Accelerated Algebra and Calculus

Solution to Week 7 Assignment

Q4(i), (ii)

We use the Euclidean algorithm to compute $\gcd(51, 36)$:

$$\begin{aligned} 51 &= 1 \cdot 36 + 15 \\ 36 &= 2 \cdot 15 + 6 \\ 15 &= 2 \cdot 6 + 3 \\ 6 &= 2 \cdot 3 + 0. \end{aligned}$$

This shows that $\gcd(51, 36) = 3$. Now using these equations,

$$\begin{aligned} 3 &= 15 - 2 \cdot 6 \\ &= (51 - 36) - 2(36 - 2 \cdot 15) \\ &= 51 - 36 - 2 \cdot 36 + 4(51 - 36) \\ &= 5 \cdot 51 - 7 \cdot 36, \end{aligned}$$

so we have $51x + 36y = 3$ with $x = 5$, $y = -7$.

Multiplying this equation by 2 gives $6 = 51 \cdot 10 + 36 \cdot (-14)$, so we have $51m - 36n = 6$ with $m = 10$, $n = 14$.

Lemma 3.2.1

Let A, B be finite sets with n, N elements respectively. There exists an injective map $f : A \to B$ if and only if $n \leq N$.

Proof. Let us write $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_N\}$.

- If $n \leq N$ then define $f : A \to B$ by $f(a_i) = b_i$ for each $i = 1, \ldots, n$. Since $b_1, \ldots, b_n$ are all different, it follows that if $f(x_1) = f(x_2)$ they must both equal the same b_i, hence $x_1 = x_2 = a_i$.
- Suppose $f : A \to B$ is injective. This means that $f(a_i) \neq f(a_j)$ when $i \neq j$, so the set $\{f(a_i) \mid i = 1, \ldots, n\}$ is a subset of B with n distinct elements. Hence $n \leq N$.

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