Sheet 8

8.1 Suppose that L is a semisimple Lie algebra, and α is a root. Show, by rescaling as in lectures, that we can choose a basis of $\mathfrak{sl}(\alpha)$ to be $\{e_\alpha, f_\alpha, h_\alpha\}$ such that $\alpha(h_\alpha) = 2$.

8.2 Suppose that L is semisimple, and choose a root α. Show that

$$H + \mathfrak{sl}(\alpha) = \operatorname{Ker}\alpha \oplus \mathfrak{sl}(\alpha)$$

as $\mathfrak{sl}(\alpha)$-modules.

8.3 Using the formula $\dim L = \dim H + |\Phi|$ from lectures, or otherwise, deduce that there can be no semisimple Lie algebras of dimension 4 or 5.