Thursday, 15*th March*, 2007 10.10 *a.m. to* 10.55 *a.m.*

University of Glasgow

Department of Mathematics

Mathematics 2N - Number Theory and Cryptography

Class Test

Candidates should attempt ALL questions.

1. The Division Theorem shows that an integer a can be written in the form $3k + r$, with k an integer, and $r = 0, 1$ or 2.
   Use this to show that, for any integer a, $a(a^2 - 1)$ is divisible by 3. **7**

2. For any integer n, let $d_n = \gcd(9a+5, 7a+2)$. Prove that, for any n, $d_n = 1$ or 17. **4**

3. Show that the equation

$$121x + 77y = 1000$$

   has no solutions in integers. **4**

4. Use the Euclidean Algorithm to find integers x, y such that

$$91x + 105y = \gcd(91, 105).$$

   Hence find the general solution of the equation

$$91x + 105y = 700.$$ **15**

END]