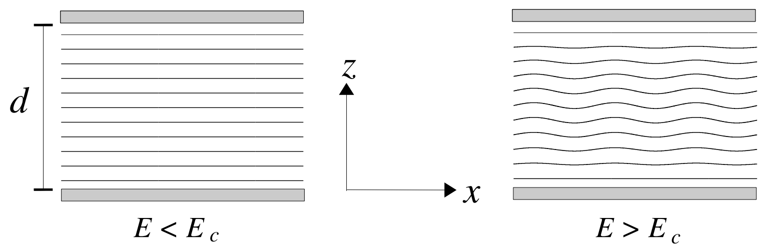

Figure 1: A schematic diagram of the undulations of the smectic layers under the influence of an electric field.

Generating random networks

Alan Taylor, Strathclyde

Large, sparse networks appear across a variety of scientific disciplines, including bioinformatics, computer science and the social sciences.

Investigating such networks typically reduces to tasks in matrix computation, and these problems are challenging due to the size and lack of regular structure in the resulting matrices. In order to test the robustness and efficiency of matrix computation methods, it is desirable to have a large number of synthetic test matrices that capture key properties of real life systems. The random network toolbox for MATLAB, under development by Taylor and Higham, implements various random graph models, that is, algorithms for randomly assigning edges in a network, that aim to produce realistic connectivity patterns.

The history of random graph models dates back to the seminal 1950s work by Erdös and Rényi, and received substantial impetus from a 1998 paper in Nature by Watts and Strogatz that introduced the concept of a *small world* network and a 1999 paper in Science by Barabási and Albert that focuses on the *scale free* property.

In this talk, we will provide some background into random graph models and then discuss their efficient implementation in MATLAB.

The module of parking functions

Richard Vale, Glasgow

I will describe a fun combinatorial object called the module of parking functions, and explain how it relates to representation theory, in particular to the celebrated $(n+1)^{n-1}$ Theorem of Haiman.

Periodic Disturbances in Cylindrically Layered SmA

Alan J. Walker, Strathclyde

The dynamic equations for cylindrically layered SmA liquid crystals are considered. We aim to see how the sample distorts under an imposed periodic disturbance. The stability of such a system is studied; anticipated physical considerations suggest a stable, decaying perturbation and this shall be confirmed mathematically.

We first introduce the dynamical equations for any curvilinear coordinate system using work by I. W. Stewart (to be published), and use energy formulations and descriptions from various liquid crystal literature. The relevant dynamic equations in cylindrical coordinates are then derived using the theory from Stewart and a decaying perturbation ansatz is introduced for the velocity vector $\mathbf{v}$, pressure p and undulation u of the form

$$\begin{pmatrix} v_x \\ v_z \\ p \\ u \end{pmatrix} = \begin{pmatrix} f_x(r,z) \\ f_z(r,z) \\ f_p(r,z) \\ f_u(r,z) \end{pmatrix} \exp(\omega t + i q_\theta \theta),$$

as considered in similar fluid mechanics literature.

The individual solutions for v_x, v_z, p and u are derived along with a restriction for the decay parameter ω. Stability is confirmed and the nature of the disturbed SmA layers is revealed.

A discussion of future work on decoupling the layer normal $\mathbf{a}$ and director $\mathbf{n}$ is given, along with the possibility of using the technique introduced by Stewart for a time-dependent stability analysis of parabolic and Dupin cyclides.

Bioconvection in a suspension of green algae

Rosie Williams, Glasgow

Bioconvection refers to the phenomenon of pattern formation in shallow suspensions of motile microorganisms at constant temperature. External influences such as light intensity may influence pattern formation, causing a taxis response in the cells i.e causing them to move in a certain direction. Here, I outline the previous models used for describing this phenomenon in suspensions of green algae (*Chlamydomonas spp.*) with a view to modelling the effects of gyrotaxis (a balance between gravitational and viscous effects)combined with phototaxis (directional movement in response to light). I will propose further modifications and improvements that I intended to incorporate in a new, rational model.

Participants

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