Classical and Quantum Integrability meeting

Glasgow 11-12 March 2016

Programme

Friday 11th March 2016

1:00 – 2:00pm Alexander Veselov (Loughborough)

2:00 – 3:00pm Ralph Willox (Tokyo)

3:00 – 3:30pm *Tea/Coffee break*

3:30 – 4:30pm Uwe Grimm (Open University)

4:30 – 5:30pm Martin Hallnäs (Loughborough)

All the talks in Boyd Orr building (next to Mathematics building) in 507

Saturday 12th March 2016

9:00 – 10:00am Georgios Papamikos (Kent)

10:00 – 10:30am *Tea/Coffee break*

10:30 – 11:30am Alexey Silantyev (Leeds)

11:30 – 12:30pm Anastasia Doikou (Heriot – Watt)

All the talks in Mathematics building in 203

Titles and Abstracts

Anastasia Doikou

"Classical integrable defects as quasi Bäcklund transformations: an algebraic setting"

We consider the algebraic setting of classical defects in discrete and continuous integrable theories. We derive the ''equations of motion'' on the defect point via the space-like and time-like description. We exploit the structural similarity of these equations with the discrete and continuous Bäcklund

transformations. The equations are similar, but not exactly the same to the Bäcklund transformations. We consider specific examples of integrable models to demonstrate our construction, i.e. the Toda chain and the sine-Gordon model.

Uwe Grimm

"Aperiodic Order"

The talk will present a general introduction to the theory of aperiodic order. While this area has been inspired by the discovery of quasicrystals (which are aperiodically ordered solids) in the 1980s, there has since been a lot of interest in the mathematics of aperiodic order. The talk will introduce the notions of cut and project sets (or model sets) and of inflation tilings, and discuss some of their properties. In particular, this concerns the diffraction measure and the spectrum of the associated dynamical system (under translation action), and how these are related to each other. At the end of the talk, a few comments on possible links to integrable systems will be given.

Martin Hallnäs

"Complex exceptional orthogonal polynomials"

In this talk I will focus on polynomials given as Wronskians of classical Hermite polynomials, which are naturally labelled by partitions. For the special class of so-called double partitions, Gomez-Ullate et al. recently showed that the corresponding polynomials are orthogonal and dense in the space of all polynomials with respect to a certain inner product, but in contrast to their classical counterparts have some degrees missing (so-called exceptional orthogonal polynomials). I will describe how their results can be generalised to all partitions by considering complex contours of integration and non-positive but Hermitian products. If time permits, I will also present a Laurent version of exceptional orthogonal polynomials, related to monodromy-free trigonometric Schrodinger operators.

The talk is based on recent joint work with W.A. Haese-Hill and A.P. Veselov

Georgios Papamikos

"Darboux transformations for the vector sine-Gordon and related structures"

We present the vector sine-Gordon equation together with its Lax representation and its reduction group. Then we construct Darboux transformations invariant under the same reduction group and we derive the corresponding Backlund transformations. We use the Darboux transformation to construct a related vector Yang-Baxter map and an integrable vector differential-difference equation on the sphere. We will briefly discuss the dressing method and the construction of soliton solutions for the vSG.

This is a joint work with Dr J.P. Wang (Kent) and Prof A.V. Mikhailov (Leeds)

Alexey Silantyev

“Calogero-Moser spaces and KP hierarchy for the cyclic quiver”

We start from the well-known relationship between rational solutions of the usual KP hierarchy and Calogero-Moser spaces (completed symmetrized phase spaces of the rational Calogero-Moser systems) of the type A_n. These spaces can be obtained from the one-vertex quiver Q with one loop arrow: one needs to take the (deformed) preprojective algebra for the framed Q and consider the space of its representations with fixed dimension. We generalised this relationship to the B_n type and, more generally, to the complex refection groups $S_n\ltimes\Gamma^n$, where $\Gamma=\mathbb Z/m\mathbb Z$ is the cyclic group (A_n and B_n cases correspond to $m=1$ and $m=2$ respectively). As it is known from the works of Etingof and Ginzburg the Calogero-Moser spaces of this type are obtained in the same way, but with Q being the cyclic quiver. This gives us rational solutions of some hierarchies, which we constructed by using the Cherednik algebra for the group Γ. This is a joint work with Dr. Oleg Chalykh.

Alexander Veselov

“Burchnall-Chaundy polynomials and Dodgson’s condensation method”

The Burchnall-Chaundy polynomials $P_n(z)$ are determined by the differential recurrence relation
$$P_{n+1}'(z)P_{n-1}(z)-P_{n+1}(z)P_{n-1}'(z)=P_n(z)^2$$
with $P_{-1}(z)=P_0(z)=1.$ The fact that this recurrence relation has all solutions polynomial is not obvious and is similar to the integrality of Somos sequences and the Laurent phenomenon.
We discuss this parallel in more detail and extend it to the difference equation $$R_{n+1}(z+1)R_{n-1}(z-1)-R_{n+1}(z-1)R_{n-1}(z+1)=R^2_n(z),$$
related to Dodgson’s condensation method for computing determinants.
As a corollary we have a new form of the Burchnall-Chaundy polynomials in terms of the initial data $P_n(0)$, which is shown to be Laurent.

The talk is based on a joint work with Ralph Willox (J. Phys. A 48 (2015) 205201).

Ralph Willox

“Ultradiscrete inverse scattering and an elementary linearization of the Takahashi-Satsuma box-ball system”

A couple of years ago a novel, inverse scattering-type, technique was discovered for solving the Cauchy problem for the ultradiscrete KdV equation over the real numbers. When restricting the initial values for this system to the set {0,1}, the action-angle variables that arise in this approach turn out to be related in a rather simple way to those that can be obtained from an algorithm proposed by T. Takagi for calculating the Kerov–Kirillov–Reshetikhin map in the context of rigged-configurations. We introduce a simple way to understand the rigged configuration of $A_1^{(1)}$-type and give an elementary proof of the linearization of the associated box-ball system, in terms of these action-angle variables.

This talk is based on joint work with S. Kakei (Rikkyo University), S. Tsujimoto (Kyoto University) and J.J.C. Nimmo (Glasgow University).