Asymptotic reduction of cardiac excitation models

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ILLUSTRATION ON NOBLE’S MODEL (1962)

Parameter embedding P1 for Noble’s model

Figure: (a–b) The caricature approximations (thin lines) of the right-hand side functions of the Noble model (2) (thick lines).

Embed a first parameter \( \epsilon \) to simplify functional form as shown above. Take the limit \( \epsilon \to 0 \). The functions of the caricature Noble model simplify to

\[
g(E) = g_0(E) = g_0(E-E_0) + g_1(E)^2 + g_2(E^3),
\]

(2a)

\[
g(E) = g_0(E-E_0) + g_1(E)^2 + g_2(E^3).
\]

(2b)

\[
g(E) = g_0(E-E_0) + g_1(E)^2 + g_2(E^3).
\]

(2c)

where

\[
G(E) = g_0(E)W + g_1(E),
\]

(3)

\[
W(E) = m_0^2(E)k_0(E) + n_0^2(E),
\]

(4a)

\[
\frac{df}{dt} = \frac{1}{E_0} + \frac{1}{E_0} - \frac{1}{E_0}.
\]

(4b)

\[
\frac{df}{dt} = \frac{1}{E_0} + \frac{1}{E_0} - \frac{1}{E_0}.
\]

(4c)

Deviation of asymptotics from full problem

Advantage: The deviation of the parametric embedding (caricature model) from the original model (Noble) can be measured exactly via continuation in \( \epsilon \).

RESULTS AND POTENTIAL APPLICATIONS

Analytical approximation to APD and CV restitution

Figure: Restitution curves of the Noble model, (a) in Cartesian and (b) logarithmic coordinates. Insets in panel (a) show selected features magnified. Lines in all plots as described by the legend in panel (b).

Non-Tikhonov asymptotic structure of detailed cardiac models

Figure: Pre/post-front voltage selection in the four different models, (a) Barkley, (b) FitzHugh-Nagumo, (c) Caricature Noble, (d) Beuter-Reuter.

Summary of important results

1. A consistent procedure for reduction of cardiac electrophysiological models.
2. Cardiac models have a non-Tikhonov asymptotic structure. Care is needed.
3. Successful application to the problem of CV-restitution curves.
4. Successful application to the problem of breakup and self-terminating propagation action potentials.

Some future problems

1. Initiation of propagating action potentials
2. Efficient numerical methods based on asymptotic reduction
3. Higher dimensions
4. More realistic models

References