The notions of Hopf algebroids and their cyclic (co)homology, Hopf-cyclic (co)homology, incorporate concepts of generalised symmetries in noncommutative geometry (i.e., the noncommutative analogue of groupoids and Lie algebroids) and their associated (co)homologies.

Among the various existing approaches, we focus on the definition of Hopf algebroids by Böhm-Szlachányi, which understands a Hopf algebroid as consisting of two bialgebroid structures (generalising the notion of a bialgebra), and an antipode intertwining these structures. This approach allows for a systematic description of the cohomology theory: we explain how the Hopf-cyclic cohomology fits into the monoidal category of Hopf algebroid modules and show that it descends in a canonical way from the cyclic cohomology of corings. We also develop a dual cyclic homology theory for Hopf algebroids, obtained by cyclic duality (in the sense of Connes) and a generalised Hopf-Galois map (in the sense of Schauenburg) canonically associated to the Hopf algebroid. Such a map is required to mediate between the involved bialgebroid module and comodule categories.

We then give a few new examples of Hopf algebroids such as universal enveloping algebras of Lie-Rinehart algebras, jet spaces, and convolution algebras over étale groupoids. By computing their respective cyclic theory, we establish Hopf-cyclic (co)homology as a noncommutative extension of both Lie-Rinehart (co)homology and groupoid homology.