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LETTERS

Energy flux determines magnetic field strength of planets and stars

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The magnetic fields of Earth and Jupiter, along with those of rapidly rotating, low-mass stars, are generated by convection-driven dynamos that may operate similarly[1–4] (the slowly rotating Sun generates its field through a different dynamo mechanism[5]). The field strengths of planets and stars vary over three orders of magnitude, but the critical factor causing that variation has hitherto been unclear[5,6]. Here we report an extension of a scaling law derived from geodynamo models[7] to rapidly rotating stars that have strong density stratification. The unifying principle in the scaling law is that the energy flux available for generating the magnetic field sets the field strength. Our scaling law fits the observed field strengths of Earth, Jupiter, young contracting stars and rapidly rotating low-mass stars, despite vast differences in the physical conditions of the objects. We predict that the field strengths of rapidly rotating brown dwarfs and massive extrasolar planets are high enough to make them observable.

So far, attempts to explain the magnetic field strength of natural dynamos have been largely heuristic and disparate for planets and stars. The field strength in a planetary dynamo is often attributed to the supposed balance between Lorentz (electromagnetic) and Coriolis (rotational) forces, requiring that the Elsasser number $\Lambda = \sigma B^2/(\rho\Omega)$ is of the order of one (here σ is electrical conductivity, B is r.m.s. magnetic field strength in the dynamo, ρ is density and Ω is rotation rate). This is in fair agreement with the observed field strength of Earth and some other planets[6]. However, Λ falls in the range 0.1–100 in different geodynamo models[7]. For stellar dynamos, the equipartitioning of magnetic and kinetic energy is sometimes assumed to be the guiding principle controlling the field strength[5]. The geodynamo probably operates in the whole of the fluid outer core, but in the Sun, much of the magnetic field generation is supposedly localized at the tachocline[5], a thin layer of intense shear between the convecting outer region and the deeper radiative zone. Fully convective stars, such as mature stars of less than 0.35 solar masses (M dwarfs) and T Tauri stars (very young contracting stars with moderate mass), often have stronger magnetic fields than the Sun and their dynamo may resemble that of planets.

Rotation strongly influences the dynamo. For stars with moderate and low mass, the X-ray luminosity (a proxy for the magnetic flux) increases with rotation rate up to some threshold value, where it saturates[8]. Direct measurements of the field strength by the magnetic broadening of spectral lines confirm the saturation for M dwarfs[9]. The magnetic field topology, which is small-scale at the surface of the slowly rotating Sun, becomes more large-scale with prominent dipole contributions when rotation is fast and the star is fully convective[1,2]. In dynamo simulations of fully convective stars, the scale and strength of the field increase with rotation rate[3], but the strength levels off in the most rapidly rotating cases. Geodynamo model studies support the existence of two regimes: for slow rotation, the magnetic field is small-scale and weak; for fast rotation, it is dipole-dominated and its strength is independent of rotation rate[7,10,11].

In ref. 7, a scaling theory for the field strength of planetary dynamos has been presented which is based on the (thermodynamically) available energy flux; in the case of thermal flux, this is the part that can be converted to magnetic energy to sustain it against ohmic dissipation. To test if the same scaling rule applies to the field strength in stellar dynamos, we generalize it to also cover cases of strong density stratification. We restrict our study to objects in the rapidly rotating regime, where in incompressible geodynamo models[7] the magnetic energy density was found to depend on density and convected energy flux q_c, but not (or very weakly) on magnetic diffusivity and rotation rate (that is, the field is saturated):

$$B^2/(2\mu_o) \propto f_{\rm ohm}\rho^{1/3}(q_c L/H_T)^{2/3} \qquad (1)$$

Here μ_o is permeability, $f_{\rm ohm} \leq 1$ is the ratio of ohmic dissipation to total dissipation, L is the length scale of the largest convective structures (in the geodynamo, this is the thickness $D = R - r_i$ of the convective shell with outer radius R and inner radius r_i) and $H_T = c_p/(\alpha g)$ is the temperature scale height with c_p the heat capacity, α the thermal expansivity and g the acceleration due to gravity. For stars we adopt the common assumption that L is of the order of the density scale height H_ρ. To account for the strong variations of density and scale height with radius, we assume that the mean squared magnetic field $\langle B^2\rangle$ is obtained by taking the average of equation (1) over the volume V of the spherical shell. We normalize density with its mean value $\langle\rho\rangle$ and q_c with a reference value q_o, for which we take the bolometric flux at the outer boundary (except for Earth's core, see below):

$$\langle B\rangle^2/(2\mu_o) = c f_{\rm ohm}\langle\rho\rangle^{1/3}(Fq_o)^{2/3} \qquad (2)$$

Here c is a constant of proportionality, and the averaging of radially varying properties has been condensed into the efficiency factor F:

$$F^{2/3} = \frac{1}{V}\int_{r_i}^{R}\left(\frac{q_c(r)}{q_o}\frac{L(r)}{H_T(r)}\right)^{2/3}\left(\frac{\rho(r)}{\langle\rho\rangle}\right)^{1/3} 4\pi r^2\,{\rm d}r \qquad (3)$$

where we set $L = \min(D, H_\rho)$.

F must be calculated for each object separately. For Earth's core, simplifying approximations are made, such as $L = D$, constancy of density and thermodynamic properties, and linear variation of gravity with radius, $g = g_o r/R$. A significant part (perhaps all) of the flux at Earth's core–mantle boundary is transported by conduction. At greater depth, the convected portion is larger and augmented by compositional driving of convection, which we treat as enhanced effective heat flux. We take the effective flux on the inner boundary $q_{i,c}$ to define the reference flux as $q_o = q_{i,c}(r_i/R)^2$. Two options for the variation of q_c

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with radius are considered: constancy of total flux $4\pi r^2 q_c$ as used in many geodynamo models, or a decrease to zero on the outer boundary. With these assumptions, equation (3) can be evaluated analytically. Setting $r_i/R = 0.35$, we obtain $F = 0.88\alpha g_o R/c_p$ for constant total flux and $F = 0.45\alpha g_o R/c_p$ when $q_c(R) = 0$. For $\alpha = 1.35 \times 10^{-5}$, $g_o = 10.7$, $R = 3.48 \times 10^6$ and $c_p = 840$ in SI units[12], F in the Earth's core is obtained as 0.52 or 0.27, respectively.

In Fig. 1 we compare results of geodynamo simulations (ref. 7 and this work) with a non-dimensional form of the scaling equation (2). The spread in the non-dimensional flux q^* relates to a variation of the rotation rate by a factor of 1,000 and a variation of the flux for fixed rotation rate by a factor of 100. The good agreement confirms the independence of the field strength from the rotation rate and the variation with the 2/3 power of the flux. It provides the constant of proportionality as $c = 0.63$. Data for zero flux on the outer boundary (Supplementary Information) are collapsed with those for constant total flux when the difference in the efficiency factor F is accounted for. Results from a stellar dynamo model with moderate density stratification[3] agree with the scaling law, suggesting that it may also be applicable to stars and that the dynamo mechanisms in this model

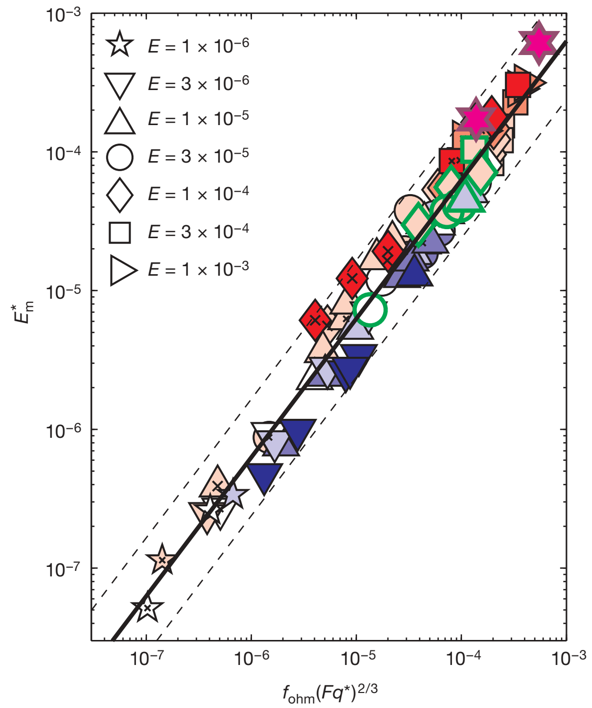

Figure 1 | Scaling law versus results from dynamo models. The non-dimensional form of equation (2) is obtained by dividing by $\langle\rho\rangle\Omega^2R^2$, resulting in a non-dimensional energy density $E_m^* = \langle B^2\rangle/(2\mu_o\langle\rho\rangle\Omega^2R^2)$ and flux $q^* = q_o/(\langle\rho\rangle\Omega^3R^3)$; the non-dimensional mean density is unity. Black-edged symbols are for models with radially constant total flux (ref. 7 and this work), green-edged symbols are for flux decreasing to zero at the outer radius (Supplementary Information). F is $0.88\alpha g_o R/c_p$ in the first case and $0.45\alpha g_o R/c_p$ in the second case. Only results in the strongly rotational regime are included, which requires that the local Rossby number[7,11] be less than 0.12. The Ekman number $E = \nu/(\Omega D^2)$, where ν is viscosity, varies between 10^{-3} and 10^{-6}. The magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$, where η is magnetic diffusivity, is colour-coded; white means Pm = 1, different shades of red indicate values progressively larger than 1, and blue values less than 1. The pink hexagrams are the two most rapidly rotating cases from a set of dynamo models for fully convecting stars with a polytropic equation of state[3] with Pm = 1 and $E = 1.6 \times 10^{-4}$ and 0.8×10^{-4}, respectively. Here $F = 1.48$ is calculated by numerically integrating the reference star model (Supplementary Information). The slope of the fitting line is set to one (if unconstrained, the least-squares slope is 1.02). Dashed lines, 3σ standard error.

and in our geodynamo models are similar. We note that if the magnetic field strength is strictly independent of rotation rate and of magnetic (and any other) diffusivity, for dimensional reasons the exponents for density and heat flux in equation (2) must necessarily be 1/3 and 2/3, respectively.

We numerically integrate equation (3) for structural models of Jupiter and stars. For Jupiter, we use the adiabatic model with gradual metallization[13], assuming that the top of the dynamo region is at 0.84 planetary radii. With convection as the only means of heat transport and with luminosity varying with radius proportional to $T\mathrm{d}m/\mathrm{d}r$, where T is temperature and m is the mass inside radius r, we obtain $F = 1.19$. We use a stellar evolution code[14], which provides density, temperature, luminosity and convected flux as function of radius, to generate models in the range of 0.25 to 0.7 solar masses for ages between 1.2 and 20 Myr and for masses of 0.25 and 0.30 solar masses up to 4.5 Gyr. The resulting F factors lie in the range 0.69–1.22.

We compare the predictions of our scaling law with the magnetic fields of Earth, Jupiter and two groups of rapidly rotating stars whose surface field strength has been determined spectroscopically. One is the classical T Tauri stars[15] and the other is a set of old M dwarfs[16], from which we select those with a projected rotational velocity $v\sin(i) \geq 3\,\mathrm{km\,s^{-1}}$ (here v is the actual velocity, and i is inclination). To estimate their mean internal field strength B from the observed mean surface field B_s, we multiply the latter by a factor of 3.5, the typical ratio found in our geodynamo simulations. Additionally, we include some M stars whose large-scale field has been inferred from Zeeman–Doppler tomography[2]. Here the total surface field B_s is usually unknown. We use the dipole field strength B_{dip} and multiply by factors $B_s/B_{dip} \approx 7$ found at EV Lac and YZ CMi (Supplementary Tables 4 and 6) and $B/B_s = 3.5$ to obtain B. Also, for planets the total field strength at the top of the dynamo is unknown. The dipole field strengths at the dynamo surface are 0.26 mT and 1.0 mT at Earth[17] and Jupiter[18], respectively. In our geodynamo simulations, we find a typical ratio B/B_{dip} of around seven for dynamos with an Earth-like magnetic power spectrum, which we apply to estimate the internal field strength of the planets.

The agreement with the theoretical prediction is remarkable for the different groups of rapidly rotating objects (Fig. 2), which span more than eight orders of magnitude in (equivalent) bolometric flux. For comparison, we also include stars with radiative cores and slow rotation[19]; as expected they fall below the prediction (green and yellow bars). The validity of some assumptions may be questioned for dynamos with strong density stratification—for example the use of H_ρ for the length scale L in equation (3) or the application of scaling factors between internal field and surface field derived from incompressible models. However, the latter are unlikely to differ vastly and even when we assume $L = D$, the F factor for stars increases only from one to five. Hence we consider the scaling law as robust on an order of magnitude scale. We conclude that dynamos in rapidly rotating stars and planets are basically similar, and that a single principle controls their magnetic field strength.

Some T Tauri stars in our sample may have formed a small radiative core. The observations for rapidly rotating old stars that are too massive to be fully convective[19] (orange symbols in Fig. 2) also agree with our field strength scaling law. Thus, the essential condition for its applicability is probably rapid rotation.

Although magnetic fields have been measured at other planets in the Solar System, the scaling law is either hard to test or not applicable at these locations: Mercury is a slow rotator and may hence fall into the non-dipolar dynamo regime[20], the dynamos in Saturn[21] and Mercury[20] probably lie below a stably stratified conducting layer of unknown thickness, and those in Uranus and Neptune may operate in a thin shell overlying a stable region[22].

Stars, particularly old M dwarfs, cluster in a narrow range of $\langle\rho\rangle^{1/3}(Fq_o)^{2/3}$ because the decrease in bolometric flux is balanced by an increase in density. This explains why rapidly rotating stars with rather different luminosities all have magnetic surface fields of

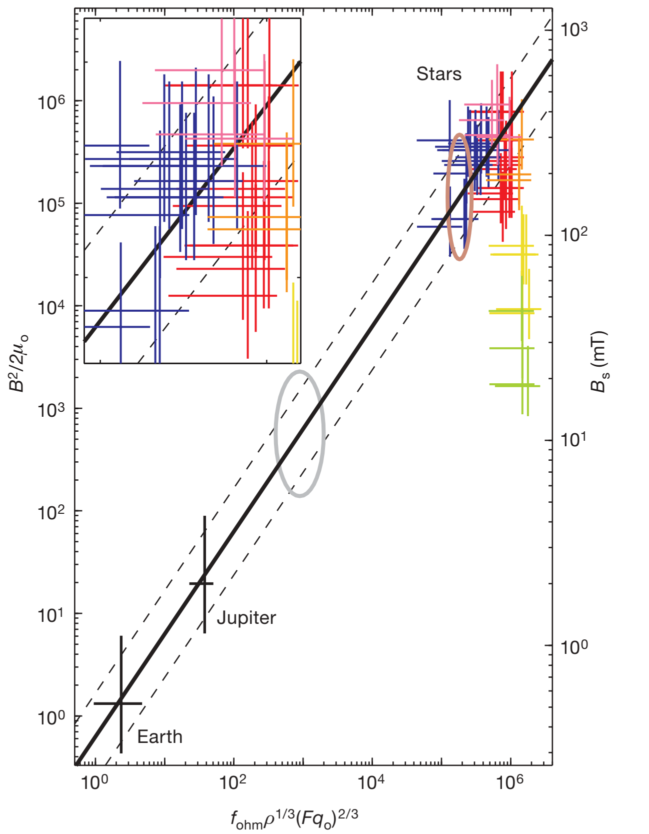

Figure 2 | Scaling law versus magnetic fields of planets and stars. Magnetic energy density in the dynamo versus a function of density and bolometric flux (both in units of $\mathrm{J\,m^{-3}}$). The scale on the right shows r.m.s. field strength at the dynamo surface. The heat flow from Earth's core is uncertain[12,26] but is in the range 30–100 $\mathrm{mW\,m^{-2}}$. The effective convected flux including compositional convection is about twice as large (Supplementary Information); we use $q_o = 100\,\mathrm{mW\,m^{-2}}$, $\langle\rho\rangle = 10^4\,\mathrm{kg\,m^{-3}}$ and $F = 0.35$. For Jupiter[27], $q_o = 5.4\,\mathrm{W\,m^{-2}}$ and $\langle\rho\rangle = 1{,}330\,\mathrm{kg\,m^{-3}}$. For stars we assume $F = 1$. For T Tauri stars[15] (in blue) and old M dwarfs (in red where data for total field is known[16], and in pink where the large-scale field was observed[2]), q_o is obtained from the effective surface temperatures[15,16,28]. Stars of 0.6–1.1 solar masses[19] are shown in green for rotation periods $P > 10$ d, yellow for $4\,\mathrm{d} < P < 10$ d and orange for $P < 4$ d. Where relevant stellar data are not quoted, we use model-based relationships between spectral subclass, mass and luminosity[29,30]. We assume $f_{ohm} \approx 1$ as a nominal value. The bar lengths show estimated uncertainty rather than formal error (Supplementary Information). Black lines show the rescaled fit from Fig. 1 with 3σ uncertainties (solid and dashed lines, respectively). The stellar field is enlarged in the inset. Brown and grey ellipses indicate predicted locations of a brown dwarf with 1,500 K surface temperature and an extrasolar planet with seven Jupiter masses, respectively.

some tenths of a tesla. Even for a typical 1-Gyr-old brown dwarf of 0.05 solar masses[23] with an effective temperature of 1,500 K and $\langle\rho\rangle = 90{,}000\,\mathrm{kg\,m^{-3}}$, a surface magnetic field of the order of 0.1 T is expected (brown ellipse in Fig. 2). Magnetic fields have not been detected at brown dwarfs so far, but our estimate suggests that a search might well be productive. For young (1–3 Gyr) giant extrasolar planets of 5–10 Jupiter masses, which should have 20–200 times Jupiter's intrinsic luminosity at a similar radius[23], the expected field strength is 5–12 times larger than that at Jupiter's surface (considering also the shallower depth of the dynamo). Another estimate[24] based on the Elsasser number rule arrived at similar maximum values, but only for rotation periods <5 h, which we do not require. The presence of such strong fields improves the prospects for detecting radio emissions from these planets[25]. From the high-frequency cut-off in the radio spectrum, the surface field strength can then be determined[25].

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Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

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