ARTIN/Integrable Systems Workshop

University of Glasgow, Department of Mathematics, Room 516

23-24 April, 2010

Programme

Friday, 23.04.2010

2:00–3:00	**V. Dotsenko**	*Compatible associative products and trees*
3:00–4:00	**J. Stokman**	*Spectral analysis of Ruijsenaars' q-difference operators*
4:00 – 4:30	Tea/Coffee Break	
4:30–5:30	**A. Hone**	*Integrable maps with the Laurent property*

7:00 **Dinner**

Saturday, 24.04.2010

9:00-10:00	**David Johnston, Bart Vlaar, Alexey Silantyev**	*Poster Presentations*
10:00 – 10:30	Tea/Coffee Break	
10:30-11:30	**S. Morier-Genoud**	*Graded algebras generalizing the octonions*
11:30–12:30	**A. Sevastyanov**	*Q-deformations of W-algebras and generalized Gelfand-Graev representations for quantum groups*

Abstracts

Vladimir Dotsenko (Dublin)

Compatible associative products and trees

Two associative products on the same vector space are called compatible if every their linear combination also gives an associative product. Algebras with two compatible associative products were recently studied by Odesskii and Sokolov (whose motivation came from integrable systems); they obtained a classification in the "semisimple" case. We consider another extreme case, that of free algebras with two compatible products. It turns out that these algebras admit an elegant combinatorial construction in terms of planar rooted trees. I shall present this construction and explain why it works.

Andy Hone (Kent)

Integrable maps with the Laurent property

The Laurent phenomenon is a surprising feature of many rational recurrences, and birational maps, whereby all of the iterates are Laurent polynomials in the initial data. It has appeared in a wide variety of contexts, ranging from Dodgson's condensation method for determinants to solvable lattice models in statistical mechanics, and from elliptic divisibility sequences in number theory to discrete Hirota equations. More recently, the Laurent property has arisen as a central feature of Fomin and Zelevinsky's theory of cluster algebras, which has provided an explanation for many of the previously known examples. After a survey of the Laurent property, the connection with integrable maps is considered. Having described a family of recurrences arising from quivers that are periodic with respect to cluster mutations, it is explained how associated Poisson algebras and integrable maps arise. (The latter is current work in progress with Allan Fordy.)

Sophie Morier-Genoud (Paris)

Graded algebras generalizing the octonions

(Joint work with V. Ovsienko)

The algebra of quaternions is not commutative. However, viewed as a graded algebra over $(Z_2)^2$ or $(Z_2)^3$, the algebra of quaternions becomes graded-commutative. More generally, any Clifford algebra is an associative graded-commutative algebra. We will show that this property completely characterizes (simple) Clifford algebras. The classical algebra of octonions is neither commutative nor associative, but it also becomes graded-commutative and graded-associative over $(Z_2)^3$. We will introduce a series of algebras generalizing the octonions in the same way as Clifford algebras generalize the quaternions. We will discuss the main properties of these algebras and mention numerous applications. In particular, we will obtain explicit solutions for the Hurwitz-Radon sum of squares problem.

Alexey Sevastyanov (Aberdeen)

Q-deformations of W-algebras and generalized Gelfand-Graev representations for quantum groups

For any simple complex Lie algebra and any element s in the Weyl group of g we introduce a q-deformed W-algebra $W_q(g, s)$ associated to the corresponding Drinfel-Jimbo quantum group and to the Weyl group element s. This algebra plays the same role in the representation theory of the quantum group as the usual W-algebras in the representation theory of g. In particular, for each Weyl group element s one can define the corresponding category of generalized Gelfand-Graev representations of the quantum group, and this category is equivalent to the category of finitely generated representations of $W_q(g, s)$. This is a quantum group analogue of the Skryabin correspondence. In the talk we shall only consider for simplicity the case $g = sl(n)$.

Jasper Stokman (Amsterdam)

Spectral analysis of Ruijsenaars' q-difference operators

The relativistic quantum trigonometric Calogero-Moser system is a quantum integrable many body system whose quantum conserved integrals are Ruijsenaars' commuting q-difference operators. For the corresponding spectral problem, one needs to consider the Ruijsenaars' operators as linear operators on symmetric Laurent polynomials on a complex torus. The spectral problem is then solved by the celebrated symmetric Macdonald polynomials. In this talk I will address the spectral problem of the Ruijsenaars' commuting q-difference operators, now viewed as linear operators on meromorphic functions on a complex torus.

The motivation is two-fold. On the one hand, we expect the results to be relevant to the analysis of relativistic versions of the quantum hyperbolic Calogero-Moser system (i.e. particles on the line instead of on the circle). On the other hand, we expect the results to be an important step in the development of harmonic analysis on quantum noncompact symmetric spaces.

I will in particular derive a natural q-analogue of Harish-Chandra's c-function expansion of the spherical function. A fundamental role in the talk is played by the affine Hecke algebra.