

SCHOOL OF MATHEMATICS AND STATISTICS

Mathematical Modelling II (Continuum Mechanics)

Prof. X. Y. Luo

At the *microscopic* level matter is discontinuous. It is composed of molecules and atoms, which are themselves composed of elementary particles. However, in most situations encountered in real life we are concerned with volumes of matter with dimensions large compared with these particles and in practice we need not be concerned with individual atoms, only with their behaviour in some *average* sense. Continuum mechanics deals with the behaviour of bodies of solid and fluid matter on a *macroscopic* scale. It disregards the microscopic structure and treats matter as being smoothly distributed throughout the body under consideration. Quantities such as density and velocity are regarded as continuous (or piecewise continuous) functions of position. In this course we develop the mathematical apparatus required for the analysis of the flow of fluids and the deformation of solids. The theory is then applied to a number of prototype problems with the emphasis on elasticity theory. An understanding of continuum mechanics is of fundamental importance in many applications, including meteorology, seismology, chemical engineering, biomechanics, structural mechanics, aircraft design, etc.

Aims of the course

To provide an introduction to the basic theoretical ideas in continuum mechanics using the theory of Cartesian tensors and to apply the theory in the solution of boundary-value problems in elasticity theory.

Pre-requisites

The usual prerequisite for MM2 is MM1, but students without MM1 may take MM2 if they have taken Mathematical Methods 2 and have background knowledge of mechanics from, for example, Dynamics 1 or a physics courses.

Recommended reading

There is no requirement to purchase a textbook since no book covers all aspects of the course adequately. There are many books, however, which are suitable for parts of the course. The following is a partial list of some useful texts; many others with similar titles can be found in the University Library.

Continuum Mechanics, A.J.M. Spencer: Dover (2004)

Continuum Mechanics, P. Chadwick: Dover (2003)

Introduction to Continuum Mechanics, S. Nair: CUP (2009)

A First Course in Continuum Mechanics, O. Gonzalez and A.M. Stuart: CUP (2008)

Nonlinear Solid Mechanics, G.A. Holzapfel: Wiley (2001)

Non-Linear Elastic Deformations, R.W. Ogden: Dover (1997)

Course content

1. Introduction to the theory of Cartesian Tensors
2. Kinematics of deformation and motion
3. Conservation laws and the stress tensor
4. Equations of motion and equilibrium, Cauchy's theorem.
5. Constitutive laws: general principles and material symmetry
6. Isotropic functions of tensors and isotropic materials
7. Examples of constitutive laws for solids and fluids
8. Incompressibility and other constraints
9. Formulation of boundary-value problems in elasticity
10. Derivation of Navier-Stokes equations. Plane parallel flows.

Learning objectives

On completion of this course the student will be expected to know and understand the main

- aspects of the theory and should be able to
- apply differential operators to tensor functions
- formulate tensor products and write tensors in diagonal form
- calculate deformation and velocity fields
- calculate deformation and velocity gradients
- use Nanson's formula connecting area elements
- derive and use various transport formulae
- establish the equations of motion for a continuum
- understand the notion of a stress tensor
- find surface tractions and principal stresses
- determine whether given tensors are objective
- understand the differences between constitutive laws
- define what is meant by an isotropic tensor function
- formulate and solve basic equilibrium problems in elasticity
- solve plane parallel flows.