Frobenius structures and relations

University of Glasgow, 22 – 23 March 2018

Programme

Thursday 22nd March 2018

1:00 – 2:00pm Giordano Cotti (MP Bonn)

2:00 – 3:00pm Paolo Lorenzoni (Milan)

3:00 – 3:30pm *Tea/Coffee break*

3:30 – 4:00pm Georgios Antoniou (Glasgow)

4:00 – 5:00pm Vassily Gorbounov (Aberdeen)

Friday 23rd March 2018

9:00 – 9:30am David Palazzo (Glasgow)

9:30 – 10:00am Timothy Magee (Mexico)

10:00 – 10:30am *Tea/Coffee break*

10:30 – 11:30am Marta Mazzocco (Birmingham)

11:30 – 12:30pm Iain Gordon (Edinburgh)

All the talks are in Math&Stats seminar room 311B.

Titles and Abstracts of the talks

Giordano Cotti "Monodromy local moduli of semisimple coalescent Frobenius structures"

Based on a series of joint papers with B. Dubrovin and D. Guzzetti. In occasion of the 1998 ICM in Berlin, B. Dubrovin conjectured an intriguing connection between the enumerative geometry of a Fano manifold X with algebro-geometric properties of exceptional collections in the derived category $D^b(X)$. Under the assumption of semisimplicity of the quantum cohomology of X, the conjecture prescribes an explicit form for local invariants of $QH^*(X)$, the so-called "monodromy data", in terms of Gram matrices and characteristic classes of objects of exceptional collections. Frobenius manifolds appearing in the study of the conjectural relations mentioned above typically show a coalescence phenomenon at points where the Frobenius algebra is semisimple, but the operator of multiplication by the Euler vector field has not simple spectrum. On the one hand, the definition of monodromy data is based on the analytic theory of isomonodromy deformations, which a priori cannot be applied at coalescence semisimple points of $QH^*(X)$. On the other hand, it turns out that the Frobenius structure may be known only at coalescence points, which are thus the only locus where the monodromy data can actually be computed. This is the case of the small quantum cohomology of complex Grassmannians, for which the occurrence and frequency of the coalescence phenomenon is surprisingly subordinate to the distribution of prime numbers. In this talk I will firstly show how under minimal conditions the classical theory of M. Jimbo, T. Miwa and K. Ueno (1981) can be extended to describe isomonodromy deformations at a coalescing irregular singularity; I will also show how to locally describe the Frobenius structure near coalescing semisimple points, and finally, what is the "mirror counterpart" of our description in terms of exceptional collections in the derived category.

Vassily Gorbounov "Quantum integrable systems and networks"

We will show that a particular type of 5 vertex model is equivalent to networks on graphs on a plane or cylinder. It leads to very special properties of its partition functions, namely they carry a cluster algebra structure.

Iain Gordon "Parking spaces, after Armstrong, Reiner and Rhoades".

This is joint work in progress with Martina Lanini and recently Misha Feigin. I will review a recent conjecture of Armstrong, Reiner and Rhoades that gives more structure to known bijections between (various complex reflection group generalisations of) non-crossing partitions and non-nesting partitions using the representation theory of rational Cherednik algebras. I will explain how this conjecture may be related to the so-called LL morphism and Frobenius structures and give some evidence currently. There's much to do and help would be welcome!

Paolo Lorenzoni “Bi-Flat F-manifolds, complex reflection groups and integrable systems of conservation laws”

We show that bi-flat F -manifolds can be interpreted as natural geometrical structures encoding the almost duality for Frobenius manifolds without metric. Using this framework, we extend Dubrovin's duality between orbit spaces of Coxeter groups and Veselov's v-systems, to the orbit spaces of exceptional well-generated complex reflection groups of rank 2 and 3. We finally discuss some applications to integrable systems of conservation laws. The talk is based on joint works with A. Arsie.

Marta Mazzocco “Dualities in the q-Askey scheme and degenerate DAHA”

The Askey-Wilson polynomials are a four-parameter family of orthogonal symmetric Laurent polynomials $R_n[z]$ which are eigenfunctions of a second-order q-difference operator L, and of a second-order difference operator in the variable n with eigenvalue $z + z^{-1}$. Then L and multiplication by $z+z^{-1}$ generate the Askey-Wilson (Zhedanov) algebra. A nice property of the Askey-Wilson polynomials is that the variables z and n occur in the explicit expression in a similar and to some extent exchangeable way. This property is called duality. It returns in the non-symmetric case and in the underlying algebraic structures: the Askey-Wilson algebra and the double affine Hecke algebra (DAHA).
In this paper we follow the degeneration of the Askey-Wilson polynomials until two arrows down and in four diferent situations: for the orthogonal polynomials themselves, for the degenerate Askey-Wilson algebras, for the non-symmetric polynomials and for the (degenerate) DAHA and its representations.

Georgios Antoniou “Saito metric and determinant on Coxeter discriminant strata”

Let W be a finite Coxeter group and V its reflection representation. It is known that the space of orbits $M =V/W$ carries the structure of a Frobenius manifold. On M there exists a pencil of flat metrics of which the Saito flat metric η, defined as the Lie derivative of the W-invariant form g on V^*, is the key object. The metric η is compatible with the Frobenius algebra multiplication. We are interested in obtaining the Saito metric and specifically its determinant on the Coxeter discriminant strata in M. It is shown that this determinant in the flat coordinates of the form g is proportional to the product of linear factors associated to the root subsystem defining the discriminant stratum. We also find multiplicities of these factors in the determinant. The talk is based on joint work with M. Feigin and I. Strachan.

Timothy Daniel Magee “Cluster varieties, toric degenerations, and mirror symmetry”

Cluster varieties are a special class of complex varieties that come equipped with an atlas of algebraic tori and a conical volume form. They come in two flavors-- A and X. X-cluster varieties have a Poisson structure. I’ll describe a natural (partial) compactification of the X-variety respecting this Poisson structure. The strata of the compactification are X-varieties compactified in the same way. Then I’ll give a class of toric degenerations of this compactified X-variety, with strata degenerating to toric strata. The remainder of the talk will address a topic that’s more exciting but less settled-- and possibly the coherence of the discussion will drop a bit. A-varieties often have natural partial compactifications and toric degenerations as well. In cases of interest, there appears to be an interesting duality relating the two compactifications. In the toric central fibers of the pair of degenerations, it reduces to a famous construction of mirror families of Calabi-Yaus in toric varieties due to Batyrev and Borisov. This is based on on-going joint work with Lara Bossinger, Alfredo Nájera Chávez, and Juan Bosco Frías Medina.

David Palazzo “Cylindric Reverse Plane Partitions and 2D TQFT”

The ring of symmetric functions carries the structure of a Hopf algebra. The coproduct of the complete symmetric functions h_λ naturally leads to the definition of skew complete symmetric functions as weighted sums over reverse plane partitions (RPP) involving binomial coefficients. Employing the level n action of the affine symmetric group, we define cylindric complete symmetric functions as the generalization of these weighted sums to cylindric RPP.

The cylindric complete symmetric functions are shown to be h-positive, that is their expansion coefficients in the basis h_λ are non-negative integers. We describe how these expansion coefficients are related to tensor multiplicities of irreducible representation of the generalized symmetric group, and then we illustrate their connection with the fusion coefficients of a family of 2D tolopogical quantum field theory (2D TQFT).

Joint work with Christian Korff.