INTEGRABILITY IN ALGEBRA, GEOMETRY AND PHYSICS: NEW TRENDS

ALEXANDER VESELOV'S 60TH BIRTHDAY

CONGRESSI STEFANO FRANSCINI, SWITZERLAND
13-17 JULY, 2015

Scientific Committee:

Boris Dubrovin
Rei Inoue
Andrei Okounkov

Organising Committee:

Yuri Berest
Alexei Bolsinov
Oleg Chalykh
Misha Feigin
Giovanni Felder
Marta Mazzocco

Conference Schedule

Monday, July 13		
7:30-9:00	Breakfast	
9:00-9:15	Registration	
9:15-10:05	Igor Krichever	*Elliptic families of solutions of the KP equation and compact cycles in the moduli space of algebraic curves*
10:05-10:55	Toshitake Kohno	*Holonomy of braids and its 2-category extension*
10:55-11:25	Coffee break	
11:25-12:15	Victor Buchstaber	*Toric Topology and Combinatorics of Fullerenes*
12:30-14:00	Lunch	
14:45-15:00	Presentation of the CSF	
15:00-15:50	Alexander Its	*Connection problem for the tau-function of Sine-Gordon reduction of Painlevé-III*
15:50-16:20	Coffee break	
16:20-17:10	Alberto Grünbaum	*The bispectral problem as a source of non-commutative algebras of differential operators with matrix coefficients*
18:30-19:30	Reception and wine	
19:30-21:00	Dinner	
Tuesday, July 14		
7:30-9:00	Breakfast	
9:00-9:50	Alexander Bobenko	*What one can build from circles*
9:50-10:40	Evgeny Ferapontov	*On the integrability in Grassmann geometries*
10:40-11:10	Coffee break	
11:10-12:00	Alexei Penskoi	*Geometric optimization of the eigenvalues of the Laplace operator and mathematical physics*
12:30-14:00	Lunch	
15:00-15:50	Hovhannes Khudaverdian	*Compensating field in odd Laplacian*
15:50-16:20	Coffee break	
16:20-17:50	Poster session	
19:30-21:00	Dinner	

Wednesday, July 15		
7:30-9:00	Breakfast	
9:00-9:50	Sergey Novikov	*Singular solitons and isospectral deformations*
9:50-10:40	Boris Dubrovin	*On Witten–Kontsevich tau-function and its generalisations*
10:40-11:10	Coffee break	
11:10-12:00	Pavel Etingof	*Minimal support modules over Cherednik algebras and applications-I*
12:30-14:00	Lunch	
14:00-19:30	Free afternoon	
19:30-21:00	Dinner	
Thursday, July 16		
7:30-9:00	Breakfast	
9:00-9:50	Pavel Etingof	*Minimal support modules over Cherednik algebras and applications-II*
9:50-10:40	Rei Inoue	*Double affine symmetric group action on the toric network and generalized disctete Toda lattice*
10:40-11:10	Coffee break	
11:10-12:00	Alexander Sergeev	*Quantum Calogero-Moser systems and symmetric Lie superalgebras*
12:30-14:00	Lunch	
15:00-15:50	Konstantin Khanin	*On KPZ universality: when universality meets integrability*
15:50-16:20	Coffee break	
16:20-17:10	Tamara Grava	*Normal matrix models and orthogonal polynomials*
19:30-21:00	Dinner	
Friday, July 17		
7:30-9:00	Breakfast	
9:00-9:50	Leon Takhtajan	*Kähler metrics on the moduli space of punctured Riemann surfaces*
9:50-10:40	Leonid Chekhov	*Quantum cluster algebras from geometry*
10:40-11:10	Coffee break	
11:10-12:00	Sergey Dobrokhotov	*On the phase shift in the Kuzmak-Whitham ansatz*
12:30-14:00	Lunch	

Abstracts of contributions

What one can build from circles

A. Bobenko (TU Berlin)

We present several geometric constructions based on circles. These include discrete versions of confocal conics, discrete Willmore energy, conformal parametrization of closed surfaces. Analytic description of these geometries probably leads to 1D and 2D discrete integrable systems.

Toric Topology and Combinatorics of Fullerenes

V. M. Buchstaber (Steklov Mathematical Institute, Moscow)

One of the main objects of toric topology is the moment-angle functor. It assigns to each simplicial complex K a space Z_K with an action of a compact torus T^m, whose orbit space Z_K/T^m can be identified with the cone CK over K.

In the case when K is dual complex of an n-dimensional simple polytope P with m facets, the moment-angle complex Z_K has the structure of a smooth manifold Z_P with a smooth action of T^m, and the quotient Z_P/T^m can be identified with P itself.

A mathematical fullerene is a 3-dimensional simple polytope with only 5-gonal and 6-gonal 2-faces. In this case the number of 5-gonal 2-faces is 12 and there exists a fullerene with any number p_6 of 6-gonal 2-faces except $p_6 = 1$. The number of combinatorially different fullerenes with given p_6 grows fast as p_6 increases. The problem of classifying mathematical fullerenes became important after discovery of fullerenes, molecules of carbon which play important role in physics, chemistry, engineering and nanotechnology. Thanks to toric topology, we can assign to each fullerene P its moment-angle manifold Z_P. The cohomology ring $H^*(Z_P)$ is a combinatorial invariant of the fullerene P.

In our talk we shall focus upon results on the rings $H^*(Z_P)$ and their applications based on geometric interpretation of cohomology classes and their products. The multigrading in the ring $H^*(Z_P)$, coming from the construction of Z_P, plays an important role here.

The talk is based on joint works with Taras Panov and Nickolay Erohovets.

Quantum cluster algebras from geometry

L. O. Chekhov (Steklov Mathematical Institute, Moscow), M. Mazzocco

Our starting point is the geometric picture of degenerations of Riemann surfaces occurring when colliding two holes (or sides of the same hole): as the result, we obtain geodesic laminations comprising both closed curves and arcs starting and terminating at decorated bordered cusps obtained in the process of confluence of hole(s). The arcs can be identified with lambda-lengths, or cluster variables. We provide explicit coordinatization using special shear coordinates of the limiting theory. We show how skein relations and Goldman brackets for original geodesic functions transform into the Ptolemy relations for arcs in this limit. Having at least one bordered cusp we can establish a 1-1 correspondence between shear coordinates and lambda-lengths (cluster variables). We show that the induced Poisson and quantum relations for lambda-lengths are exactly quantum cluster algebras by Berenstein and Zelevinsky. We also solve the problem of quantum ordering in the expressions for quantum lambda-lengths. As a particulr case of this construction we obtain cluster algebras corresponding to the Painlevé equations.

On the phase shift in the Kuzmak-Whitham ansatz

S. Yu. Dobrokhotov (Ishlinskii Institute for Problems in Mechanics, Moscow *and* Moscow Institute of Physics and Technology).

We consider one-phase (formal) asymptotic solutions in the Kuzmak–Whitham form for the nonlinear Klein–Gordon equation and for the Korteweg– de Vries equation. In this case, the leading asymptotic expansion term has the form $X(S(x,t)/h + \phi(x,t), I(x,t), x, t) + O(h)$, where h is a small parameter and the phase $S(x,t)$ and slowly changing parameters $I(x,t)$ are to be found from the system of averaged Whitham equations. The equation for the phase shift $\phi(x,t)$ is appearing by studying the *second-order* correction to the leading term. The corresponding procedure for finding the phase shift is then nonuniform with respect to the transition to a linear (and weakly nonlinear) case. Our observation, which essentially follows from papers by R.Haberman and collaborators, is that if one incorporates the phase shift $\phi(x,t)$ into the phase and adjust I by setting $S \to S + h\phi + O(h^2)$, $I \to I + hI_1 + O(h^2)$, then the new functions $S(x,t,h)$ and $I(x,t,h)$ become solutions of the Cauchy problem for the *same* Whitham system but with modified initial conditions. These functions completely determine the leading asymptotic term in the Whitham method.

This work was done together with D.S.Minenkov.

On Witten–Kontsevich tau-function and its generalisations

B. Dubrovin (SISSA, Trieste)

A simple procedure for computing logarithmic derivatives of tau-function of any solution to the KdV hierarchy will be presented. As an application one obtains an

efficient algorithm for computation of the intersection numbers of psi-classes on the moduli spaces of stable algebraic curves of an arbitrary genus. A generalisation of the above construction to an arbitrary simple Lie algebra will also be given. The talk is based on a joint work with Marco Bertola and Di Yang.

Minimal support modules over Cherednik algebras and applications

P. Etingof (MIT, Boston)

I will consider modules over the rational Cherednik algebra from category $\mathcal{O}$, i.e., finitely generated over the subalgebra of polynomials and with locally nilpotent action of Dunkl operators. With such a module one can associate its support, a closed subvariety of the affine space. A simple module is said to have minimal support if there is no nonzero modules with smaller support.

Modules with minimal support are interesting from many points of view. First of all, they are Cohen-Macaulay modules over the polynomial subalgebra. This allows one to prove the Cohen-Macaulay property of many rings of quasiinvariants - specifically, rings of quantum integrals of deformed Calogero-Moser systems considered by O. Chalykh, M. Feigin, A. Sergeev, and A. Veselov, as well as some generalizations. Also, characters of minimally supported modules in type A have nice explicit formulas, which allows one to compute Hilbert series of quasiinvariant rings. The same character formulas can be used to compute Hilbert series of multiplicity spaces in equivariant D-modules on the nilpotent cone for the Lie algebra $\mathfrak{sl}_m$. Finally, these modules arise in the computation of the HOMFLY polynomial of the torus knot, and allow one to prove its positivity properties.

This is based on joint work with I. Losev, E. Gorsky, and E. Rains.

Lecture 1. Minimally supported modules and their characters, with application to D-modules and knot invariants.

Lecture 2. Minimally supported modules and Cohen-Macaulay properties of quasiinvariant rings.

On the integrability in Grassmann geometries

E. Ferapontov (Loughborough University)

Let V be vector space of dimension n, and $G(d, n)$ the Grassmannian of d-dimensional subspaces of V. Any subvariety X in $G(d, n)$ gives rise to a differential system $S(X)$ that governs d-dimensional submanifolds of V whose Gaussian image is contained in X. We address the problem of geometric characterisation of integrable systems of the form $S(X)$ for a particularly interesting case of fourfolds in $G(3, 5)$.

This is based on joint work with B. Doubrov, B. Kruglikov, and V. Novikov.

Normal matrix models and orthogonal polynomials

T. Grava (SISSA, Trieste)

We study the asymptotic behaviour of orthogonal polynomials related to normal matrix models with exponential weights with discrete rotational symmetry. We obtained explicit relations between the behaviour of the distribution of the zeros of the orthogonal polynomials and the limiting eigenvalues distribution of the normal matrix models. Such relations are in agreement with existing results in the literature.

The bispectral problem as a source of non-commutative algebras of differential operators with matrix coefficients

F. A. Grünbaum (UC Berkeley)

The bispectral problem was started many years ago in joint work with Hans Duistermaat. It has unexpected connections with integrable systems, monodromy properties, and many other issues.

An appropriate formulation for the case of matrix valued wave functions gives rise to non-commutative algebras of differential operators with a common family of eigenfunctions. Concrete examples will be given in the case of Calogero–Moser Spin systems. What is missing is a description of the algebra that replaces the algebraic-geometric picture in the scalar valued case, and any assistance is welcome.

I will start with a very brief introduction to the bispectral problem originating in connection with "time-and-band limiting" a problem posed by Claude Shannon.

Double affine symmetric group action on the toric network and generalized disctete Toda lattice

R. Inoue (Chiba University)

We define generalizations of discrete Toda lattice, by using the networks on a torus. Each such system has a family of commuting rational maps, which comes from two affine symmetric group action on the network. We describe the maps in terms of algebro-geometrical data, and study the initial value ploblem. It is based on a joint work with Thomas Lam and Pavlo Pylyavskyy.

Connection problem for the tau-function of Sine-Gordon reduction of Painlevé-III

A. Its (Indiana University Purdue University Indianapolis)

We evaluate explicitly, in terms of the Cauchy data, the constant pre-factor in the large x asymptotics of the Painlevé III tau-function. Our result proves the conjectural formula for this pre-factor obtained recently by O. Lisovyy, Y. Tykhyy, and the speaker with the help of the recently discovered connection of the Painlevé tau-functions with the Virasoro conformal blocks. In the talk an alternative approach to the "constant of integration problem", which is based on the Riemann-Hilbert method, will be presented. This is joint work with A. Prokhorov.

On KPZ universality: when universality meets integrability

K. Khanin (University of Toronto)

We shall introduce the phenomenon of KPZ (Kardar-Parisi-Zhang) universality. This is an extremely general set of scaling predictions for many systems in 2D disordered environment. Connections to integrable systems were actively studied in the last 5 years. This allowed to obtain many beautiful exact formulae for the limiting probability distributions. At the same time the universality problem remains largely open.

Compensating field in odd Laplacian

H. Khudaverdian (University of Manchester)

A second order operator Δ can be uniquely defined by its principal symbol S and potential U, if it acts on half-densities. The potential U is a second order compensating field, (second order connection). It compensates (gauges) the action of diffeomorphisms on the second derivatives in an operator Δ in the same way as an affine connection compensates the action of diffeomorphisms on first derivatives in the first order operator, a covariant derivative.

We consider cases of Riemannian and odd Poisson supermanifolds. If an even principal symbol S defines Riemannian structure, then one can uniquely define compensating field U via Levi-Civita connection of Riemannian metric. There is no Levi-Civita connection in a case if an odd principal symbol S defines an odd Poisson structure on supermanifold. However in this case under some retrictions (in particular if Poisson structure is symplectic one) one comes to unique compensating field.

Holonomy of braids and its 2-category extension

T. Kohno (The University of Tokyo)

We describe a generalization of holonomy representations of braid groups to higher categories. The 2-categories consist of objects, morpshims and 2-morphisms for

any pair of morpshims. Using a method of formal homology connection due to K.-T. Chen, we construct a 2-functor from the path 2-groupoid of the configuration spaces. This construction gives representations of cobordism categories of braids.

Elliptic families of solutions of the KP equation and compact cycles in the moduli space of algebraic curves

I. Krichever (Columbia University & Higher School of Economics, Moscow)

Using constructions of the theory of elliptic families of solutions of the KP equations a new bound on the dimension of complete cycles in the moduli space of stable curves of compact type is obtained.

Ruijsenaars type deformation of hyperbolic BC_n Sutherland model

I. Marshall (Higher School of Economics, Moscow)

Hamiltonian reduction is used to project a trivially integrable system on the Heisenberg double of $SU(n,n)$ to obtain s system of Ruijsenaars type on a suitable quotient space. This system possesses BC_n symmetry and is shown to be equivalent to the standard three parameter BC_n hyperbolic Sutherland model in the cotangent bundle limit.

Singular solitons and isospectral deformations

S. P. Novikov (University of Maryland & Russian Academy of Sciences (Steklov and Landau Institutes))

Smooth solutions to the famous KdV equation define isospectral deformations of Schrodinger Operator which is a base of Inverse Scattering Transform and Theory of Periodic Finite-Gap (Algebrogeometric) Solutions. A lot of exact singular solutions to KdV system is known. Do they define anything like isospectral deformation? Our joint works with Peter Grinevich made in the last years answer that question.

New commuting differential operators of rank 2 and arbitrary genus

V. Oganesyan (Moscow State University)

New commuting differntial operators of rank 2 and arbitrary genus and their common eigenfunctions will be presented. In some cases we will consider an explicit characterization of commuting differential operators of rank 2.

Geometric optimization of the eigenvalues of the Laplace operator and Mathematical Physics

A. Penskoi (Moscow State University)

The problem of geometric optimization of the eigenvalues of the Laplace operator goes back to the famous book "The theory of sound" by Lord Rayleigh. The Rayleigh's question was about finding such a shape of a drum that it produces the lowest possible sound among all the drums of the same area. It turns out that this question and its generalizations are a very important part of Spectral Geometry, connected with many branches of mathematics, including minimal surfaces, classical equations of mathematical physics and many others.

Deformations of non semisimple Poisson pencils of hydrodynamic type

A. D. Vedova, P. Lorenzoni and **A. Savoldi** (Loughborough University)

We study deformations of two-component non semisimple Poisson pencils of hydrodynamic type associated with Balinskiĭ-Novikov algebras. We show that in most cases the second order deformations are parametrized by two functions of a single variable. It turns out that one function is invariant with respect to the subgroup of Miura transformations preserving the dispersionless limit and another function is related to a one-parameter family of truncated structures. In two expectional cases the second order deformations are parametrized by four functions. Among them two are invariants and two are related to a two-parameter family of truncated structures. We also study the lift of deformations of n-component semisimple structures. This example suggests that the deformations of non semisimple pencils corresponding to the lifted invariant parameters are unobstructed.
Reference: `arXiv 1506.02309`

Quantum Calogero-Moser systems and symmetric Lie superalgebras

A. Sergeev (Saratov State University & National Research University Higher School of Economics)

The representation thory of symmeric Lie super-algebras and corresponding spherical functons are studied in relation with the theory of deformed quantum Calogero-Moser sysems. In the special case of the symmetric pair $\mathfrak{g} = \mathfrak{gl}(n,m)$, $\mathfrak{p} = \mathfrak{osp}(n,2m)$ we establish a natural bijection between projective covers of spherically typical irreducible $\mathfrak{g}$-modules and the finite-dimensional generalized eigenspaces of the algebra of Calogero-Moser integrals $D_{n,m}$ acting on the correspnding quasi-invariants $A_{n,m}$.

A survey on the integrability theorem

J. Singh (Tata Institute of Fundamental Research, Mumbai)

In this talk, we tell the story of one of the most beautiful theorems of D-module theory, known as integrability theorem which roughly says that the singular support of a D-module on a smooth algebraic manifold is an involutive subvariety in the cotangent bundle. We also discuss the importance of its relationship to symplectic geometry and mathematical physics.

On Darboux integrability of discrete 2D Toda lattices

S. V. Smirnov (Moscow State University)

It is widely known that two-dimensional Toda lattices corresponding to simple Lie algebras are Darboux integrable, that is, they admit complete families of essentially independent integrals along both characteristics. These generalized Toda lattices are particular cases of the so-called exponential systems. Recently discrete and semidiscrete analogs of exponential systems were introduced by Habibullin and collaborators.

Darboux integrability of semidiscrete and purely discrete exponential systems corresponding to the Cartan matrices of the series A and C is proved. Complete families of integrals along both characteristics are given explicitly. Existence of integrals appears to be a direct consequence of the nature of two-dimensional Toda lattice related to Darboux-Laplace transformations. The same approach provides complete families of integrals in the continuous case as well as in the (semi)discrete one.

Kähler metrics on the moduli space of punctured Riemann surfaces

L. Takhtajan (Stony Brook University)

I will review WP and TZ metrics on the moduli space of punctured Riemann surfaces and discuss a potential for a special combination of these metrics. This is a joint work with J. Park and L.P. Teo.

List of Participants

Yuri Berest	*Cornell University, USA* berest@math.cornell.edu
Alexander Bobenko	*TU Berlin, Germany* bobenko@math.tu-berlin.de
Victor Buchstaber	*Steklov Mathematical Institute, Russia* buchstab@mi.ras.ru
Pierpaolo Calligaris	*Loughborough University, UK* P.Calligaris@lboro.ac.uk
Oleg Chalykh	*University of Leeds. UK* o.chalykh@leeds.ac.uk
Leonid Chekhov	*Steklov Mathematical Institute, Russia* chekhov@mi.ras.ru
Sergey Dobrokhotov	*Ishlinskii Institute for Problems in Mechanics, Moscow* *Moscow Institute of Physics and Technology, Russia* dobr@ipmnetnet.ru
Boris Dubrovin	*SISSA, Italy* dubrovin@sissa.it
Pavel Etingof	*MIT, USA* etingof@math.mit.edu
Misha Feigin	*University of Glasgow, UK* misha.feigin@glasgow.ac.uk
Giovanni Felder	*ETH Zürich, Switzerland* felder@math.ethz.ch
Evgeny Ferapontov	*Loughborough University, UK* E.V.Ferapontov@lboro.ac.uk
Vasily Goncharenko	*Financial University, Moscow, Russia* vasgon72@yandex.ru
Tamara Grava	*SISSA, Italy* grava@sissa.it
F. Alberto Grünbaum	*UC Berkeley, USA* grunbaum@math.berkeley.edu
Ashish Gupta	*Indian Institute for Science Education and Research, Bhopal, India* a0gupt@gmail.com
Davide Guzzetti	*SISSA, Italy* guzzetti@sissa.it

Rei Inoue — *Chiba University, Japan*
reiiy@math.s.chiba-u.ac.jp

Alexander Its — *Indiana University Purdue University Indianapolis, USA*
itsa@math.iupui.edu

Elizabeth Its — *Indiana University Purdue University Indianapolis, USA*
liza@math.iupui.edu

Konstantin Khanin — *University of Toronto, Canada*
khanin@math.toronto.edu

Hovhannes Khudaverdian — *University of Manchester, UK*
khudian@manchester.ac.uk

Alastair King — *University of Bath, UK*
a.d.king@bath.ac.uk

Toshitake Kohno — *University of Tokyo, Japan*
kohno@ms.u-tokyo.ac.jp

Igor Krichever — *Columbia University, USA & HSE, Moscow, Russia*
krichev@math.columbia.edu

Ievgen Makedonskyi — *Higher School of Economics, Moscow, Russia*
makedonskii_e@mail.ru

Ian Marshall — *Higher School of Economics, Moscow, Russia*
imarshall@hse.ru

Marta Mazzocco — *Loughborough University, UK*
m.mazzocco@lboro.ac.uk

Dario Merzi — *SISSA, Italy*
dmerzi@sissa.it

Alexander Mikhailov — *University of Leeds, UK*
a.v.mikhailov@gmail.com

Alexander Molev — *University of Sydney, Australia*
alexander.molev@sydney.edu.au

Martin Müller-Lennert — *ETH Zürich, Switzerland*
martin.mueller-lennert@math.ethz.ch

Sergey Novikov — *University of Maryland, USA*
Russian Academy of Sciences (Steklov and Landau Institutes)
novikov@umd.edu

Vardan Oganesyan — *Moscow State University, Russia*
vardan.o@mail.ru

Maxim Pavlov	*Lebedev Physical Institute, Moscow, Russia* maxim.pavlov@mtu-net.ru
Alexei Penskoi	*Moscow State University, Russia* alexei.penskoi@gmail.com
Andrea Savoldi	*Loughborough University, UK* a.savoldi@lboro.ac.uk
Alexander Sergeev	*Saratov State University, Russia* *National Research University Higher School of Economics, Russia* sergeevan@info.sgu.ru
Jyoti Singh	*Tata Institute of Fundamental Research, India* jyotijagrati@gmail.com
Sergey Smirnov	*Moscow State University, Russia* ssmirnov@higeom.math.msu.su
Ian Strachan	*University of Glasgow, UK* ian.strachan@glasgow.ac.uk
Leon Takhtajan	*Stony Brook University, USA* leontak@math.sunysb.edu
Alexander Veselov	*Loughborough University, UK* *Moscow State University, Russia* A.P.Veselov@lboro.ac.uk
Alexander Vitanov	*ETH Zürich, Switzerland* alexander.vitanov@math.ethz.ch

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