Spring Workshop on Nonlinear Mechanics

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Unified Strength Theory for Materials and Structures

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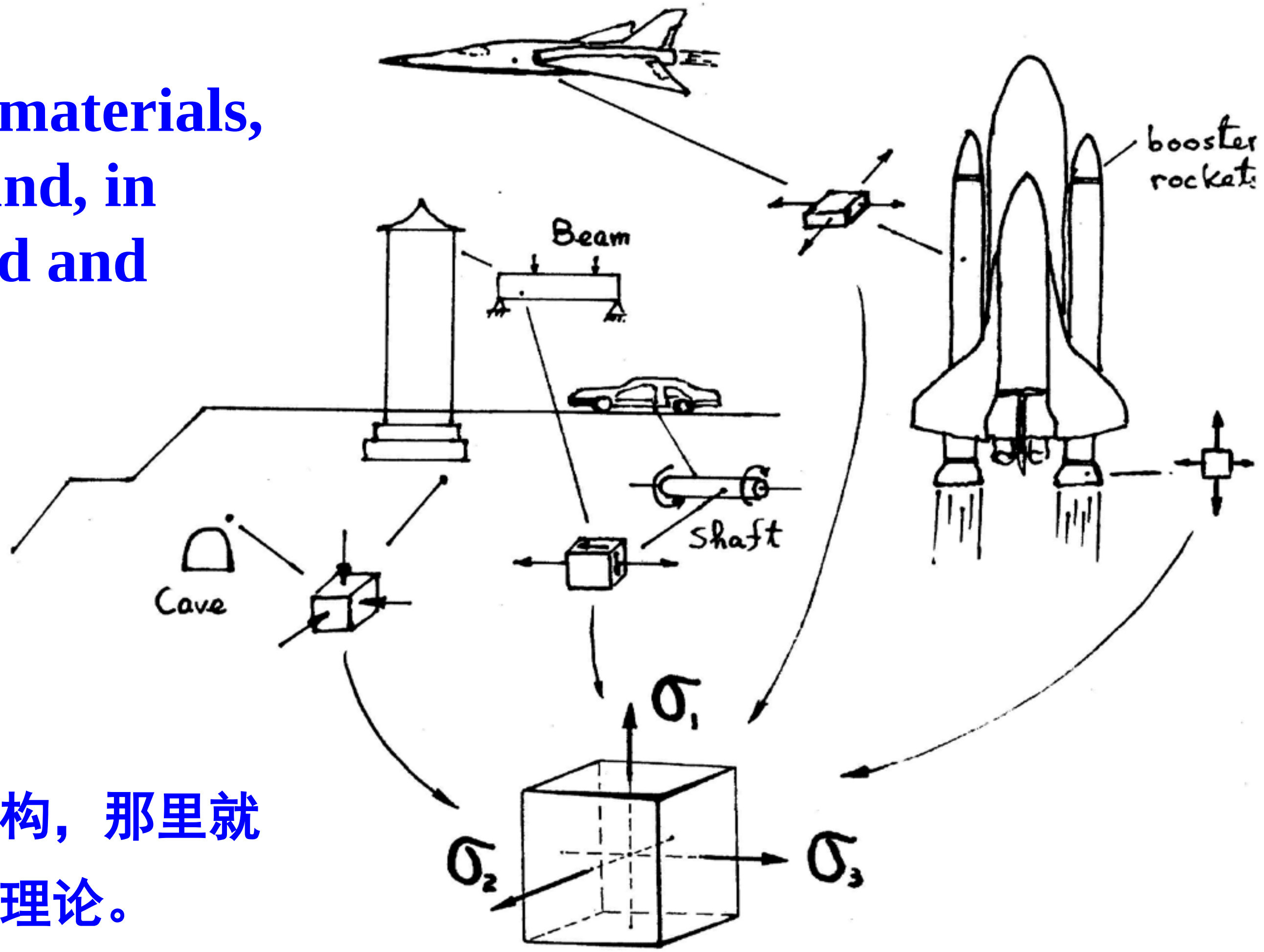
CONTENTS

Strength:

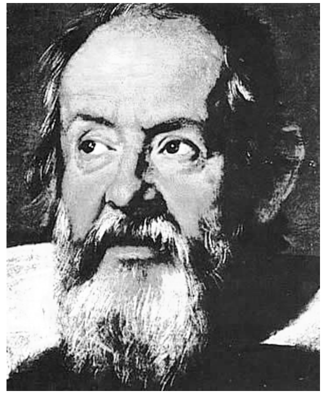
comes from various materials, structures (on Ground, in Space, under Ground and water).

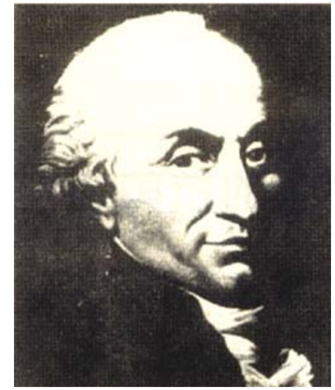
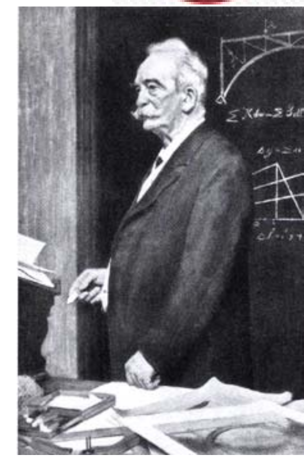


强度：

那里使用材料、建造结构，那里就有强度问题，就需强度理论。

1 Significance> Strength Theory



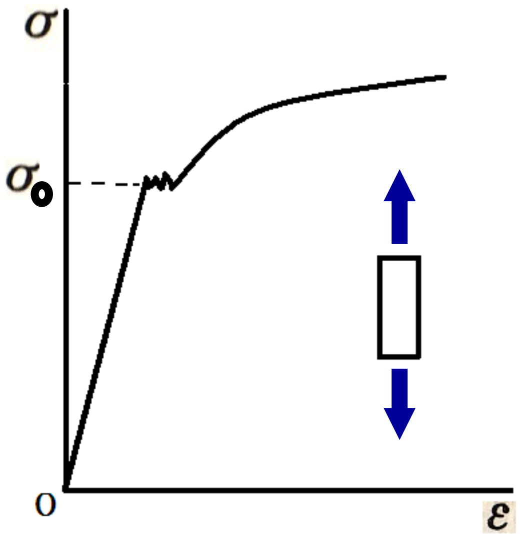
Strength theory:

- **study the material's yield or failure law under complex stress condition,**
- **provide necessary calculation criterion and safety criterion for structure design.**

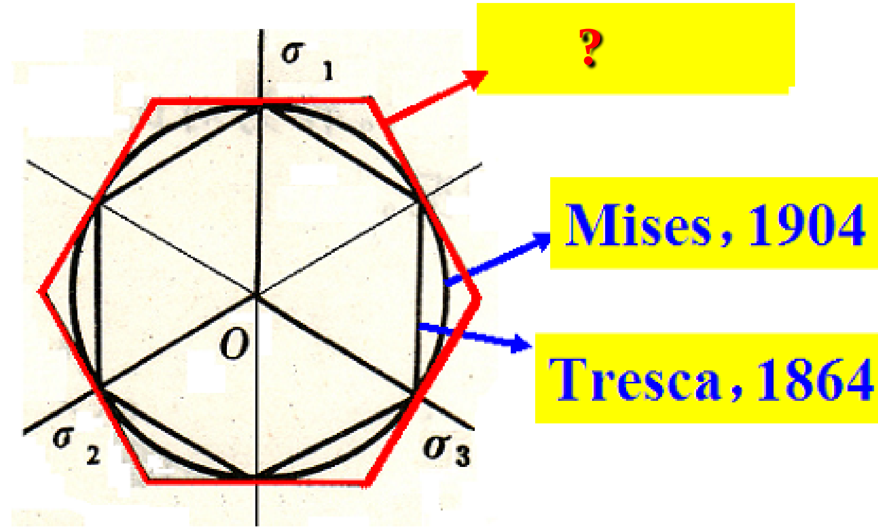
强度理论：

- **研究材料在复杂应力状态下屈服或破坏规律，**
- **为工程结构设计提供必须的计算准则和安全判据。**

1 Significance> Problem



yield point under
simple stress state



π-plane

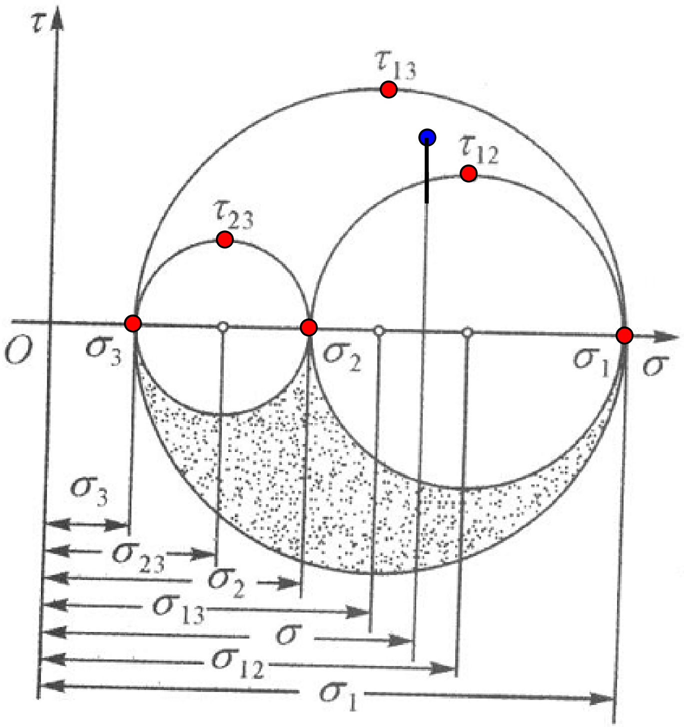
yield surface under
complex stress state

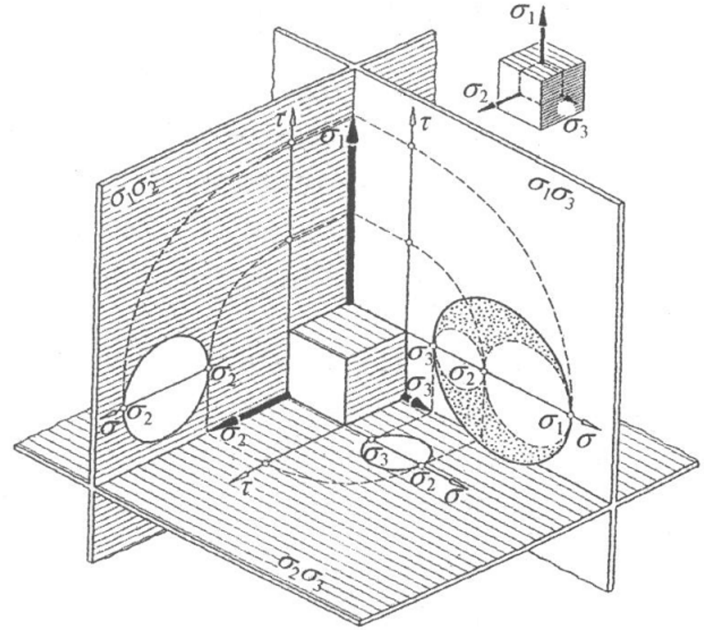
What is the meaning of outer convex limit loci ?

最大外凸极限线的意义是什么?

2 Unified Strength Theory

Stress State at One Point





$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} \qquad \sigma_{13} = \frac{\sigma_1 + \sigma_3}{2}$$

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \qquad \sigma_{12} = \frac{\sigma_1 + \sigma_2}{2}$$

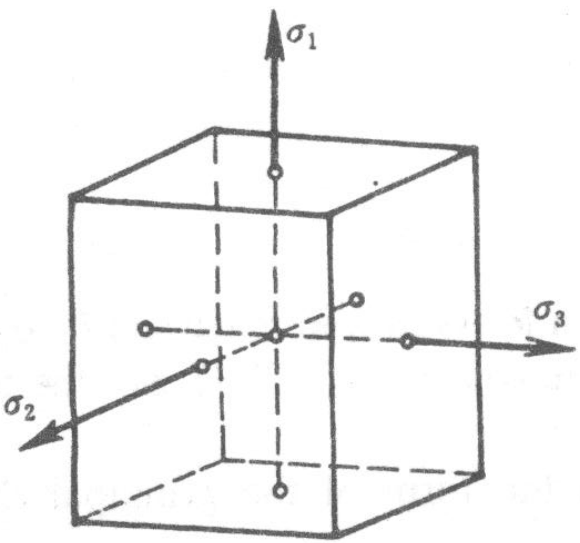
$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \qquad \sigma_{23} = \frac{\sigma_2 + \sigma_3}{2}$$

Three principal shear stresses
Three principal normal stresses

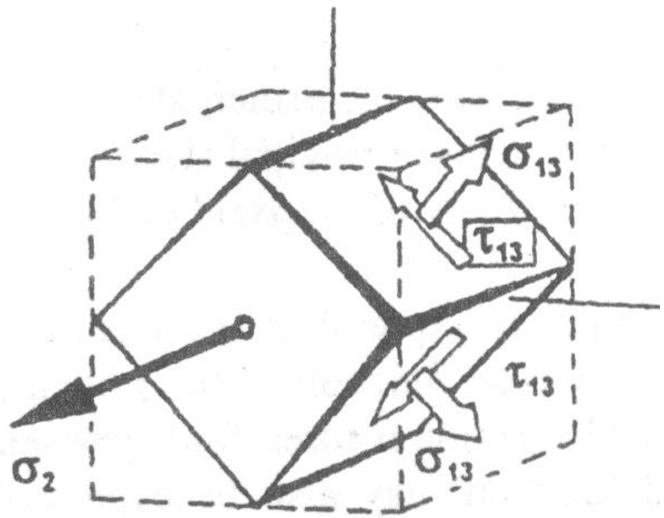
obviously $\tau_{13} = \tau_{12} + \tau_{23}$

2 Unified Strength Theory

Single-Shear Stress Concept



(a)



(b)

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_{13} = \frac{\sigma_1 + \sigma_3}{2}$$

stress state

(σ_1, σ_2, σ_3)

principal shear stress

(τ_{13}, τ_{12}, τ_{23})

$$\tau_{13} = C$$

Tresca

2 Unified Strength Theory

Twin-Shear Stress Concept

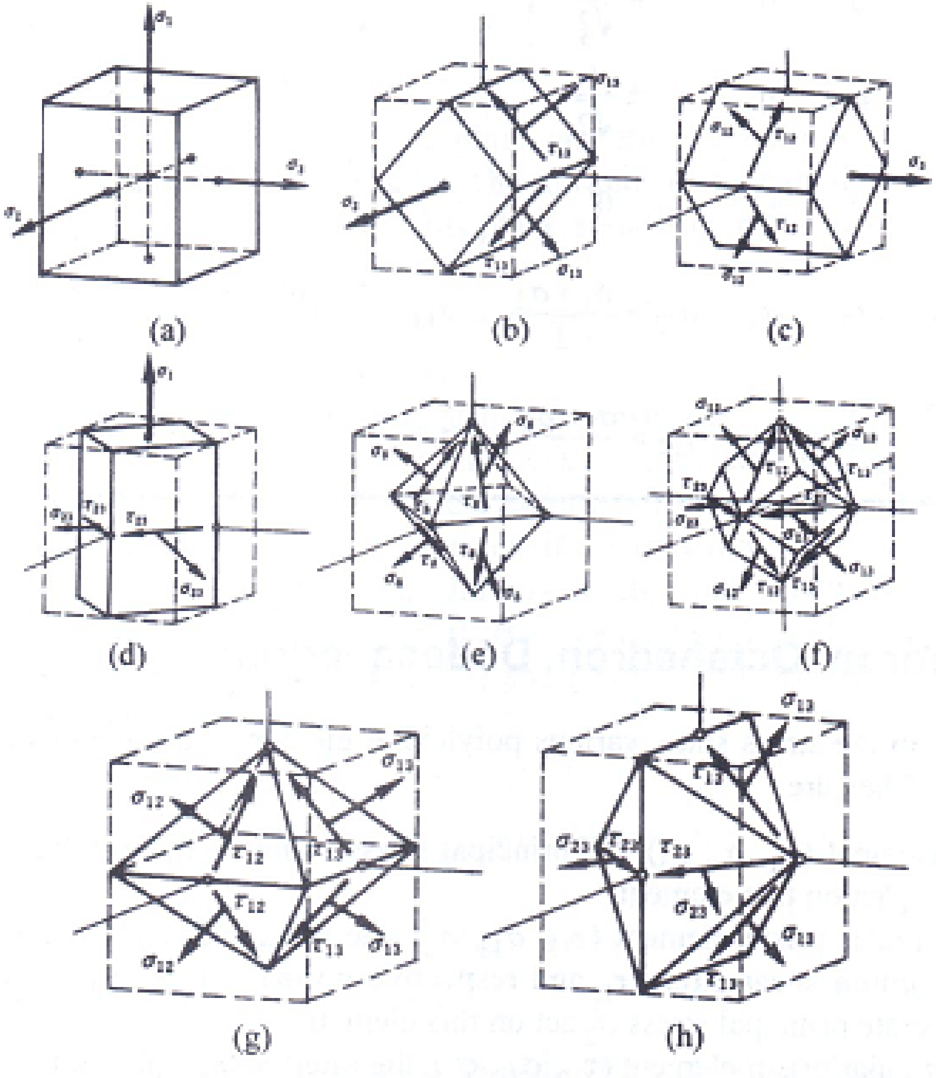
$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} \qquad \sigma_{13} = \frac{\sigma_1 + \sigma_3}{2}$$

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \qquad \sigma_{12} = \frac{\sigma_1 + \sigma_2}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \qquad \sigma_{23} = \frac{\sigma_2 + \sigma_3}{2}$$

since $\tau_{13} = \tau_{12} + \tau_{23}$

$$\begin{cases} \tau_{13} + \tau_{12} = C & \text{when} \quad \tau_{12} \geq \tau_{23} \\ \tau_{13} + \tau_{23} = C & \text{when} \quad \tau_{12} \leq \tau_{23} \end{cases}$$



Various polyhedral elements

2 Unified Strength Theory

For geo-material:

$$\begin{cases} F = \tau_{13} + \tau_{12} + \beta(\sigma_{13} + \sigma_{12}) = C & when\ F \geq F^{'} \\ F^{'} = \tau_{13} + \tau_{23} + \beta(\sigma_{13} + \sigma_{23}) = C & when\ F \leq F^{'} \end{cases}$$

where β 、C can be determined with material properties, σ_c σ_t

$$\beta = \frac{1-\alpha}{1+\alpha}, \qquad C = \frac{2\sigma_t}{1+\alpha}, \qquad (\alpha = \sigma_t / \sigma_c)$$

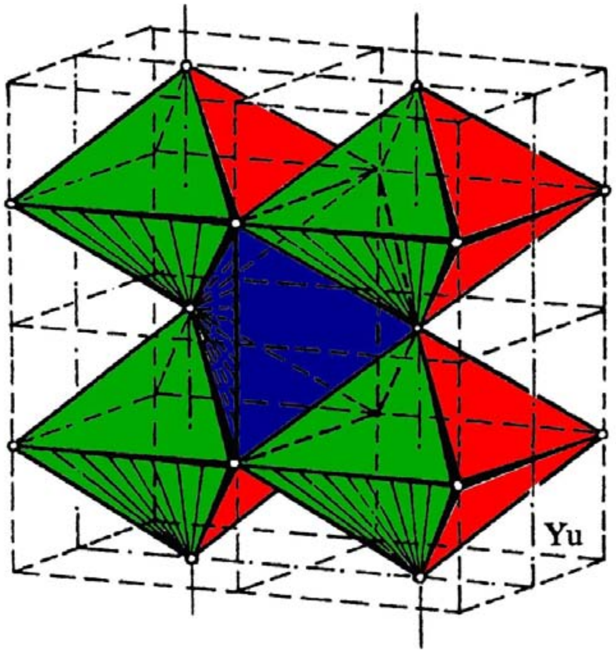
Convert to pincipale stress state

$$F = \sigma_1 - \frac{\alpha}{2}(\sigma_2 + \sigma_3) = \sigma_t \qquad \text{when} \quad \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

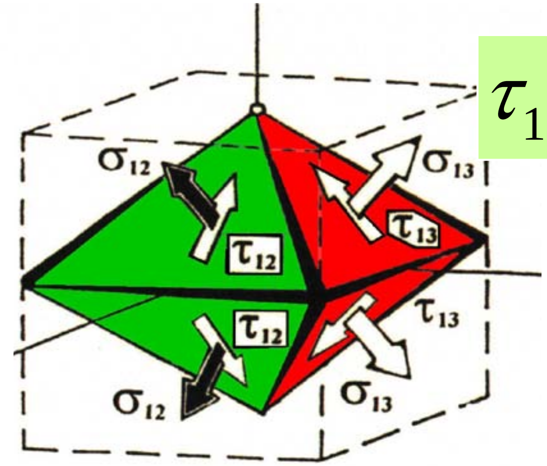
$$F' = \frac{1}{2}(\sigma_1 + \sigma_2) - \alpha\sigma_3 = \sigma_t \qquad \text{when} \quad \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

2 Unified Strength Theory

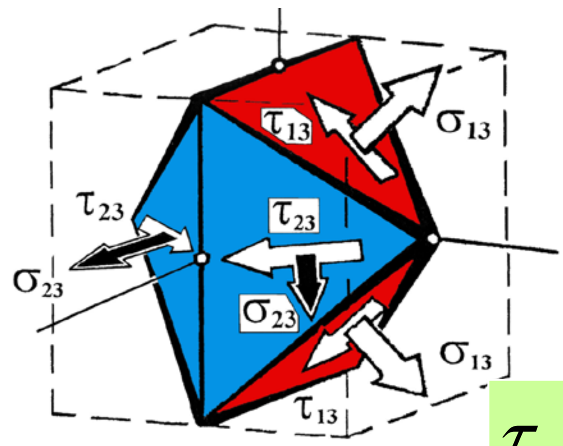
Mechanical Model of the Unified Strength Theory



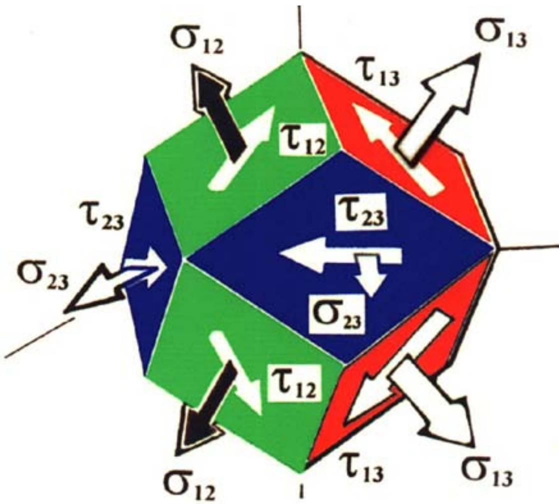
continuum, space filled



τ_{13} , τ_{12}



τ_{13} , τ_{23}

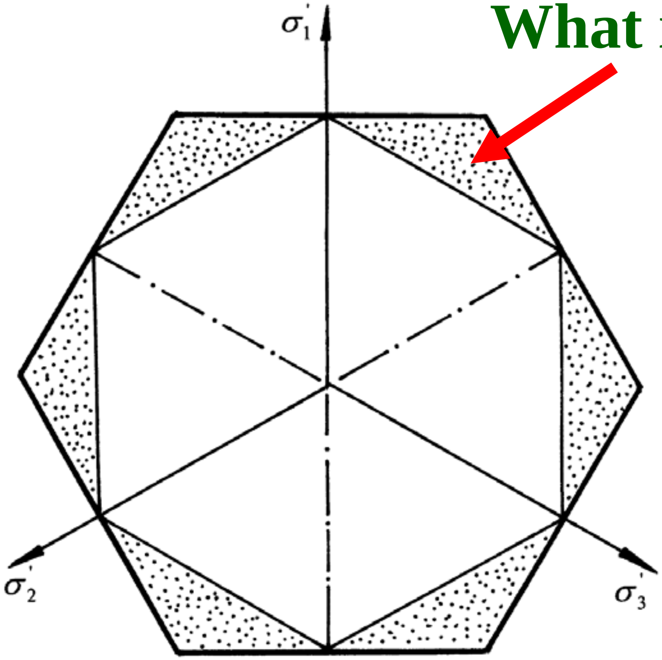


τ_{13}, τ_{12}, τ_{23}

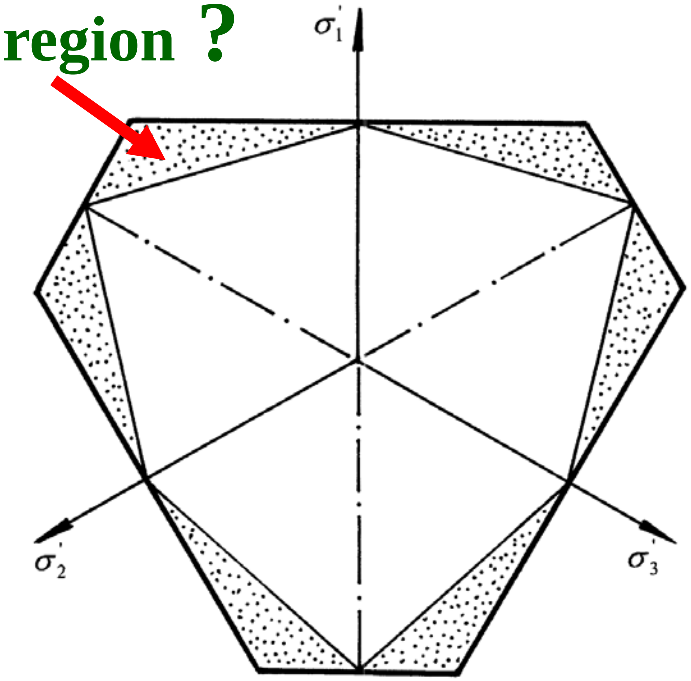
2 Unified Strength Theory

Bounds and region

What is for the region ?



yield loci for metal material



limit loci for geo-material

2 Unified Strength Theory

A parameter b is introduced, and the unified strength theory is modeled as:

$$\begin{cases} F = \tau_{13} + b\tau_{12} + \beta(\sigma_{13} + b\sigma_{12}) = C & \text{when } F \ge F' \\ F' = \tau_{13} + b\tau_{23} + \beta(\sigma_{13} + b\sigma_{23}) = C & \text{when } F \le F' \end{cases}$$

where C、β、b could be determined by tension σ_t、 compressionσ_c and pure shear τ_0 as well as:

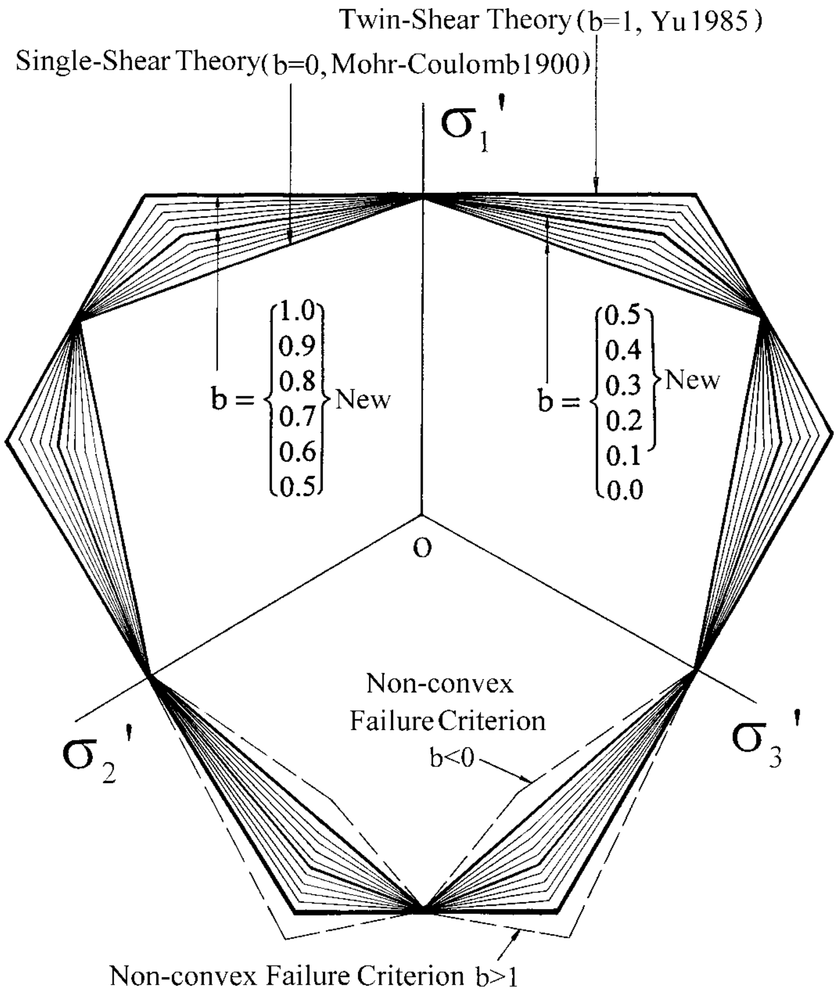
$$\beta = \frac{1-\alpha}{1+\alpha}, \qquad C = \frac{(1+b)}{1+\alpha}\sigma_t, \qquad b = \frac{\sigma_t \tau_0}{(\sigma_t - \tau_0)\sigma_c} - 1$$

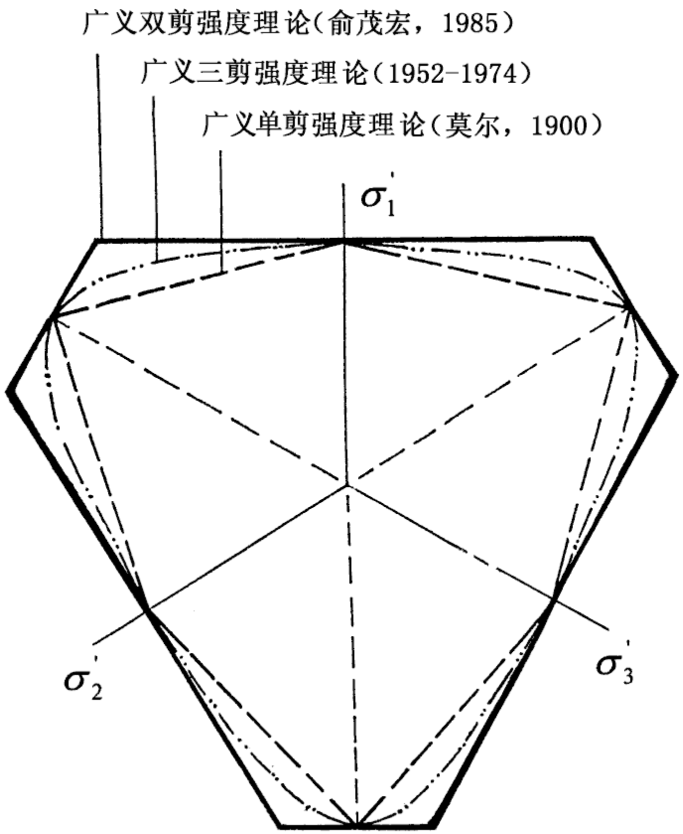
Convert to pincipale stress space:

$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_t \qquad when \quad \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \alpha\sigma_3 = \sigma_t \qquad when \quad \sigma_2 \ge \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

2 Unified Strength Theory



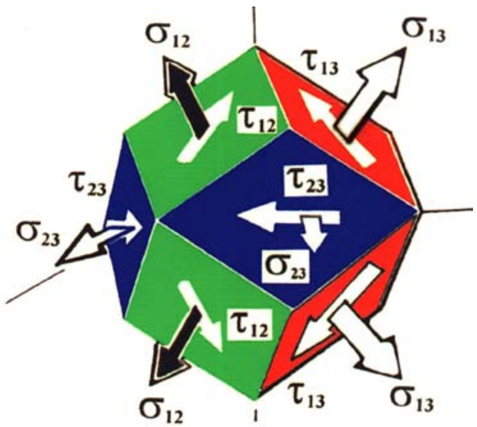


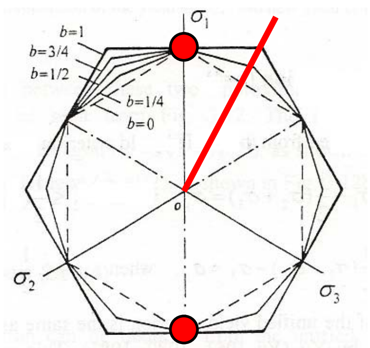
Limit Loci of the Unified Strength Theory at Deviatoric Plane

2 Unified Strength Theory

$$f = \tau_{13} + b\tau_{12} + \beta(\sigma_{13} + b\sigma_{12}) = C \qquad if \quad \tau_{12} + \beta\sigma_{12} \ge \tau_{23} + \beta\sigma_{23}$$
$$f' = \tau_{13} + b\tau_{23} + \beta(\sigma_{13} + b\sigma_{23}) = C \qquad if \quad \tau_{12} + \beta\sigma_{12} \le \tau_{23} + \beta\sigma_{23}$$

β reflects SD effect , b reflects pure shear property.



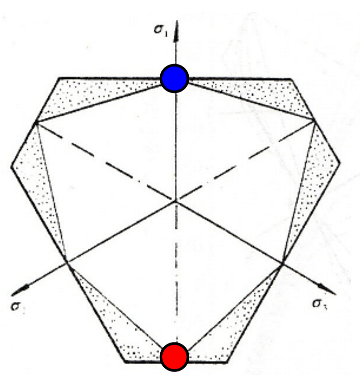


$\beta = 0$ **Metal material**

$b = 0$ ➡ $\tau_{13} = C$ **Tresca**

$b = 0.5$ ➡ $\tau_8 = C$ **Mises approximate**

$b = 1$ ➡
$$f = \tau_{13} + \tau_{12} = C \qquad if \quad \tau_{12} \ge \tau_{23}$$
$$f' = \tau_{23} + \tau_{23} = C \qquad if \quad \tau_{12} \le \tau_{23}$$
Twin-shear criterion



$\beta \neq 0$ **Geomaterial**

$b = 0$ ➡ $\tau_{13} + \beta\sigma_{13} = C$ **Mohr-Coulomb**

$b = 0.5$ ➡ $\tau_8 + \beta\sigma_8 = C$ **Druker-Prager approximate**

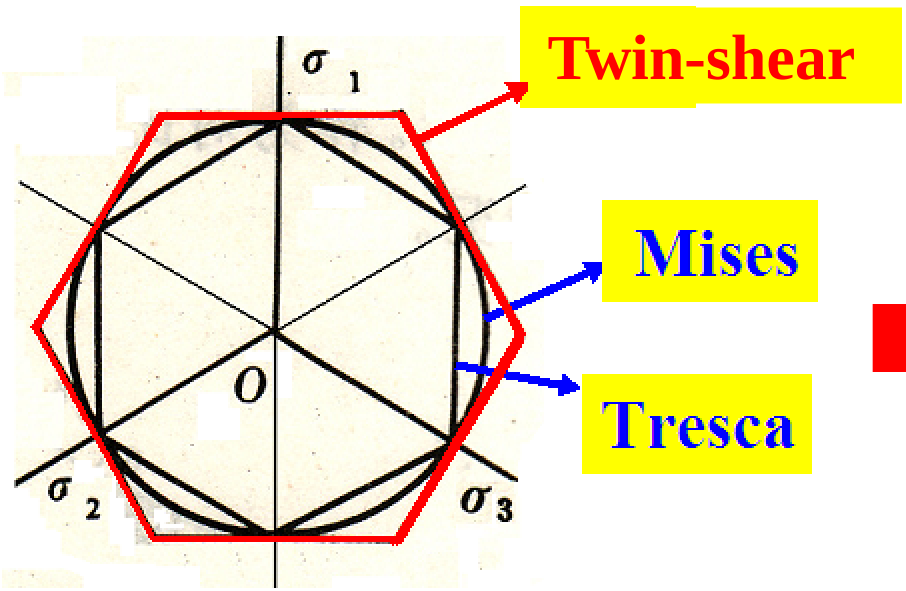
$b = 1$ ➡
$$f = \tau_{13} + \tau_{12} + \beta(\sigma_{13} + \sigma_{12}) = C \qquad if \quad \tau_{12} + \beta\sigma_{12} \ge \tau_{23} + \beta\sigma_{23}$$
$$f' = \tau_{23} + \tau_{23} + \beta(\sigma_{13} + \sigma_{12}) = C \qquad if \quad \tau_{12} + \beta\sigma_{12} \le \tau_{23} + \beta\sigma_{23}$$

Twin-shear strength theory

2 Unified Strength Theory

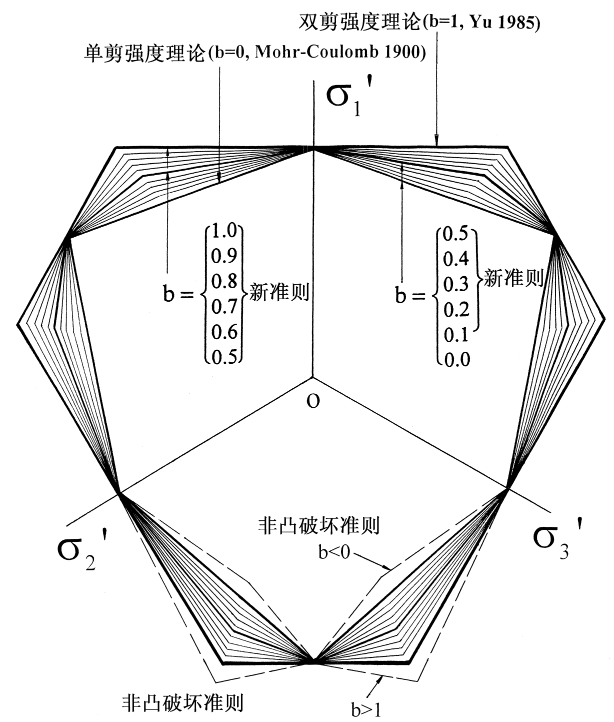
$$
\begin{aligned}
f &= \tau_{13} + b\tau_{12} + \beta(\sigma_{13} + b\sigma_{12}) = C \qquad \text{if} \quad \tau_{12} + \beta\sigma_{12} \ge \tau_{23} + \beta\sigma_{23} \\
f' &= \tau_{13} + b\tau_{23} + \beta(\sigma_{13} + b\sigma_{23}) = C \qquad \text{if} \quad \tau_{12} + \beta\sigma_{12} \le \tau_{23} + \beta\sigma_{23}
\end{aligned}
$$

Three yield criteria



π-plane

Unified strength theory



2 Unified Strength Theory

Classic Strength Theory Progress

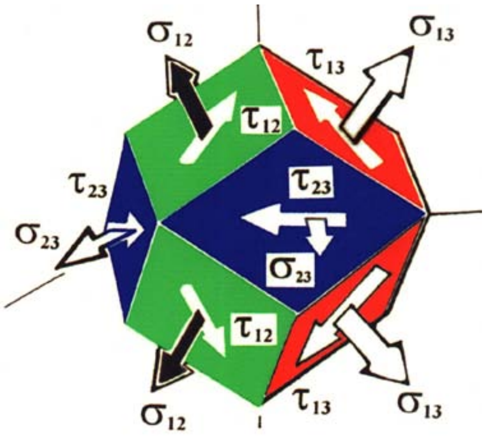
Single-shear concept
Tresca 1864 → **Mohr-Coulomb**

Three-shear concept
Mises 1904 → **Drucker-Prager**

Twin-shear concept
1962, 1982 → **Twin shear Strength Theory**
1985-1990

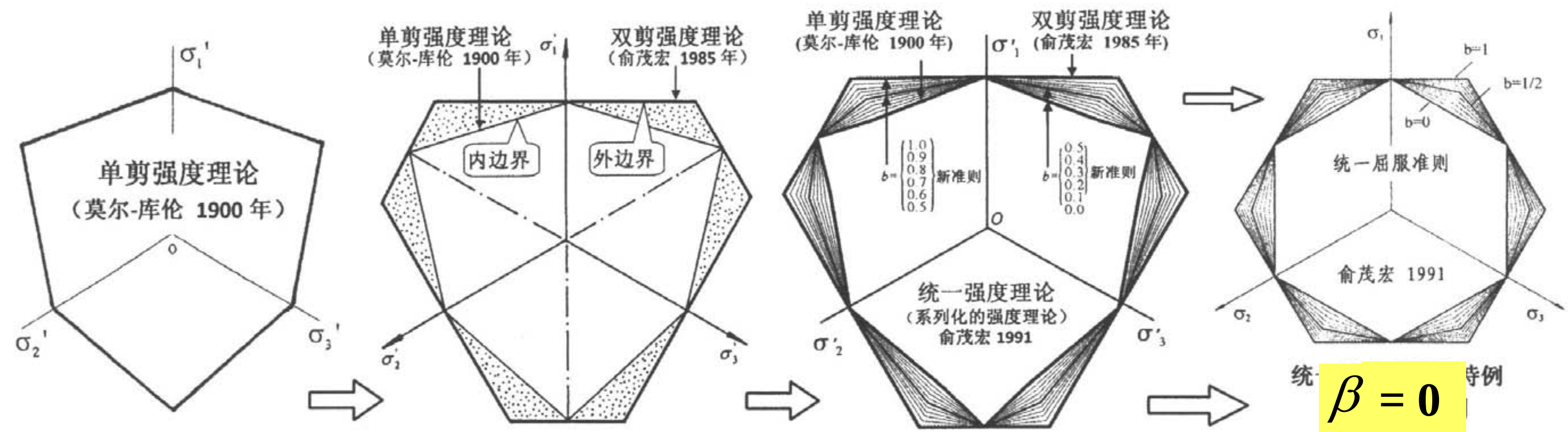
1990-

Unified Strength Theory



2 Unified Strength Theory

Classic Strength Theory Progress



单剪强度理论(1900) 双剪强度理论(1985) 统一强度理论(1991) 统一屈服准则(1991)

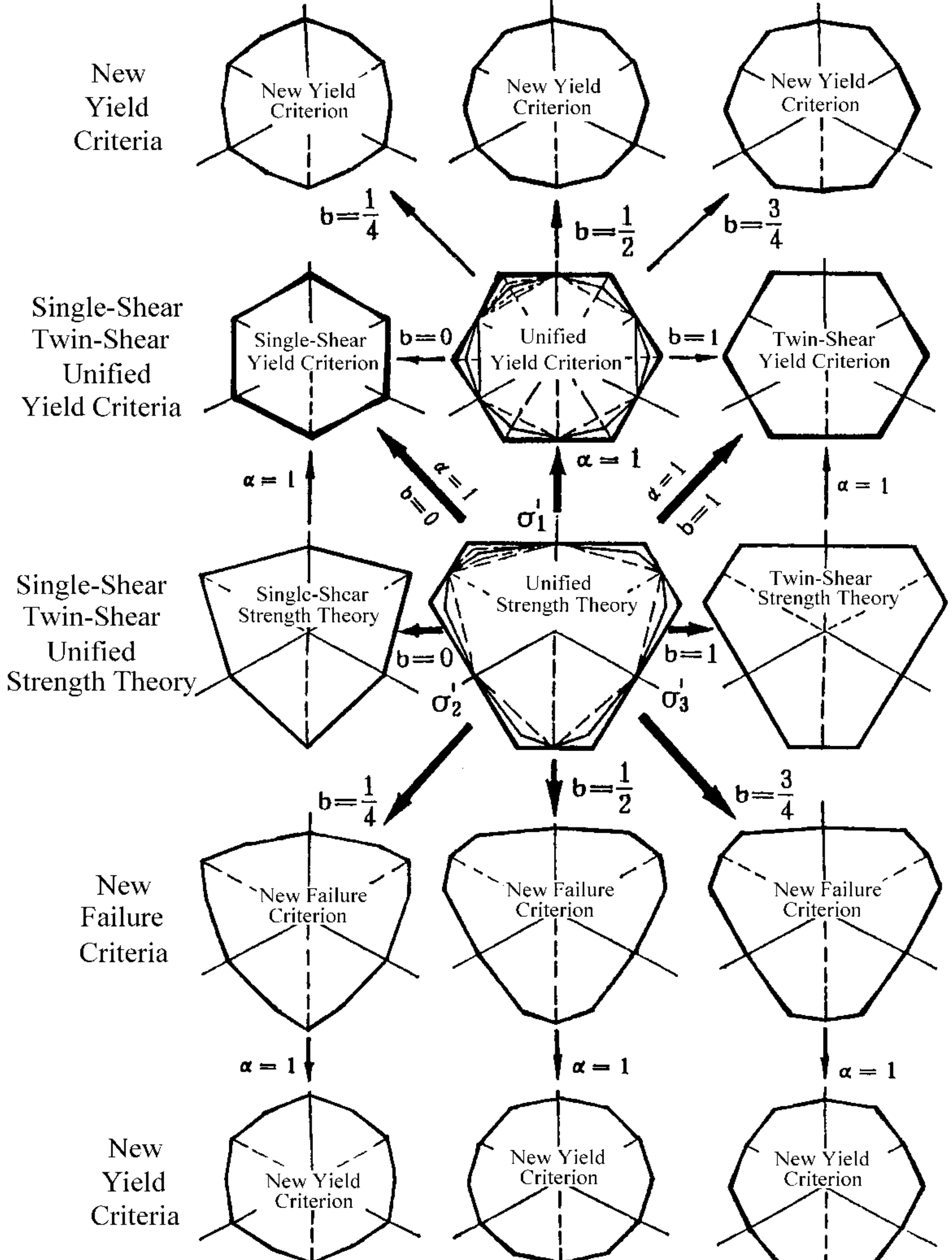
single–shear 1900 **twin–shear 1985** **unified strength theory 1991** **unified yield criterion 1991**

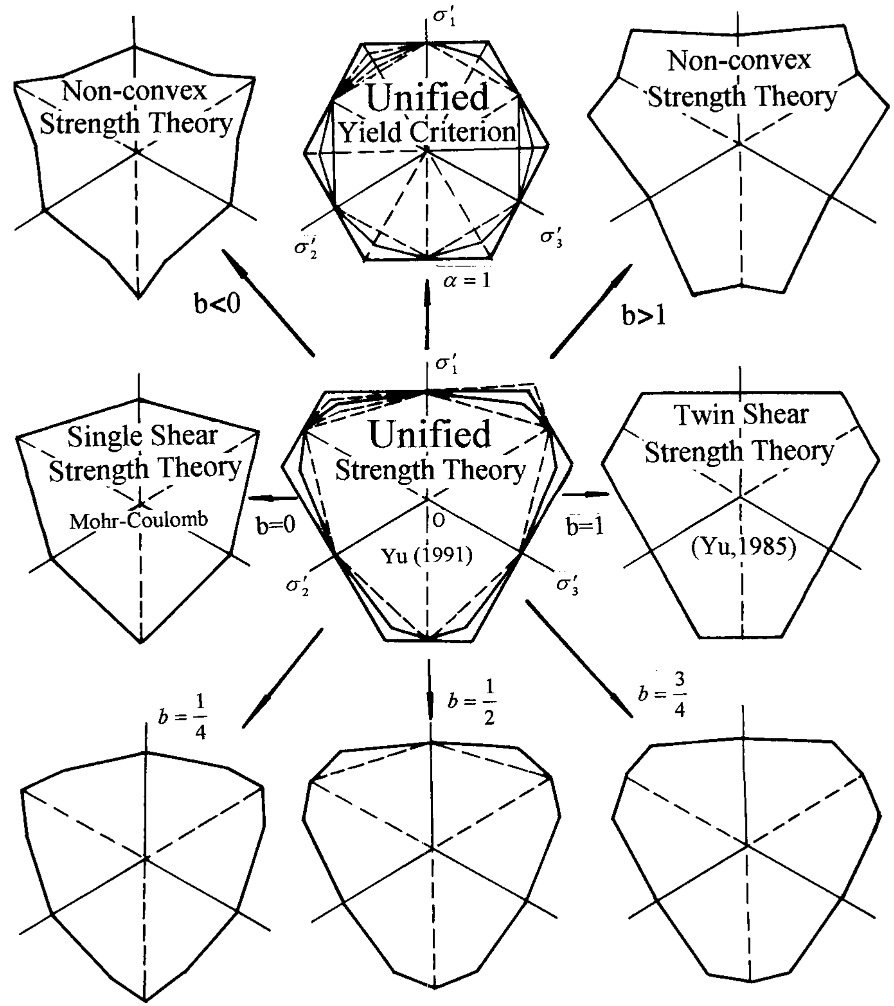
Two things were done.

Unified on π-plane， not meridian plane

Others

b<0; b>1 meaning ?

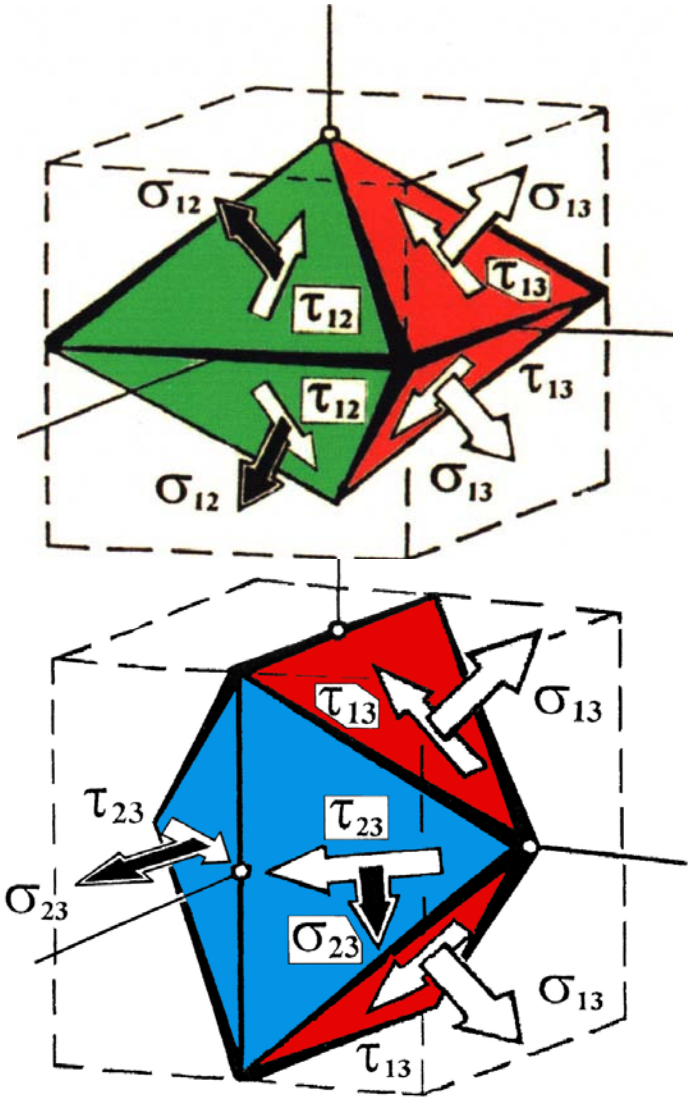


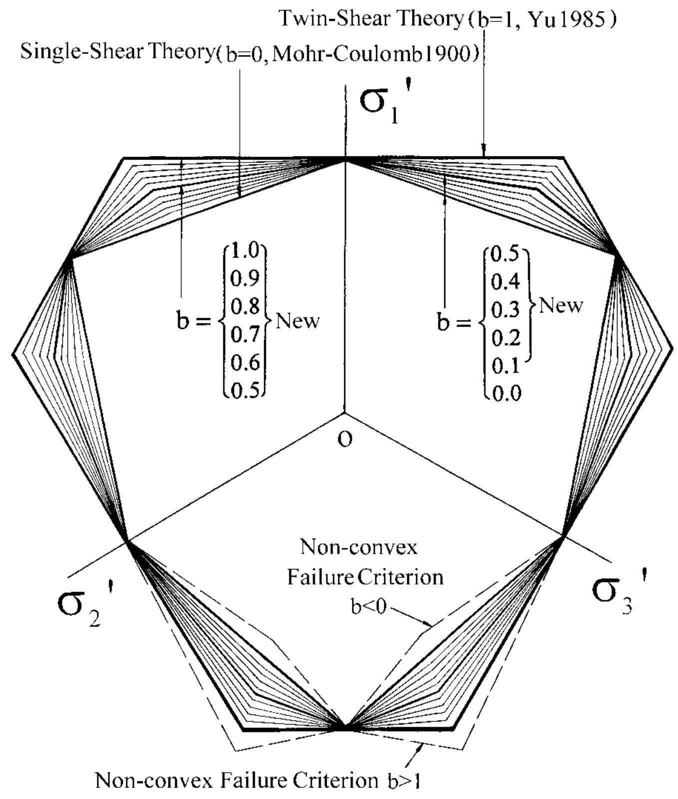


$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_t, \quad \text{when} \quad \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \alpha\sigma_3 = \sigma_t, \quad \text{when} \quad \sigma_2 \ge \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

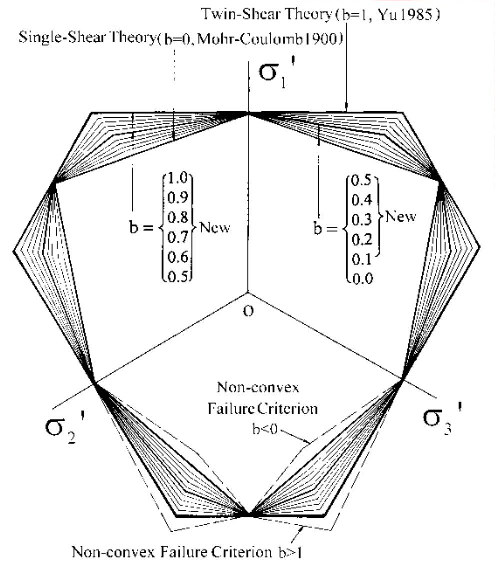
2 Unified Strength Theory> Symmetry

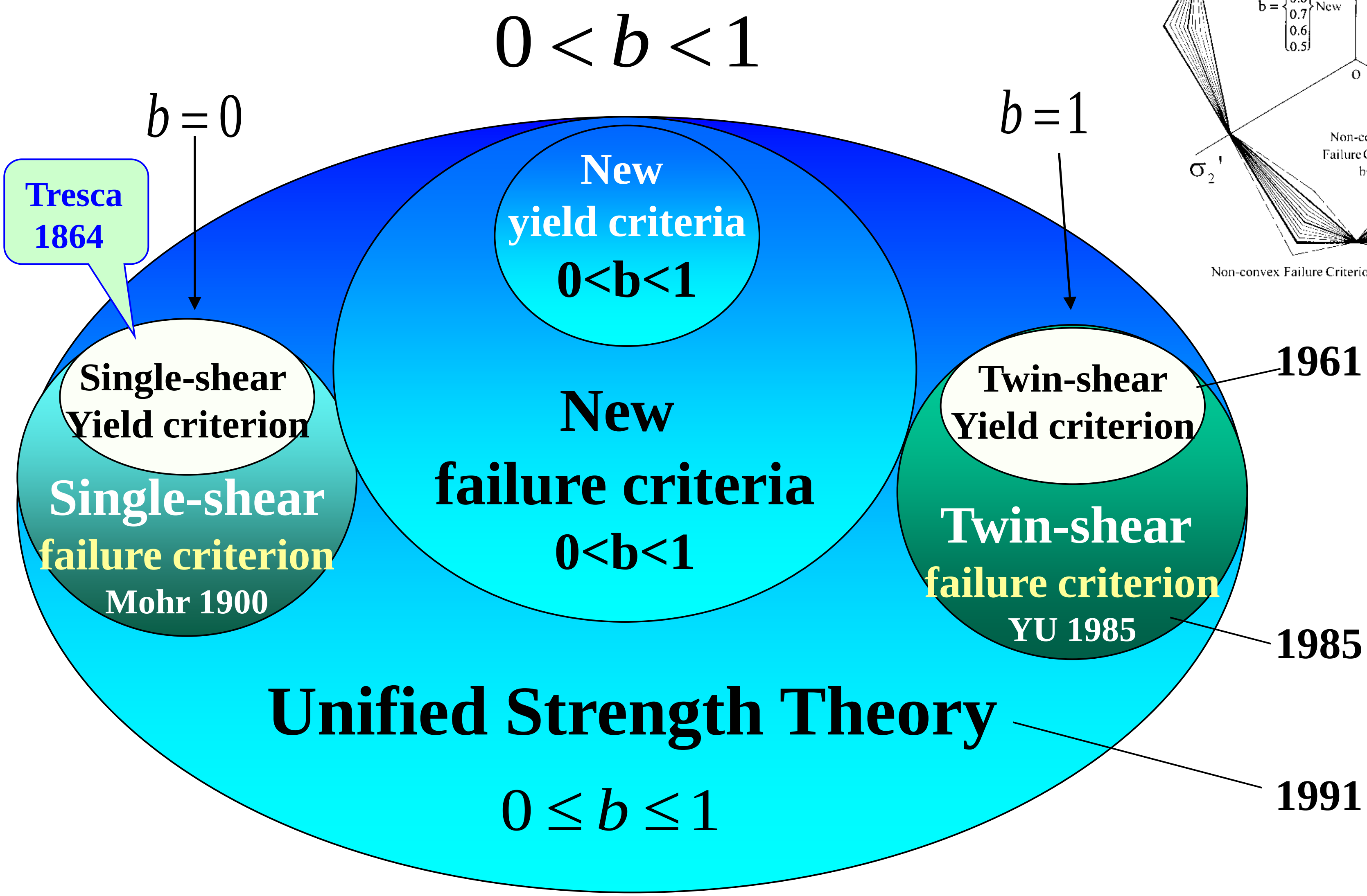




2 Unified Strength Theory>

Unification of Linear Failure Criteria





The choice of b depending on the experiments

shear stress ratio and shear strain is defined

Shear stress ratio

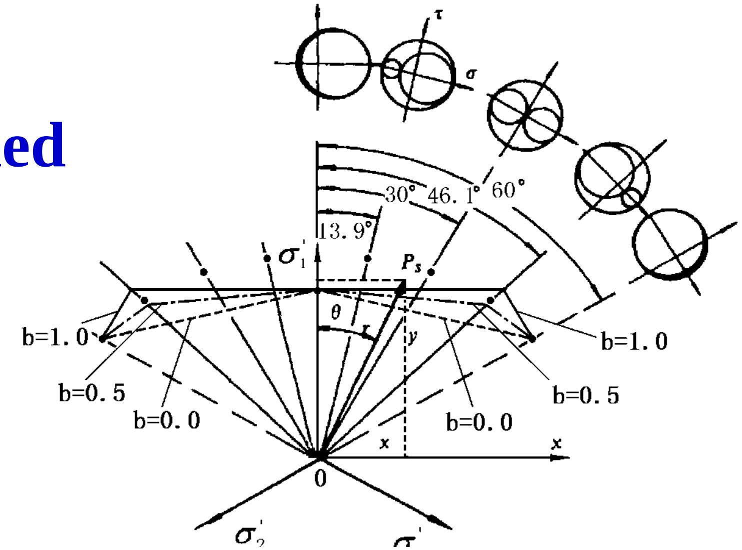
$$\frac{\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}}{\sigma_1'+\sigma_2'+\sigma_3'}$$

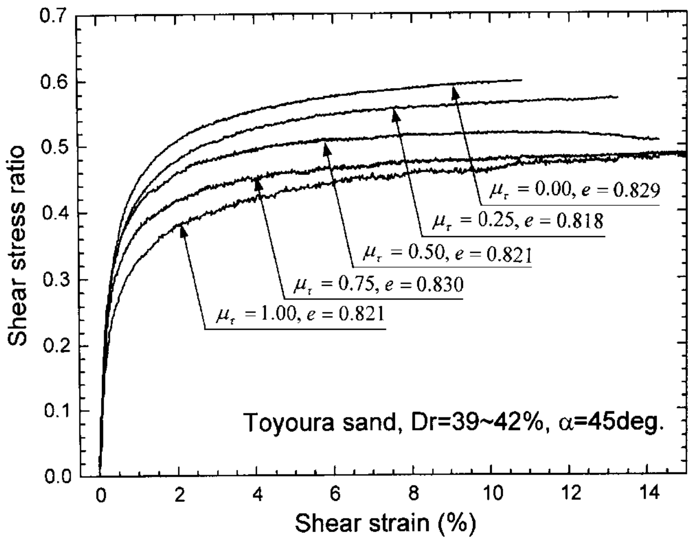
Shear strain

$$\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$$

Twin-shear stress state parameter (Yu 1998)

$$\mu_\tau=\frac{\tau_{12}}{\tau_{13}}=\frac{\sigma_1-\sigma_2}{\sigma_1-\sigma_3}$$





Experiment results of sands by Yoshimine.M.(1996)

It is convenient for analytical solution and numerical calculation.

A series of results can be obtained by using the unified strength theory, including:

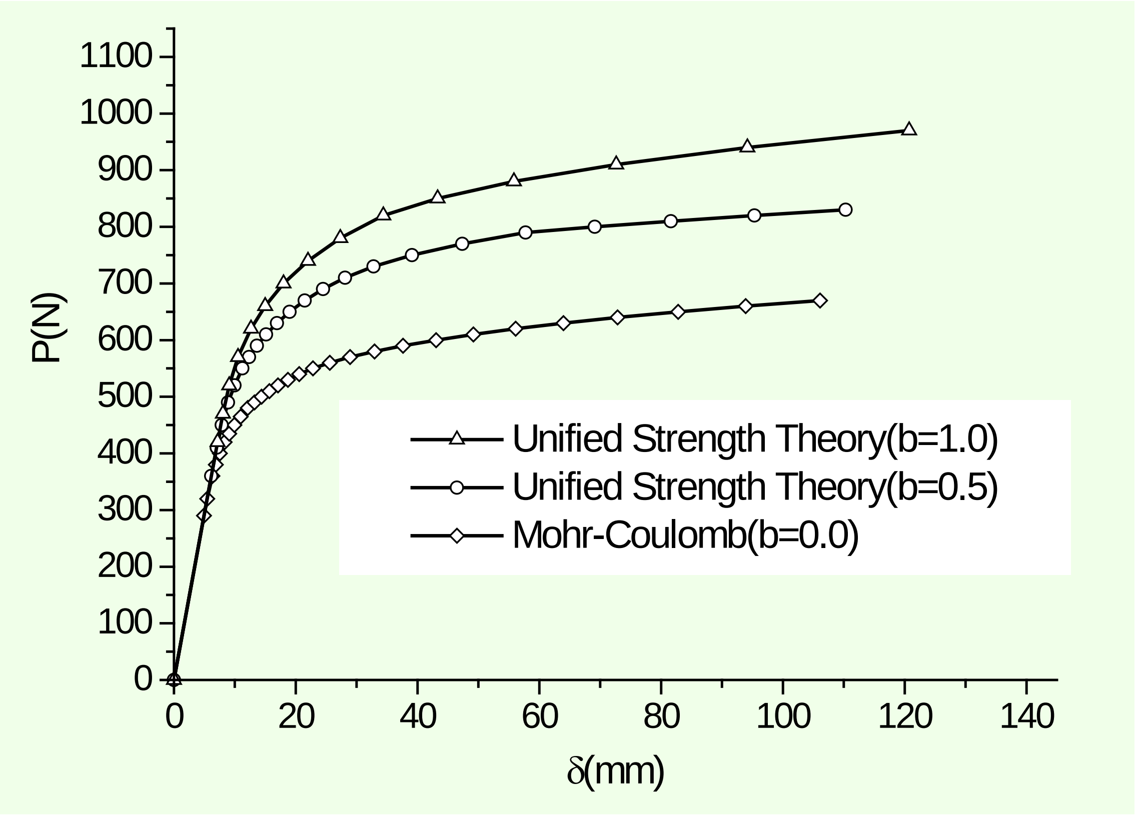
Tresca solution,

Mohr-Coulomb solution,

Twin-shear solution,

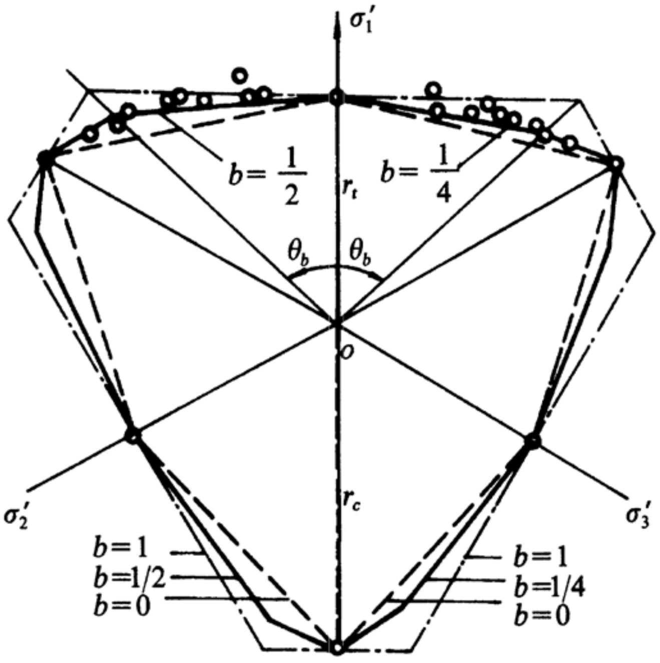
and a lot of new results.

Different b for different theory

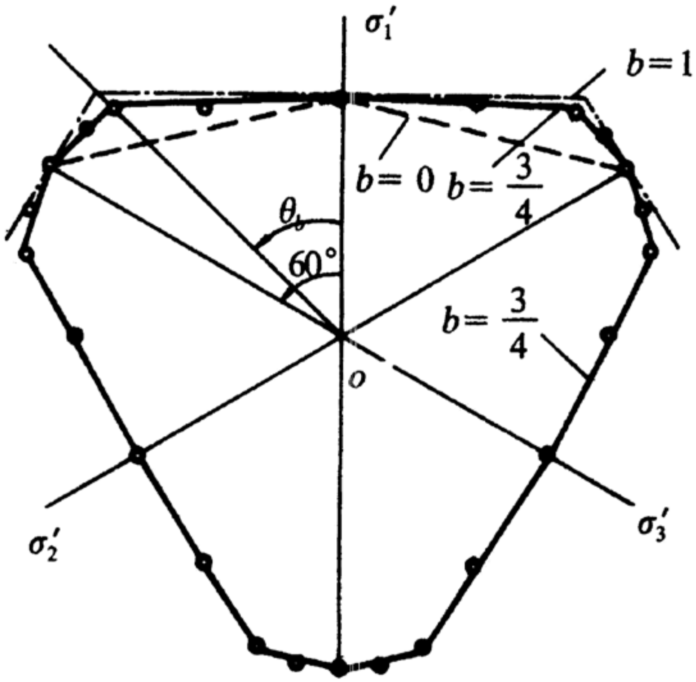


Bearing capacity for weightless soil under a plane strain strip load using unified strength theory (b=0, b=0.5 and b=1).

comparing with experimental results

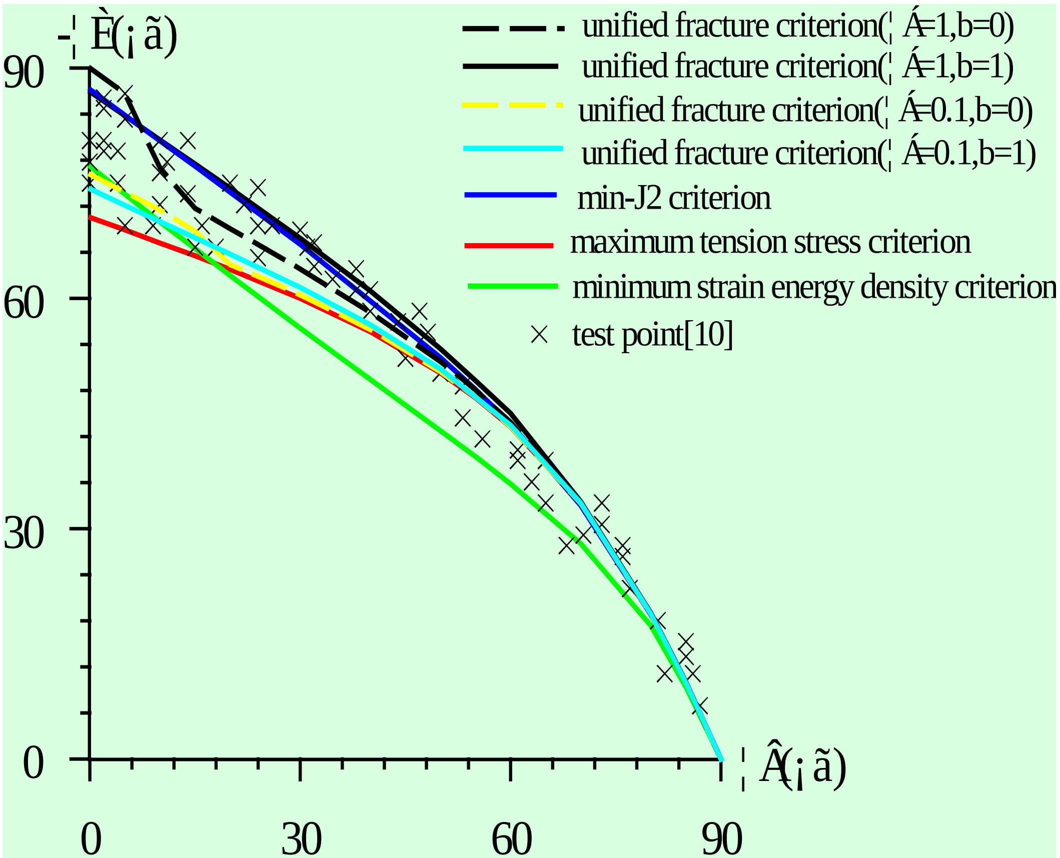


consolidation soil

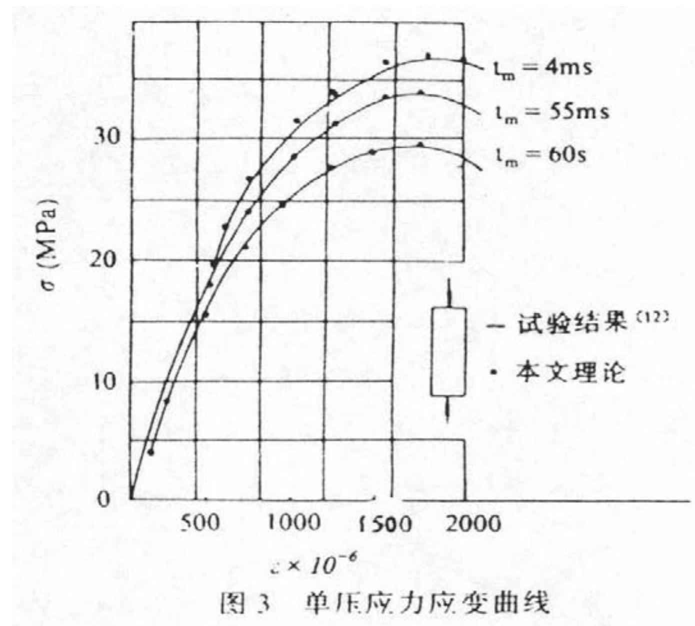


fine sand

comparing with experimental results

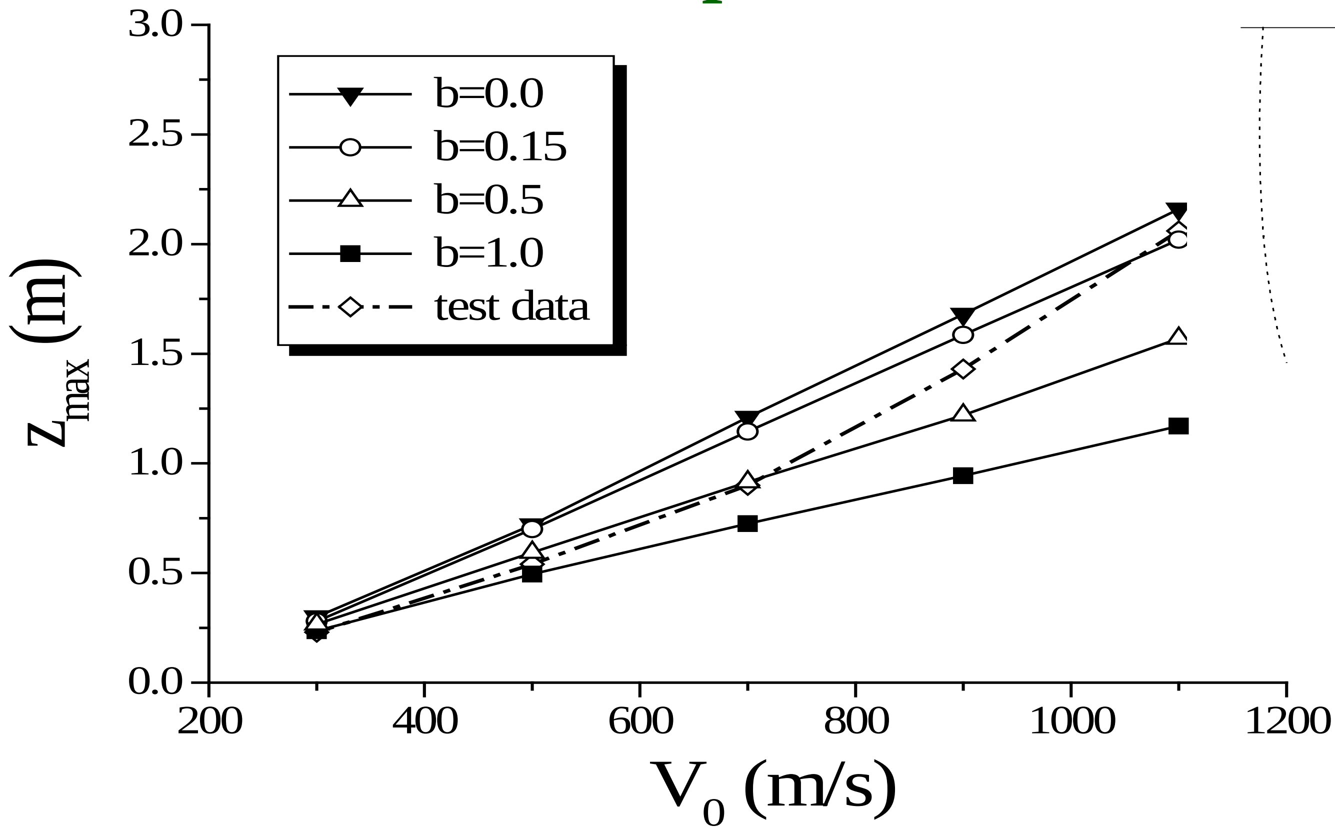


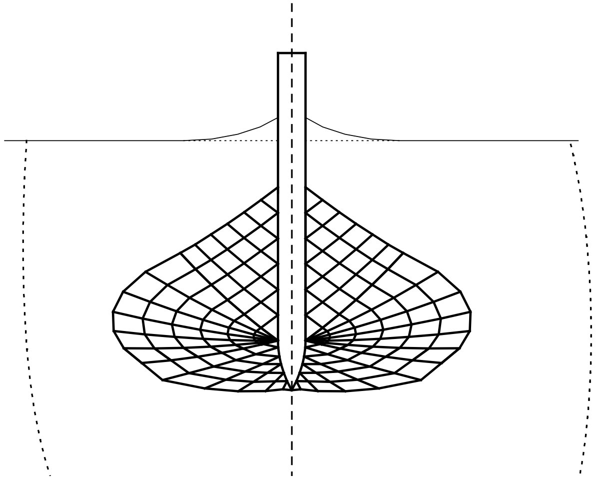
$\mu = 0.167$



Comparison of cracked angle with various fracture criteria

Normal penetration

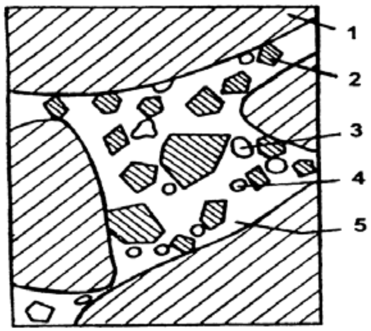




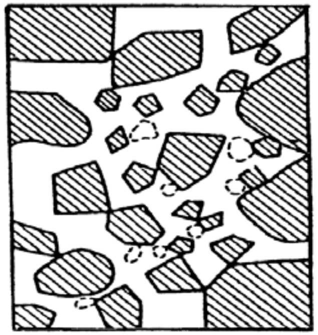
Comparison of penetration deep with test data

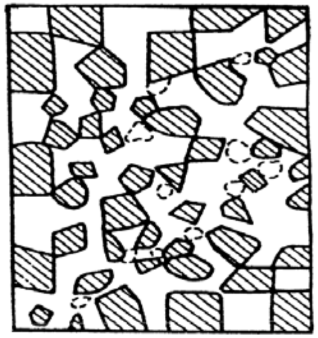
2 Unified Strength Theory > experiments

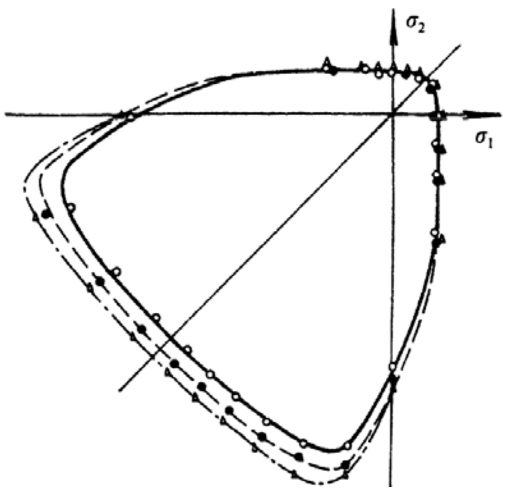
Three meso-concrete models with different aggregate gradation



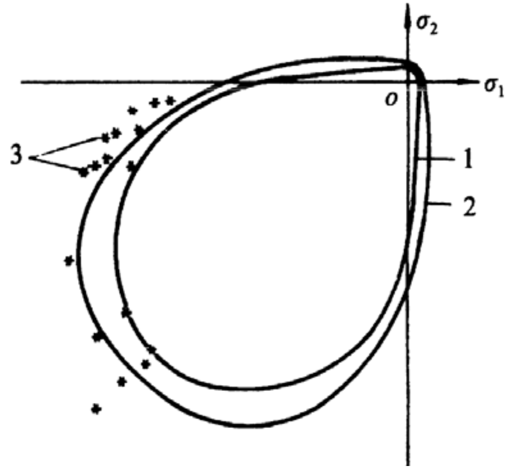
1- Large aggregates
2- Small aggregates
3- Water bubble
4- Air bubble
5- Mortar







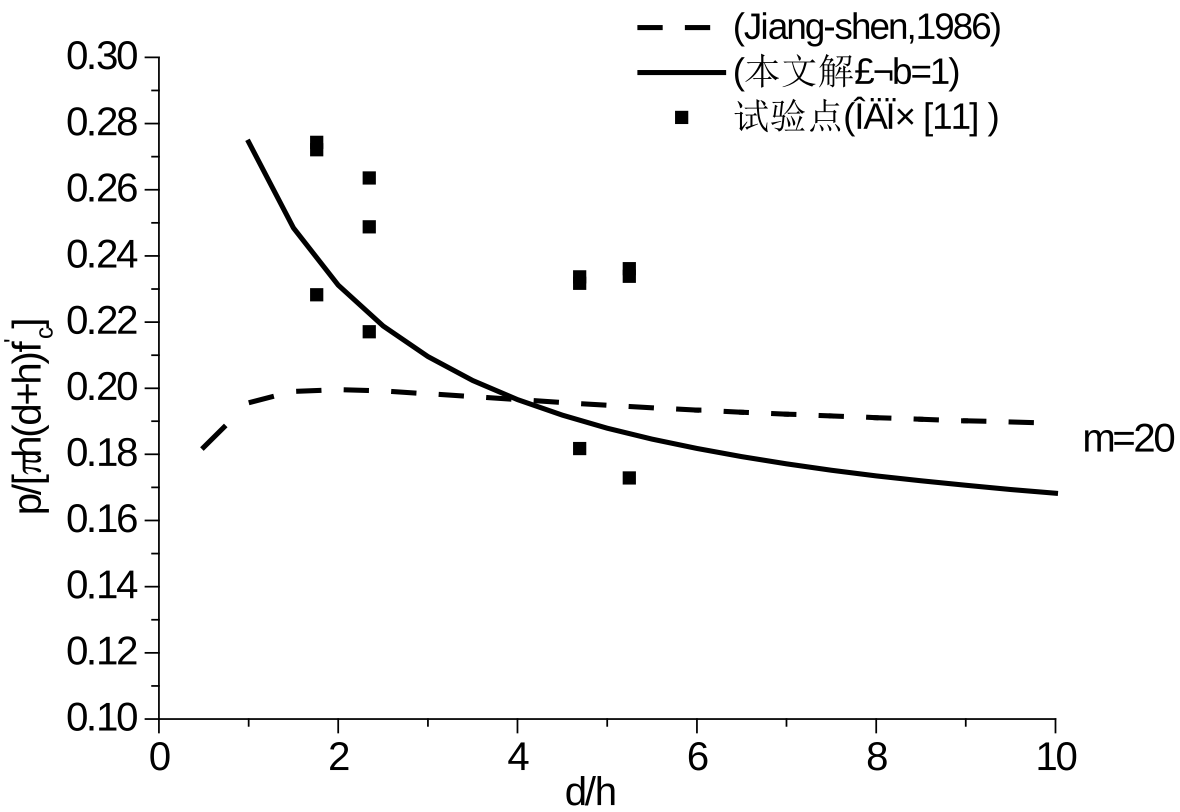
under plastic stress conditions



under plastic strain conditions

Failure loci of three meso-concrete models

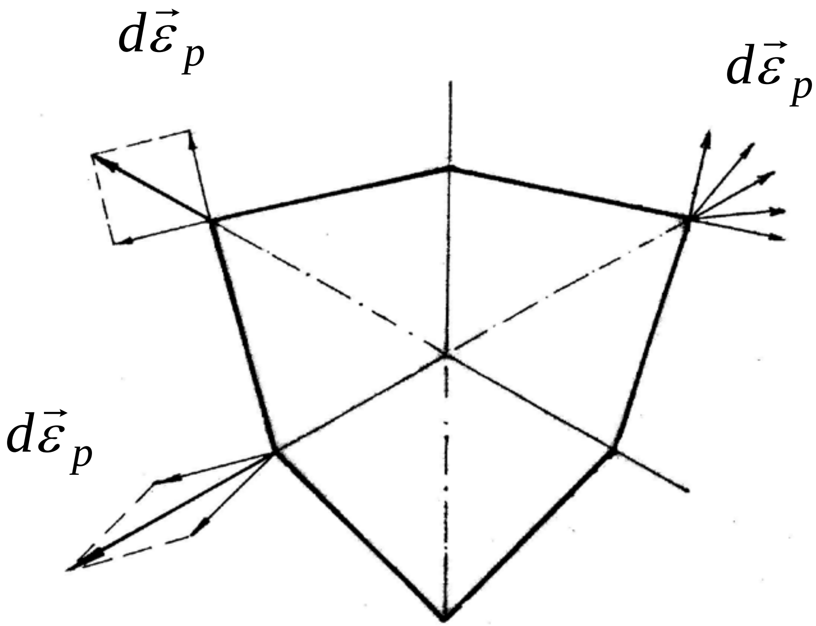
2 Unified Strength Theory > experiments



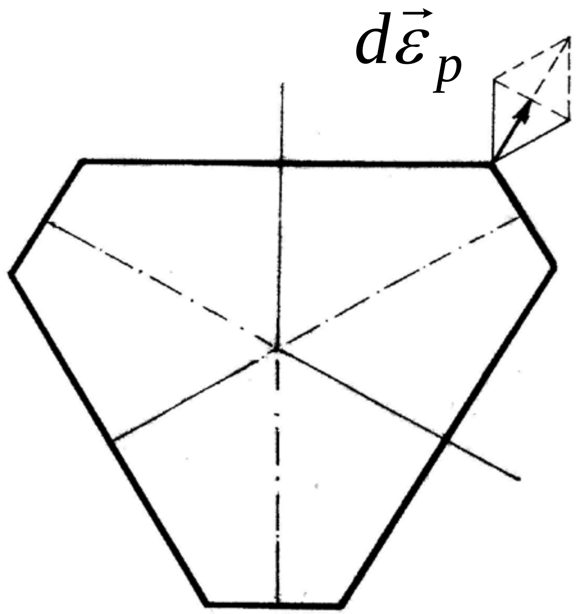
strength theory calculating value comparing with test results for concrete plate punching

3 Unified Constitutive Relation

3.1 Elasto-Plastic Constitutive Relations

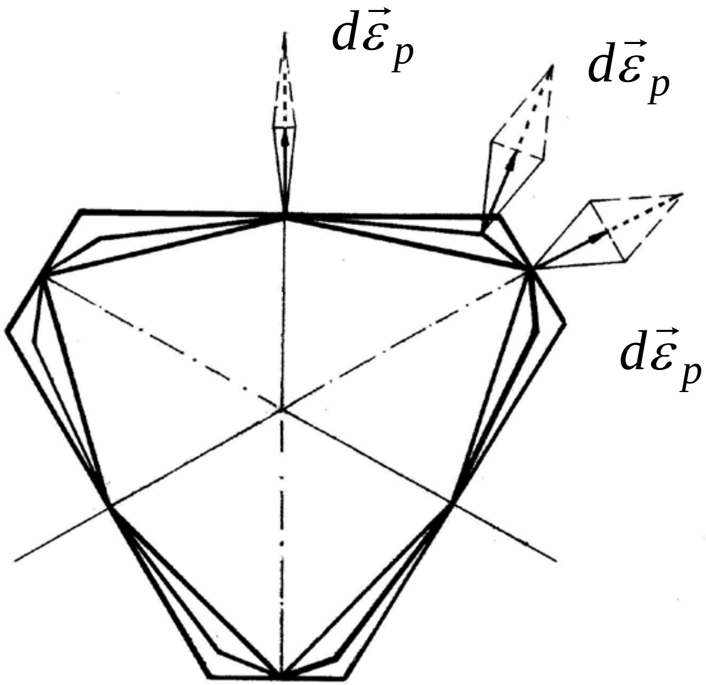


Treatment of coner singularity for single-shear theory

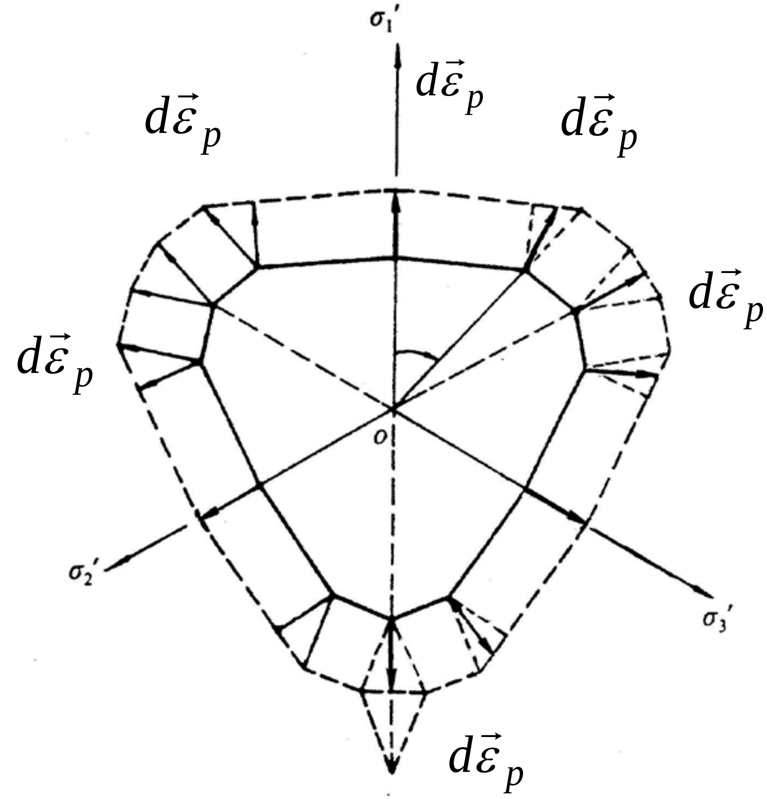


Treatment of coner singularity for twin-shear theory

3 Unified Constitutive Relation



Unified Strength Theory



Full view of platic strain increments

Coner singularity treatment of unified strength theory

3.2 Elasto-plastic FEM Analysis

The unified strength theory can be expressed by stress invariant and stress angle.

The expression can be rewritten as:

$$F=(1+\frac{\alpha}{2})\frac{2J^{1/2}}{\sqrt{3}}\cos\theta+\frac{\alpha(1-b)}{1+b}J^{1/2}\sin\theta+\frac{I_1}{3}(1-\alpha)=\sigma_t$$

$$F^{'}=(\frac{2-b}{1+b}+\alpha)\frac{J^{1/2}}{\sqrt{3}}\cos\theta+(\alpha+\frac{b}{1+b})J^{1/2}\sin\theta+\frac{I_1}{3}(1-\alpha)=\sigma_t^0$$

$$\theta=\frac{1}{3}\cos^{-1}\frac{3\sqrt{3}}{2}\frac{J_3}{\sqrt{J_2^3}}$$

3 Unified Constitutive Relation

$$[D_{ep}] = [D_e] - \frac{[D_e]\left\{\frac{\partial F}{\partial \overline{\sigma}}\right\}\left\{\frac{\partial F}{\partial \overline{\sigma}}\right\}^T [D_e]}{H' + \left\{\frac{\partial F}{\partial \overline{\sigma}}\right\}^T [D_e]\left\{\frac{\partial F}{\partial \overline{\sigma}}\right\}}$$

$$\overline{\alpha}^T = \frac{\partial F}{\partial \overline{\sigma}} = \frac{\partial F}{\partial \overline{\sigma}} \cdot \frac{\partial I_1}{\partial \overline{\sigma}} + \frac{\partial F}{\partial (J_2)^{1/2}} \cdot \frac{\partial (J_2)^{1/2}}{\partial \overline{\sigma}} + \frac{\partial F}{\partial \theta} \cdot \frac{\partial \theta}{\partial \overline{\sigma}}$$

where $\overline{\sigma} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{yx}, \tau_{xy}\}$

$$\frac{\partial \theta}{\partial \overline{\sigma}} = \frac{-\sqrt{3}}{2\sin 3\theta}\left[\frac{1}{(J_2)^{3/2}} \cdot \frac{\partial J_3}{\partial \overline{\sigma}} - \frac{3J_3}{J_2^2} \cdot \frac{\partial (J_2)^{1/2}}{\partial \overline{\sigma}}\right]$$

$$\overline{\alpha} = C_1\overline{\alpha}_1 + C_2\overline{\alpha}_2 + C_3\overline{\alpha}_3$$

where $\overline{\alpha}_1^T = \frac{\partial I_1}{\partial \overline{\sigma}}$ $\quad \overline{\alpha}_2^T = \frac{\partial (J_2)^{1/2}}{\partial \overline{\sigma}}$ $\quad \overline{\alpha}_3^T = \frac{\partial J_3}{\partial \overline{\sigma}}$

$$C_1=\frac{\partial F}{\partial I_1}=\frac{1}{3}(1-\alpha)$$

$$C_2=\frac{\partial F}{\partial (J_2)^{1/2}}+\frac{\mathrm{ctg}3\theta}{(J_2)^{1/2}}\cdot\frac{\partial F}{\partial\theta}$$

$$=(1+\frac{\alpha}{2})\frac{2}{\sqrt{3}}\cos\theta+\frac{\alpha(1-b)}{1+b}\sin\theta+\mathrm{ctg}3\theta\cdot\left[-(1+\alpha/2)\frac{2}{\sqrt{3}}\sin\theta+\frac{\alpha(1-b)}{1+b}\cos\theta\right]$$

$$C_3=-\frac{\sqrt{3}}{2\sin 3\theta(J_2)^{3/2}}\cdot\frac{\partial F}{\partial\theta}=-\frac{\sqrt{3}}{2J_2\sin 3\theta}\cdot\left[-(1+\frac{\alpha}{2})\frac{2}{\sqrt{3}}\sin\theta+\frac{\alpha(1-b)}{1+b}\cos\theta\right]$$

$$C_1^{'}=\frac{\partial F^{'}}{\partial I_1}=\frac{1}{3}(1-\alpha)$$

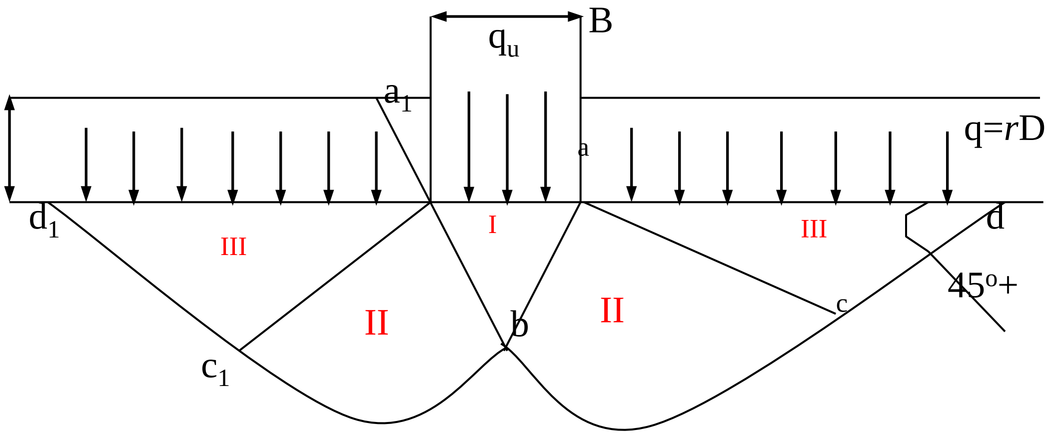
$$C_2^{'}=\frac{\partial F^{'}}{\partial (J_2)^{1/2}}+\frac{\mathrm{ctg}3\theta}{\partial(J_2)^{1/2}}\cdot\frac{\partial F^{'}}{\partial\theta}$$

$$=(\frac{2-b}{1+b}+\alpha)\frac{\cos\theta}{\sqrt{3}}+(\alpha+\frac{b}{1+b})\sin\theta+\mathrm{ctg}3\theta\cdot\left[-(\frac{2-b}{1+b}+\alpha)\frac{\sin\theta}{\sqrt{3}}+(\alpha+\frac{b}{1+b})\cos\theta\right]$$

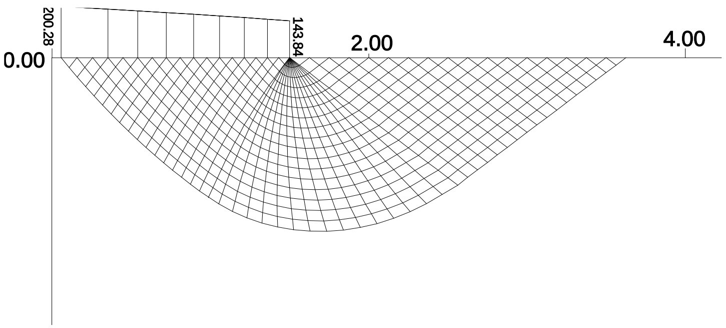
$$C_3^{'}=-\frac{\sqrt{3}}{2\sin 3\theta(J_2)^{3/2}}\cdot\frac{\partial F^{'}}{\partial\theta}=-\frac{\sqrt{3}}{2J_2\sin 3\theta}\cdot\left[-(\frac{2-b}{1+b}+\alpha)\frac{\sin\theta}{\sqrt{3}}+(\alpha+\frac{b}{1+b})\cos\theta\right]$$

$$C_1^0=\frac{1}{2}(C_1+C_1^{'})\qquad C_2^0=\frac{1}{2}(C_2+C_2^{'})\qquad C_3^0=\frac{1}{2}(C_3+C_3^{'})$$

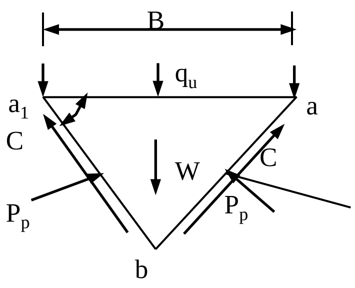
4.1 Unified Slip Field Theory for Plane Strain Problem



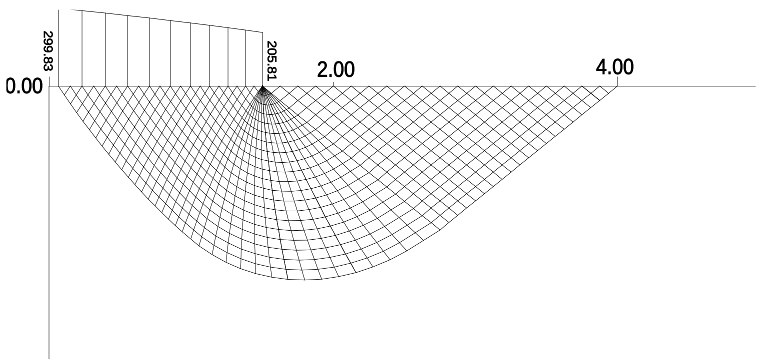
Rough footing of foundation



limit load 171.77 when b=0

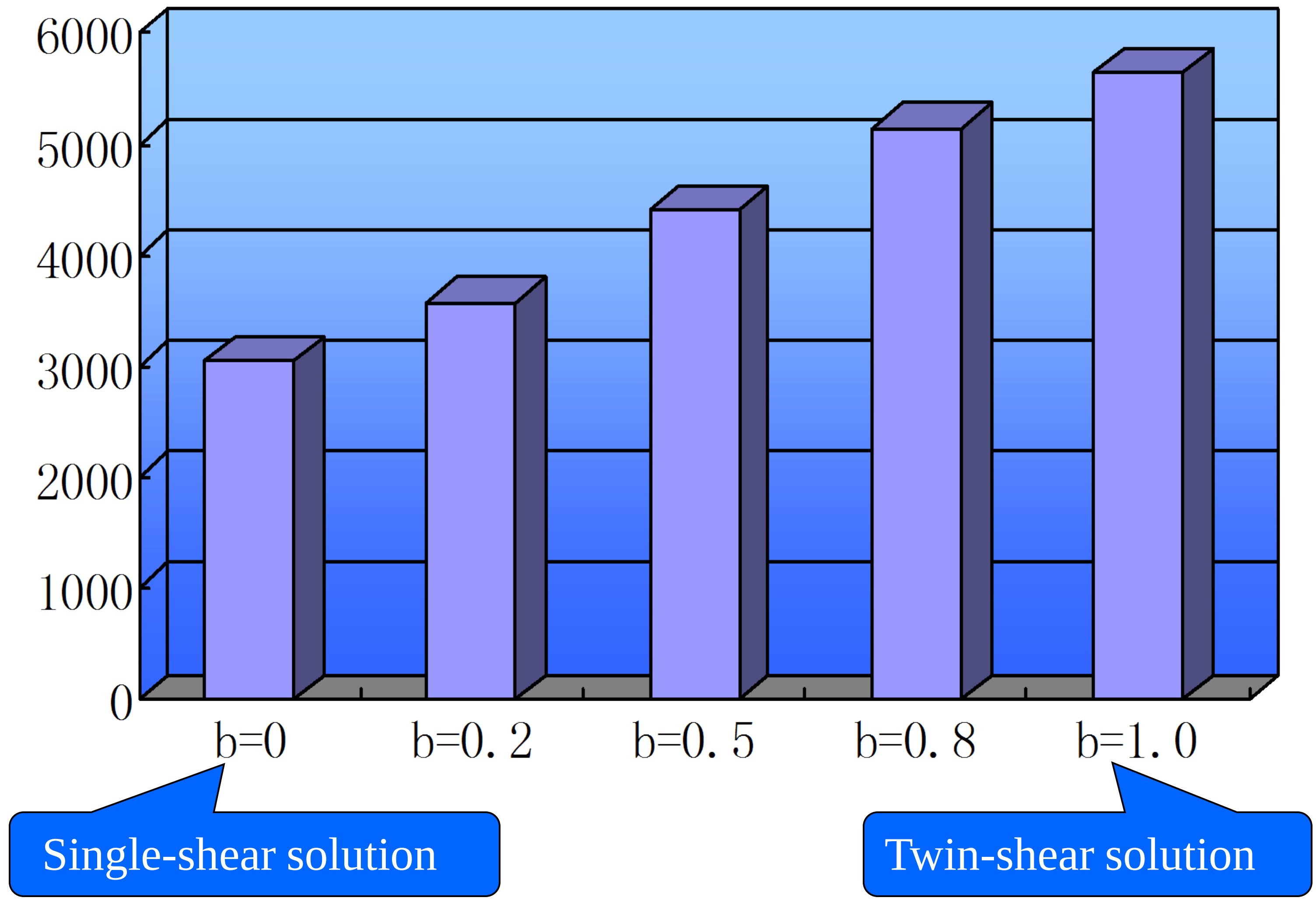


wedge-type model



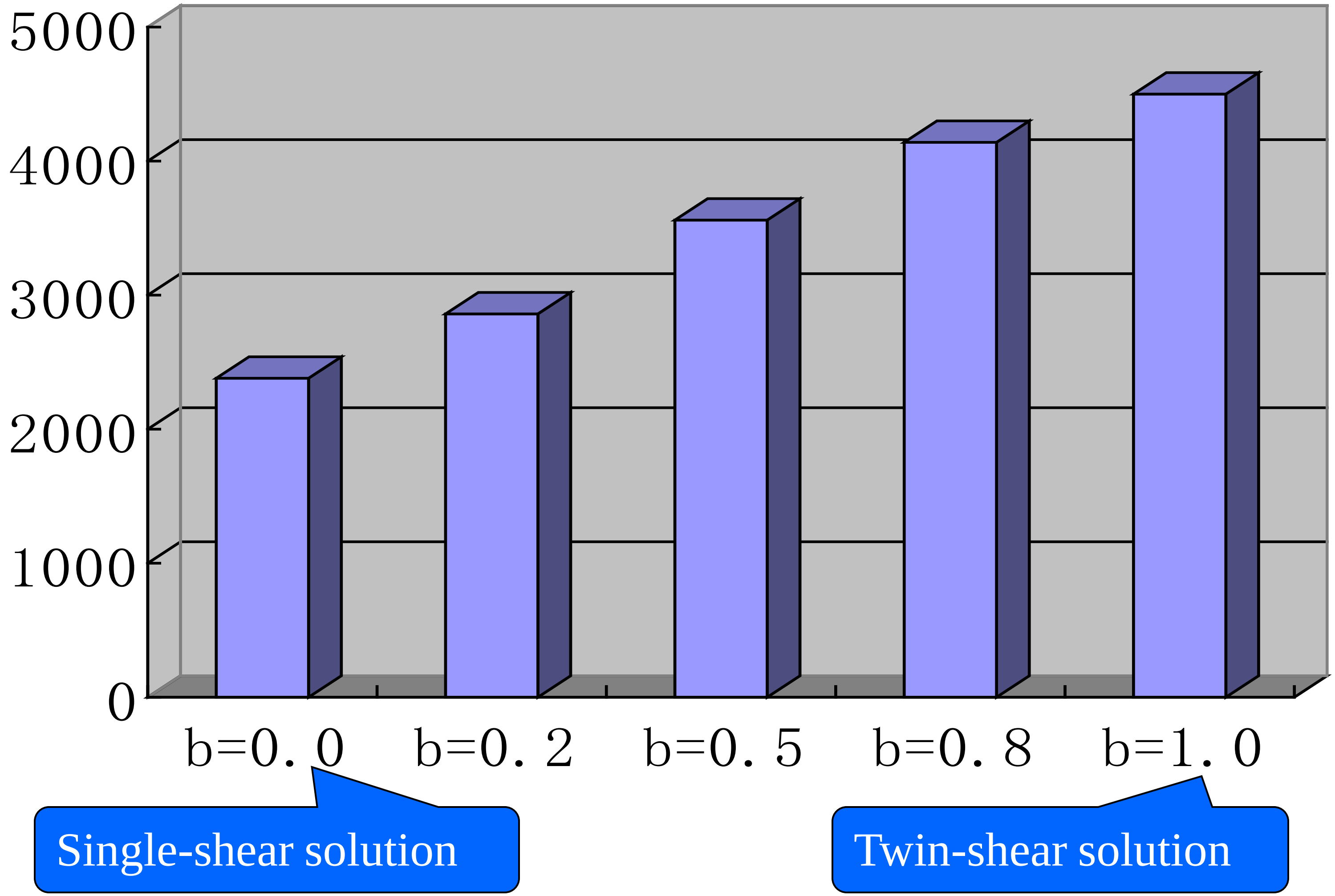
limit load 252.38 when b=1

4 Unified Characteristics line field Theory



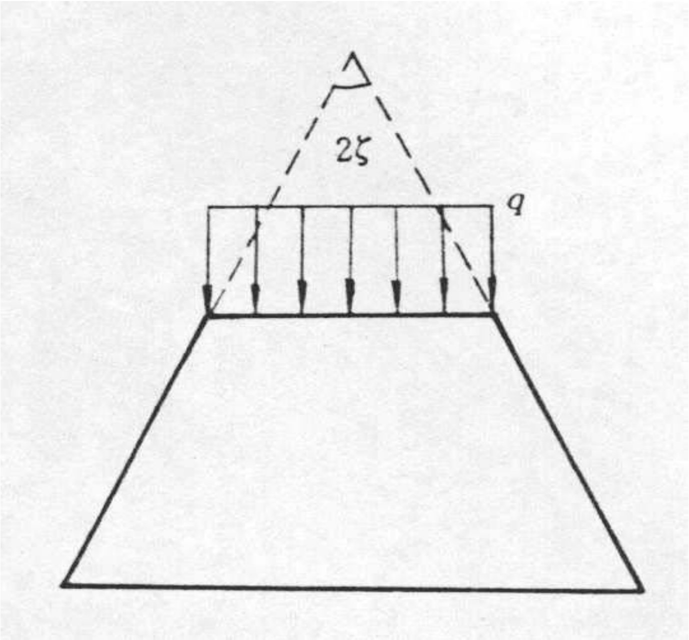
Unified solution of bearing capacity for Rough footing (基底完全粗糙)

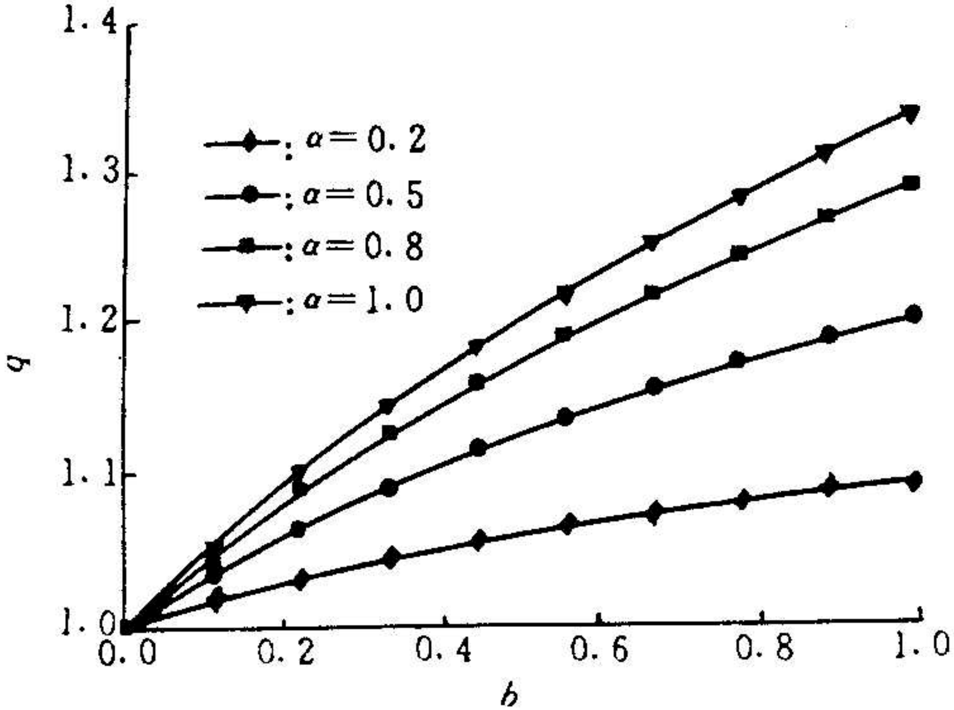
4 Unified Characteristics line field Theory



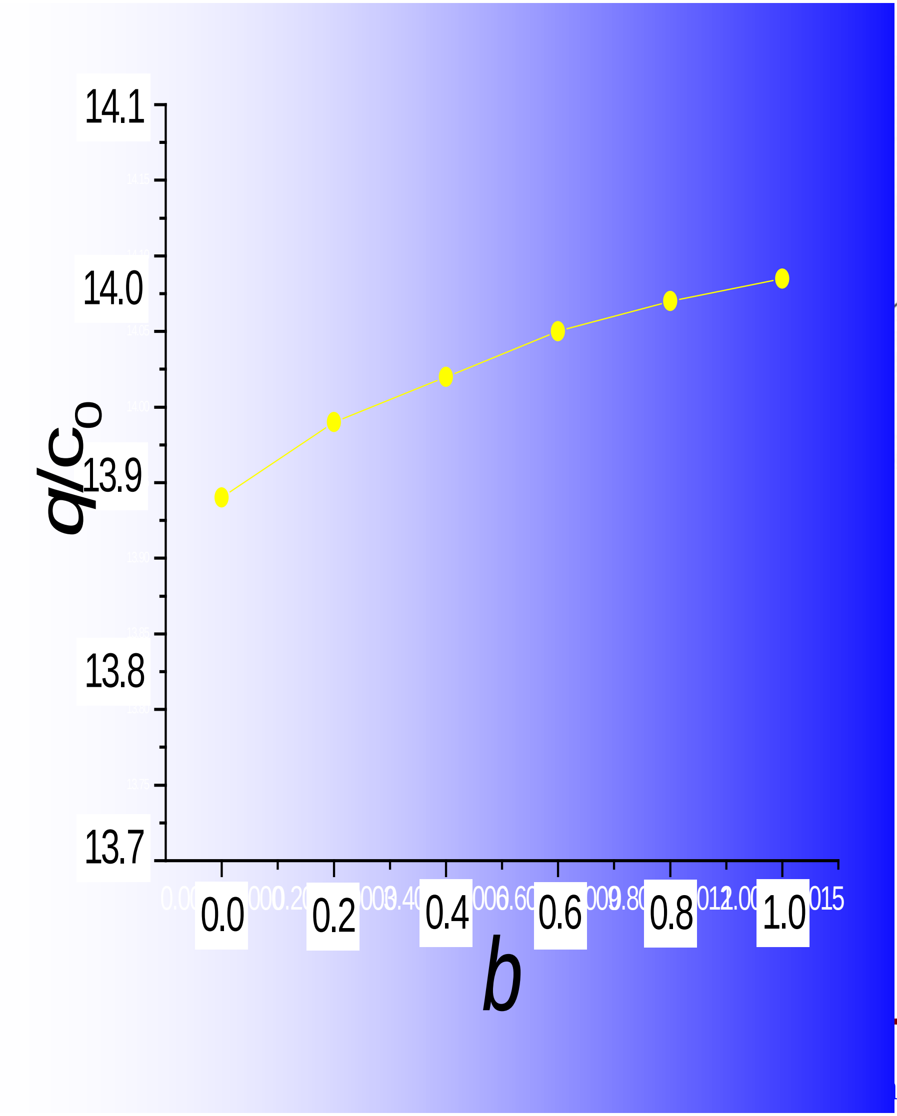
Unified solution of bearing capacity for Smooth footing (基底完全光滑)

4.2 Characteristics Line Field for Plane Stress Problem





4.3 Unified Characteristics Theory for Spatial Axisymmetric Plastic Problem



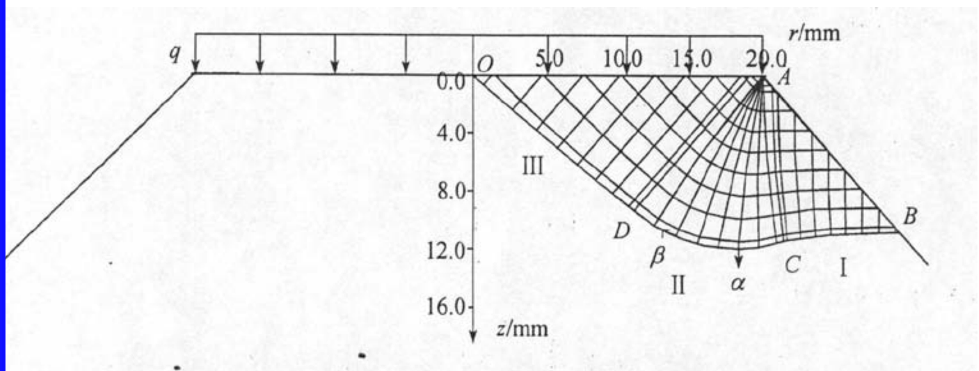
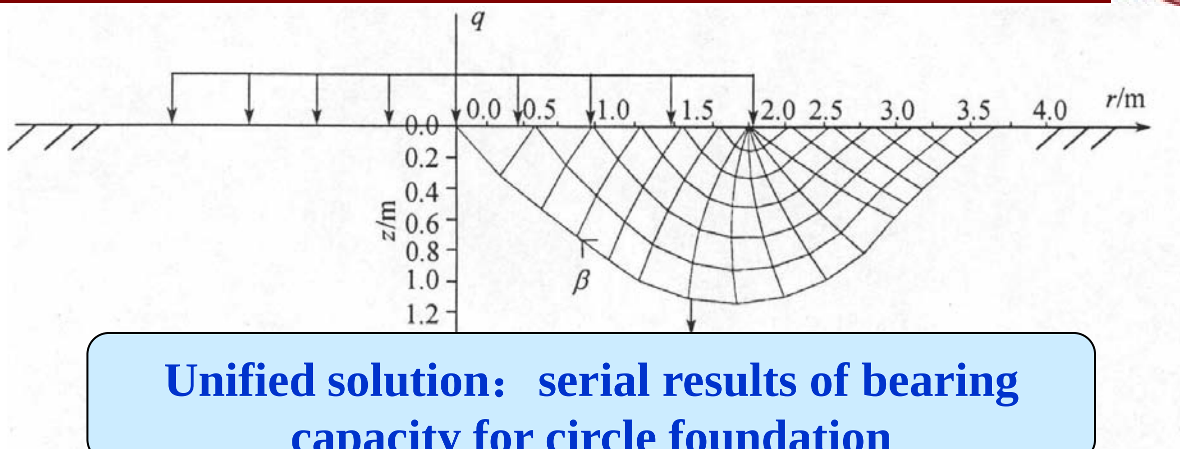


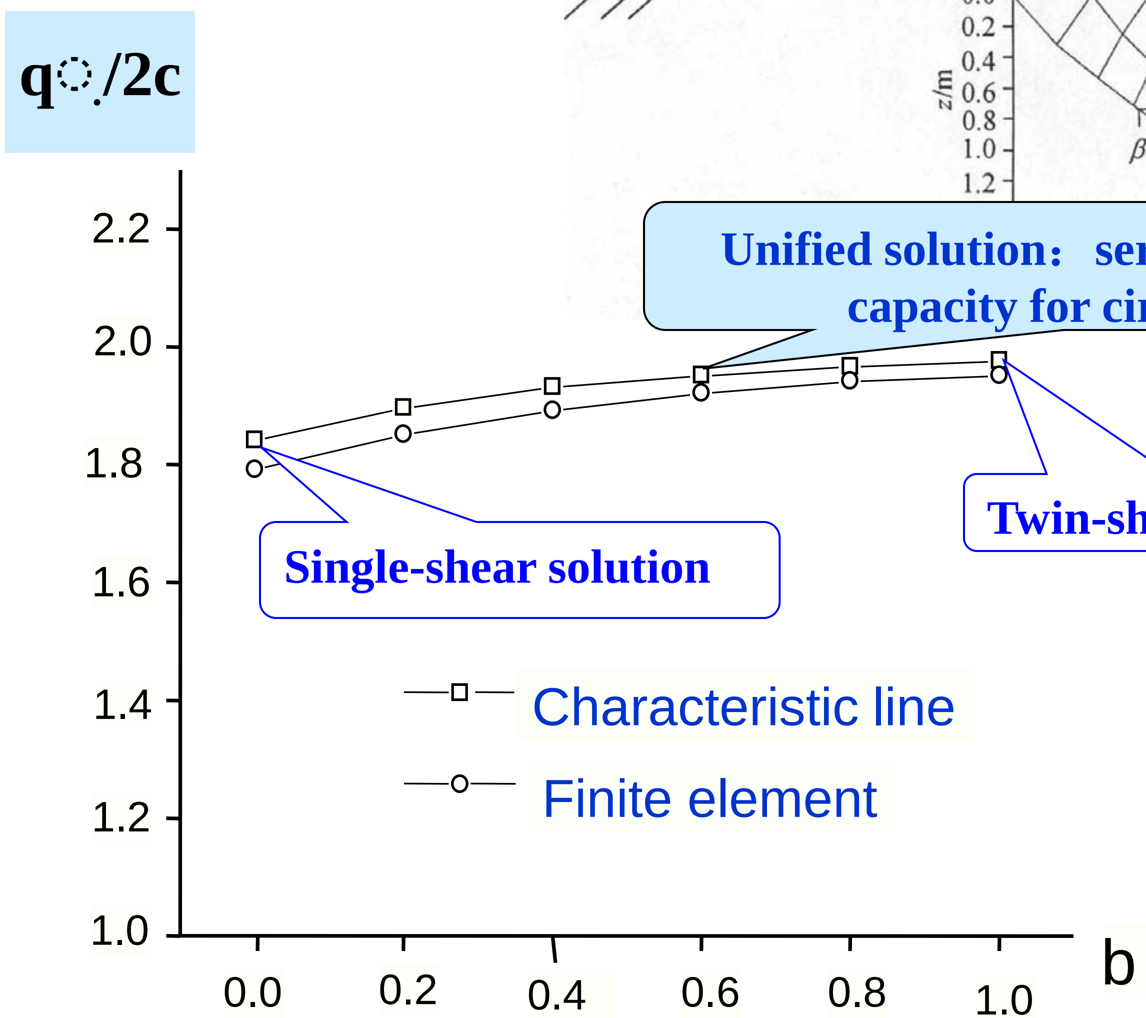
图 3 压力作用下的锥体及其空间特征线场

Yu M-H， et al.

Unified characteristics line theory of spatial axisymmetric plastic problem. *Science in China* (Series E), 2001 44(2): 207-215.

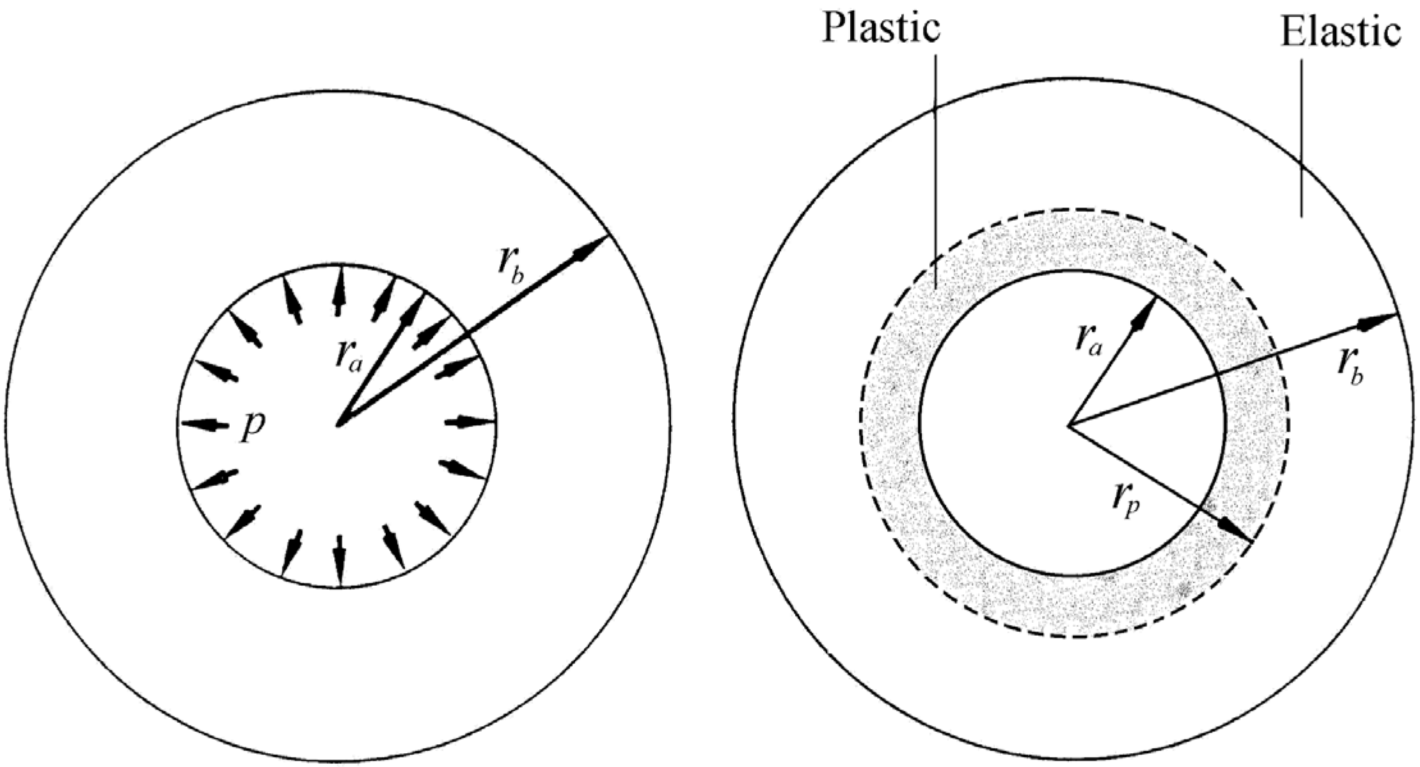
4 Unified Characteristics line field Theory



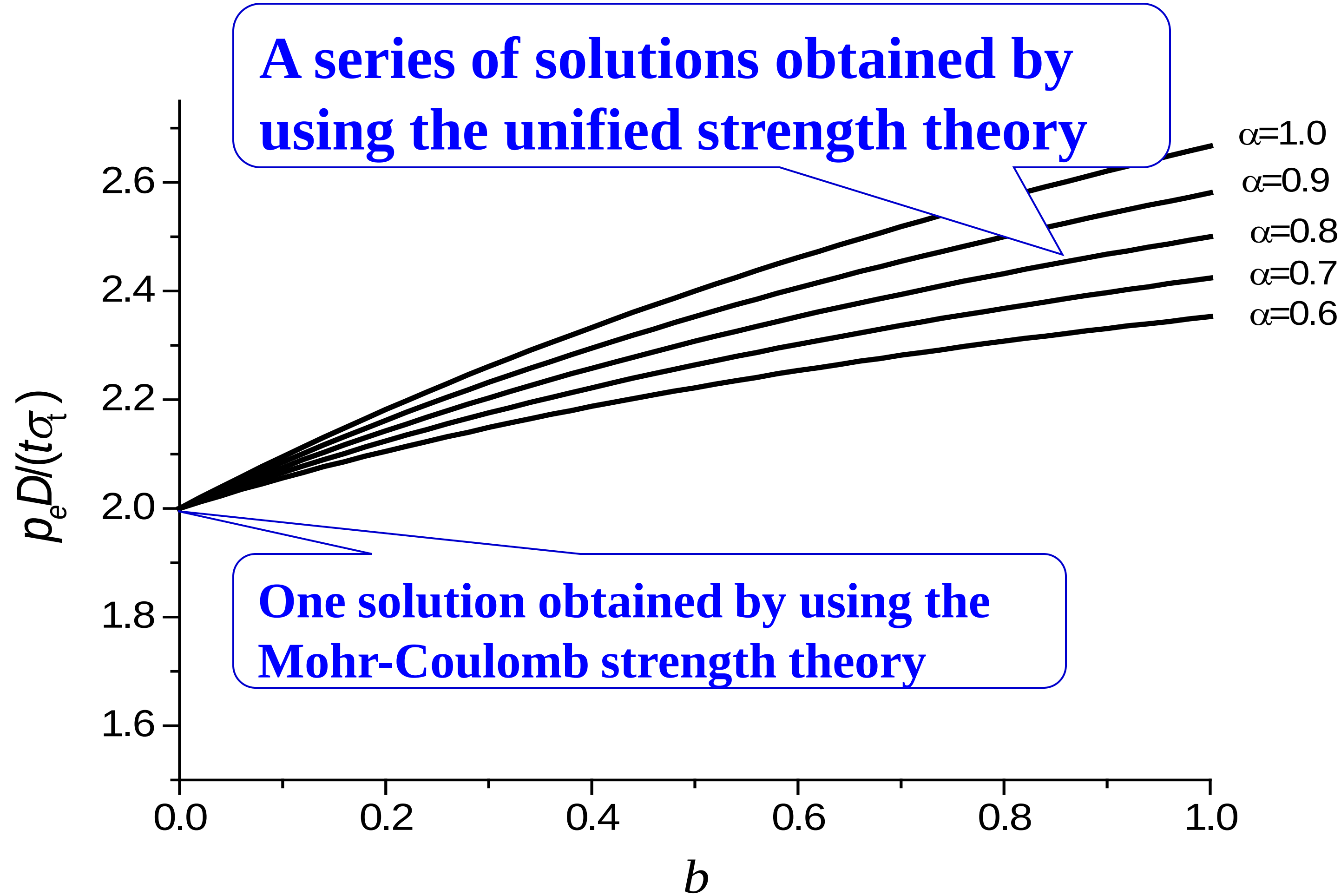


Comparison of the FEM with Characteristics field theory

Plastic Analysis of Plate



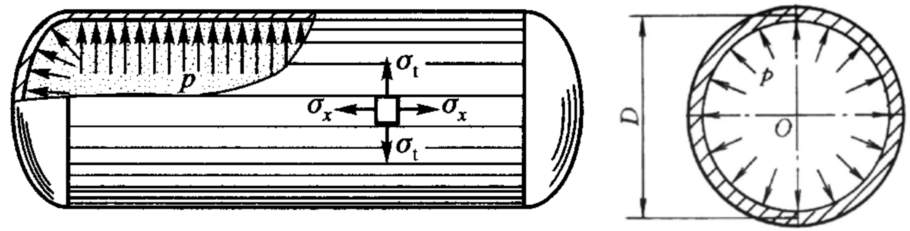
5. Applications> Analytical Solution



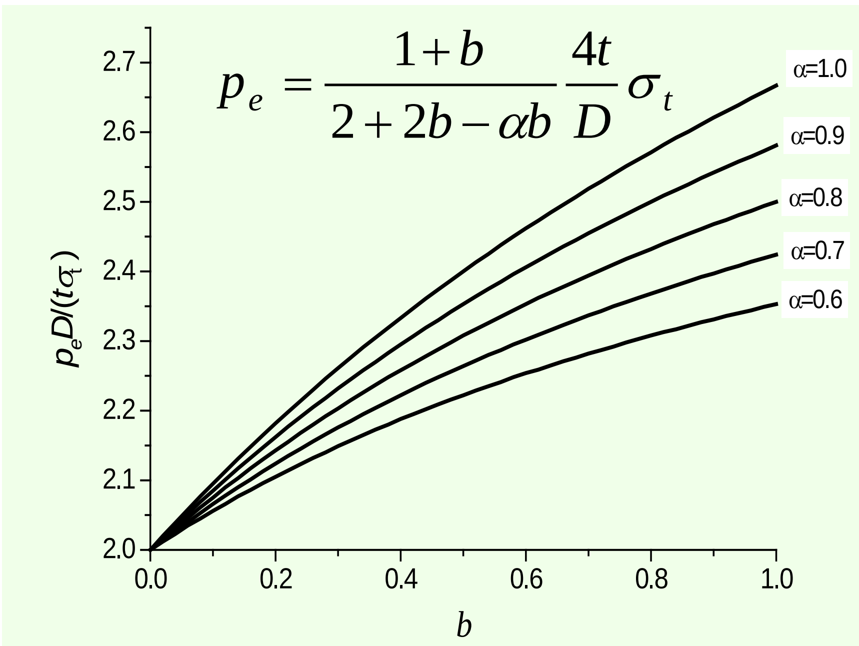
Limit pressures for thick cylinder with the parameter *b* of the *unified strength theory*

5. Applications> Analytical Solution

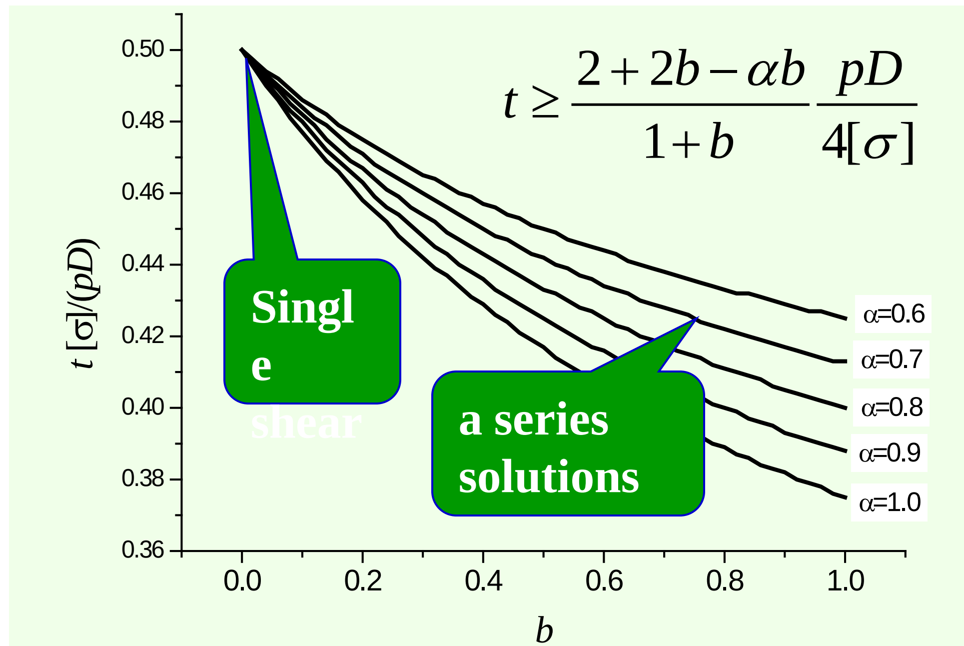
Thin-wall Pressure Vessel



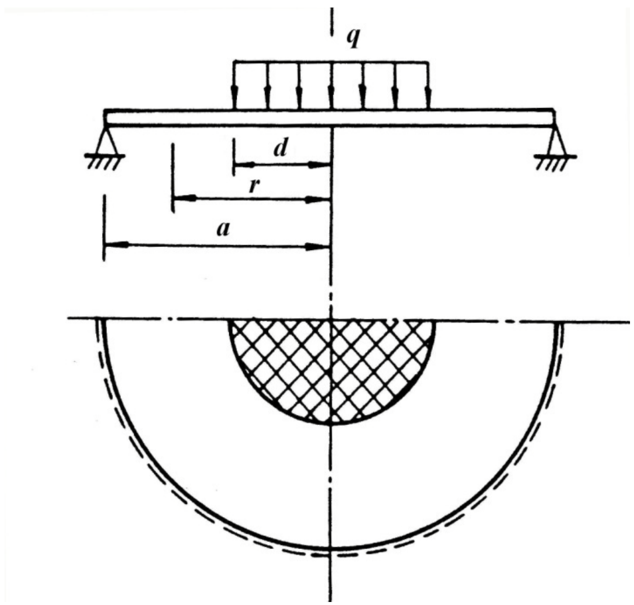
stress state

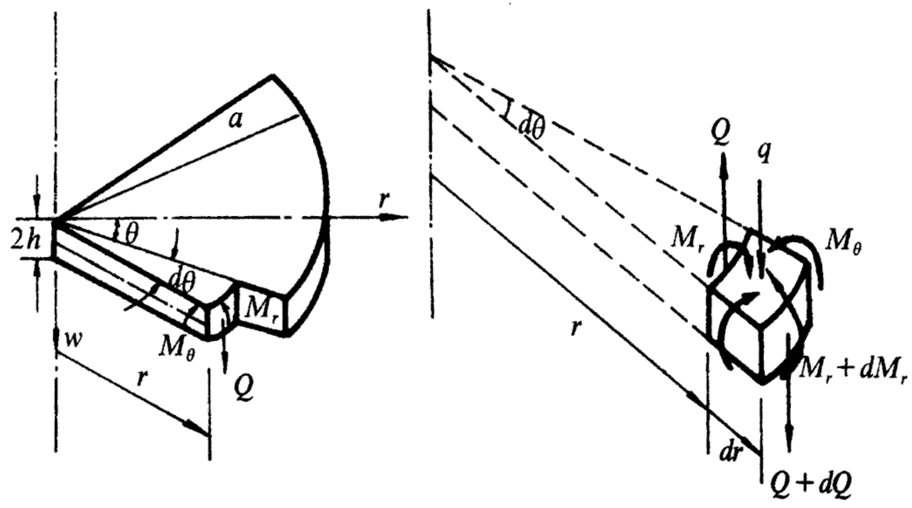


Relation of limit pressure to parameter *b*

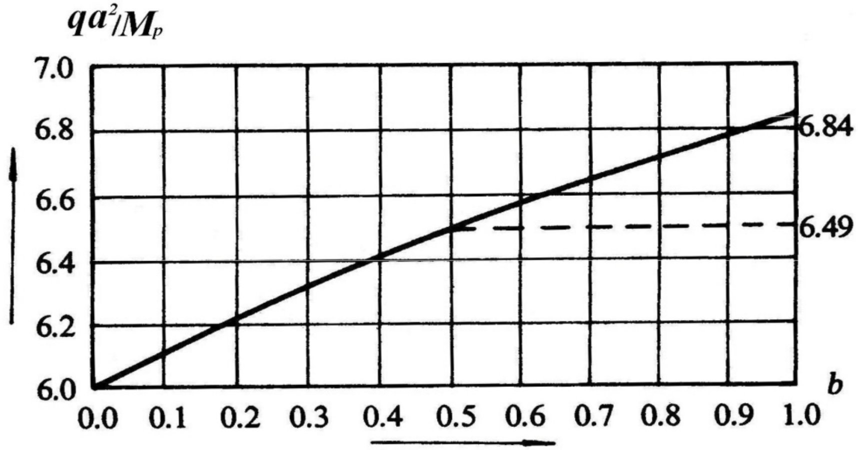


Relation of wall thickness to parameter *b*

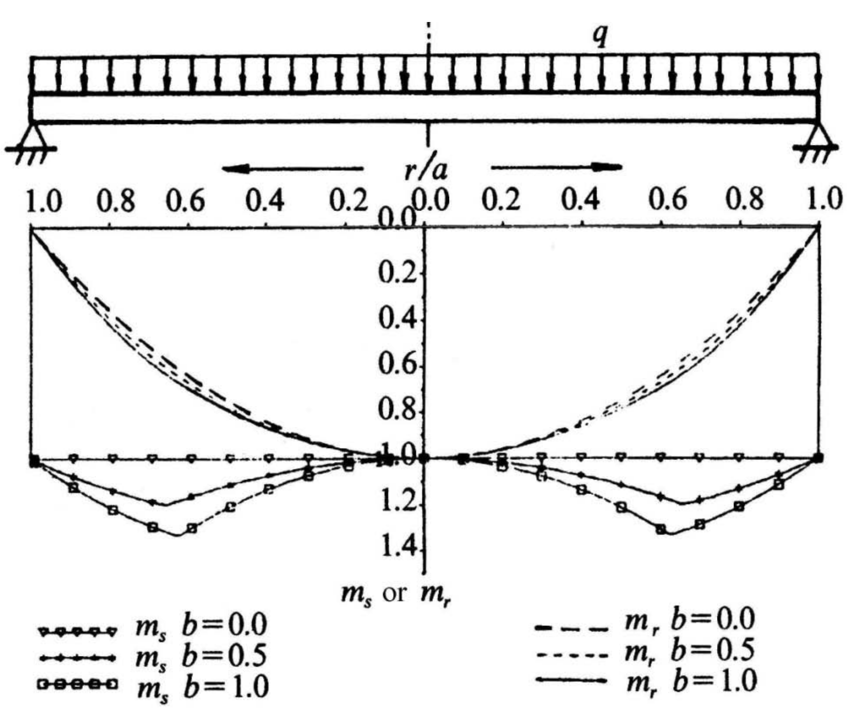




Internal forces in a circular plate element

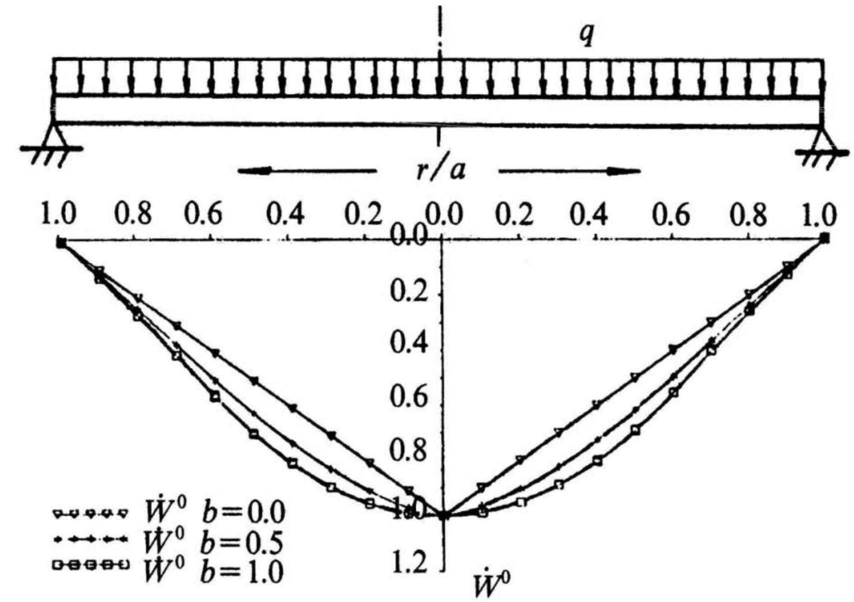


Plastic limit load of plate
(*b* from 0 to 1)



$$m_r = M_r / M_p,\ m_s = M_\theta / M_p$$

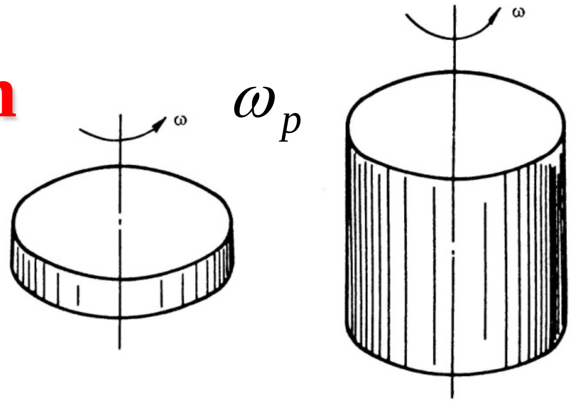
Internal moment fields with loading radius *d=a*



$$\dot{w}^0 = \dot{w} / \dot{w}_0$$

Velocity field with loading radius *d=a*

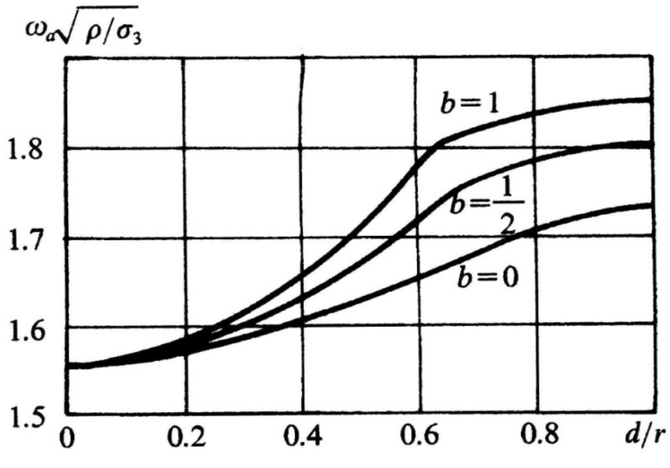
5. Applications> Analytical Solution



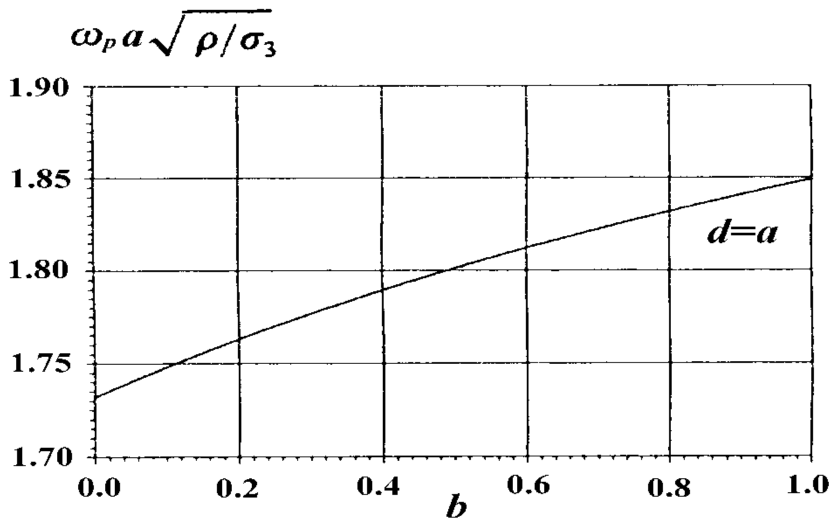
$$\omega_e = \frac{1}{2}\sqrt{\frac{8\sigma_y}{(3+\nu)\rho}}$$

$$\omega = \sqrt{\frac{3+b}{2+b}\frac{\sigma_y}{\rho}\frac{1}{r_0}}$$

Rotating discs and rotating cylinders



Relation of the angular velocity to the radius of the plastic zone



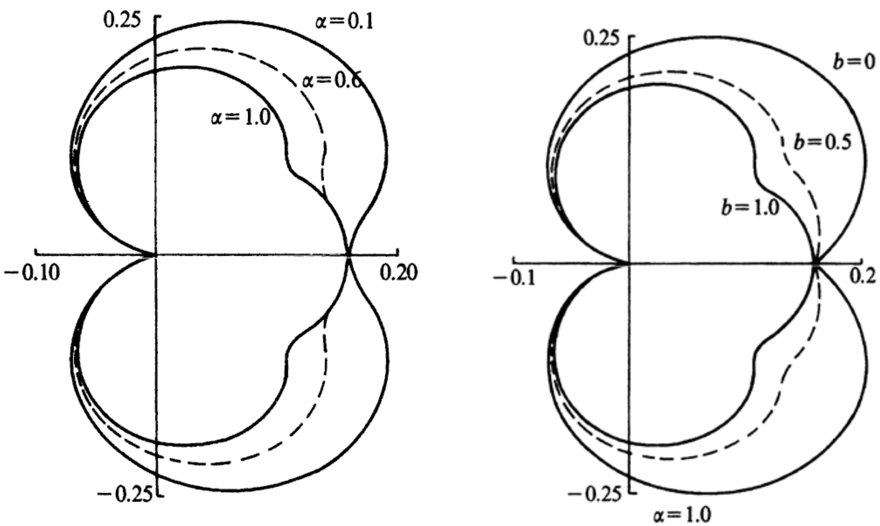
Relation of the strength theory parameter *b* to plastic limit angular velocity

5. Applications> Fracture

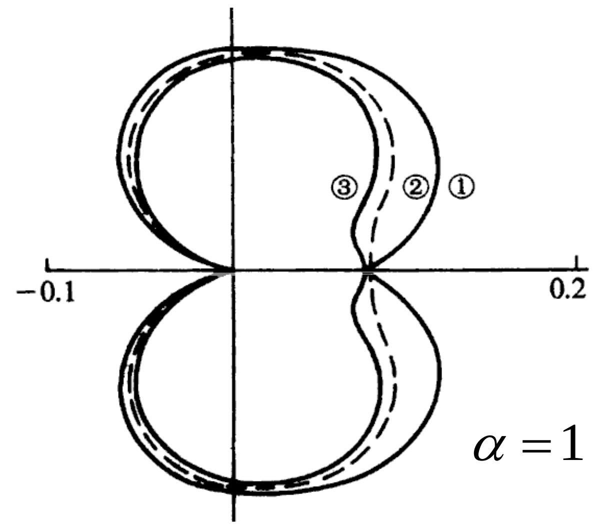
Unified solution for the plastic zone at crack tip:

$$r = \frac{1}{2\pi}\left\{\frac{K_1}{\sigma_t}\cos\frac{\theta}{2}\left[1-\frac{\alpha b}{1+\alpha}+\left(1+\frac{\alpha b}{1+b}\right)\sin\frac{\theta}{2}\right]\right\}^2 \quad \text{when} \quad \theta \geq \theta_b$$

$$r = \frac{1}{2\pi}\left[\frac{K_1}{\sigma_t}\cos\frac{\theta}{2}\left(1+\frac{1-b}{1+b}\sin\frac{\theta}{2}\right)\right]^2 \quad \text{when} \quad \theta \leq \theta_b \qquad \theta_b = 2\arcsin\frac{\alpha}{2+\alpha}$$

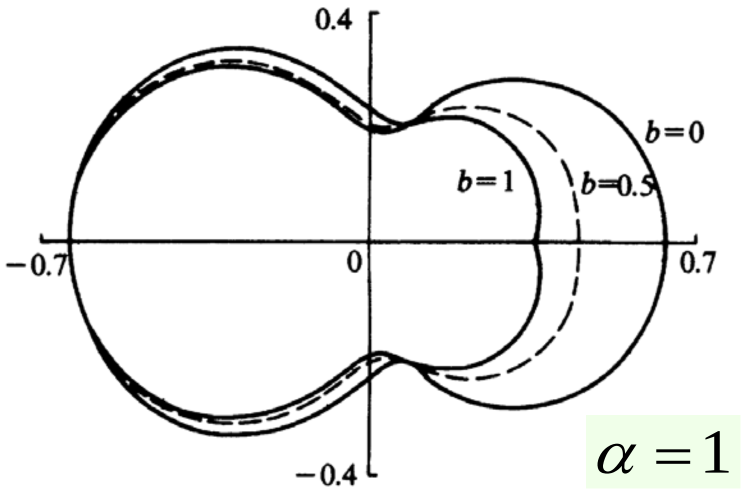


Plastic zone for mode I crack in plane stress

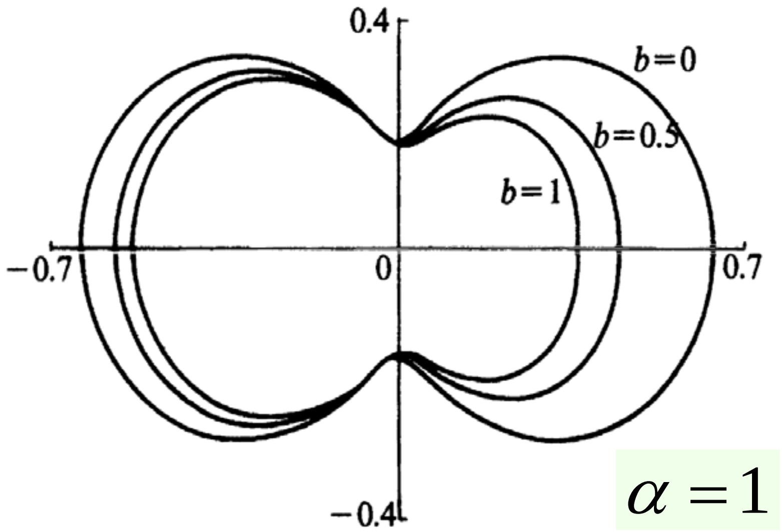


Plastic zone for mode I crack in plane strain

5. Applications> Fracture



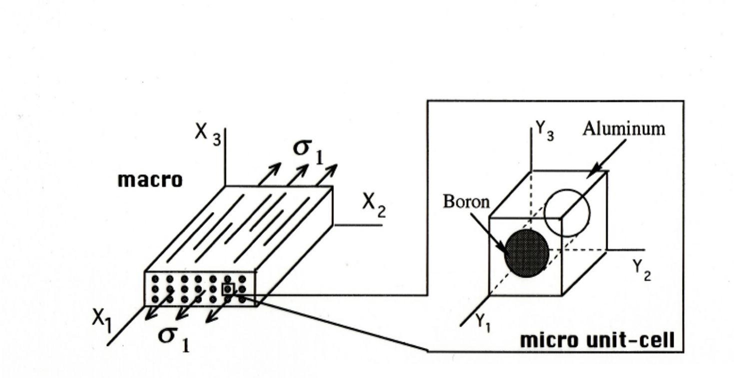
Plastic zone of crack tip for Mode II (plane stress)

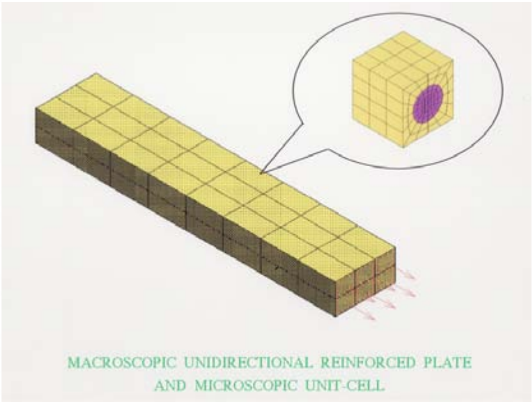


Plastic zone of Crack tip for Mode II (plane strain)

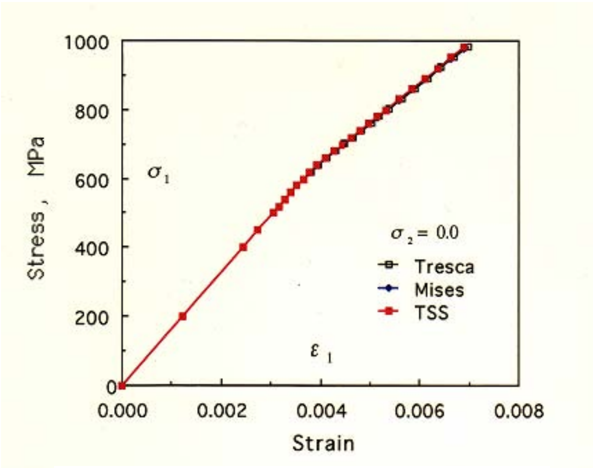
5. Applications > Multi-scale computing

A simple elasto-plastic analysis example

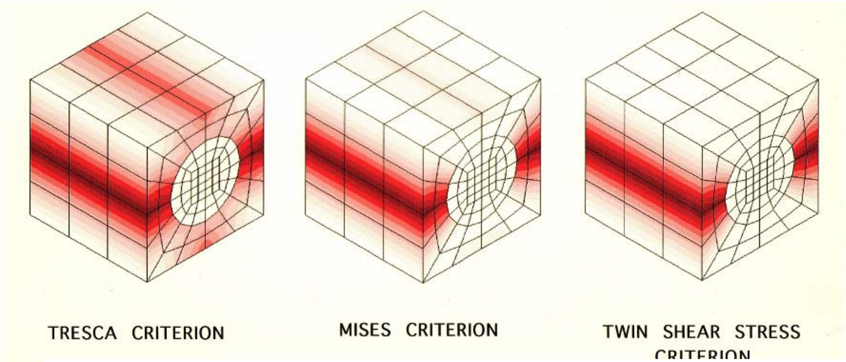




The sample model: macro and meso



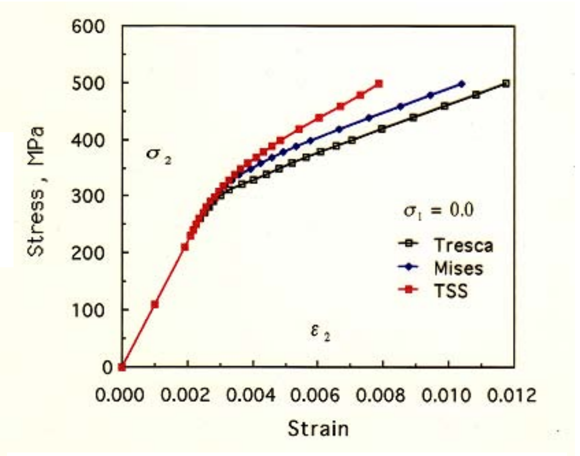
fiber direction



Tresca von Mises twin-shear

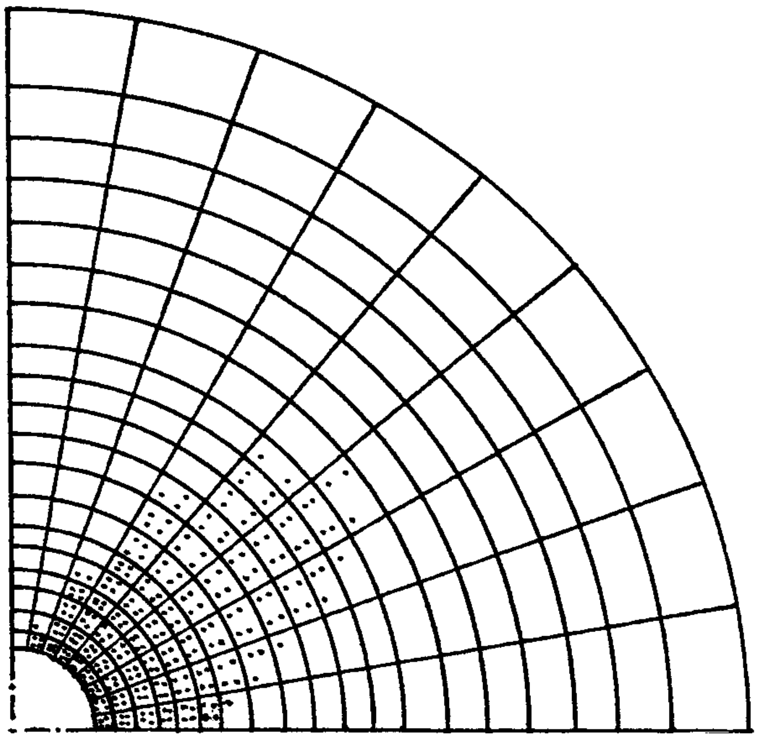
Plastic zones

Stress–strain curves

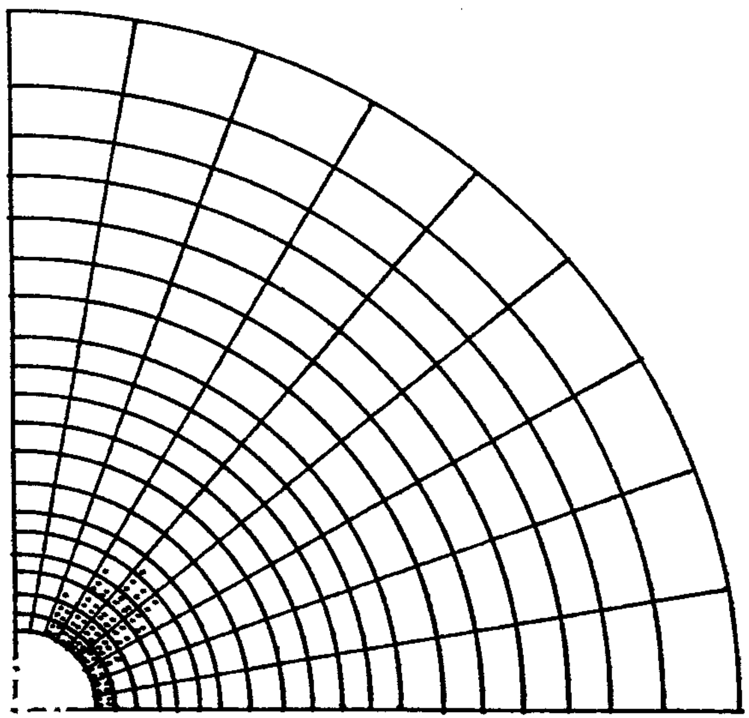


transverse direction

5. Applications> Numerical Solution



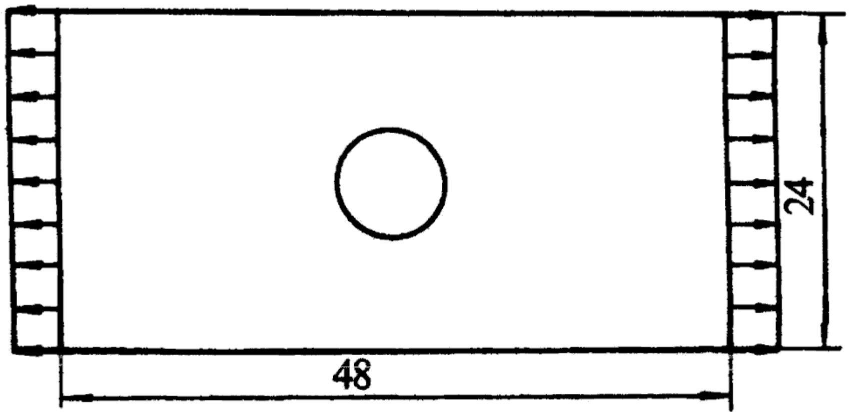
Plastic zone when b=0



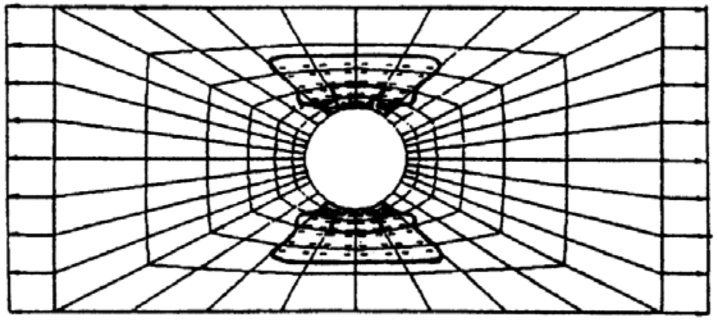
Plastic zone when b=1

5. Applications> Numerical Solution

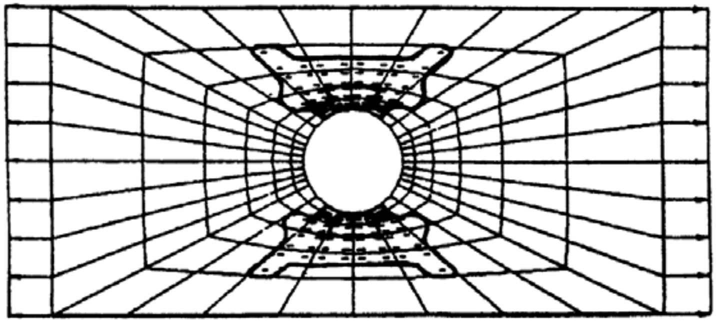
Unified Yield Criterion



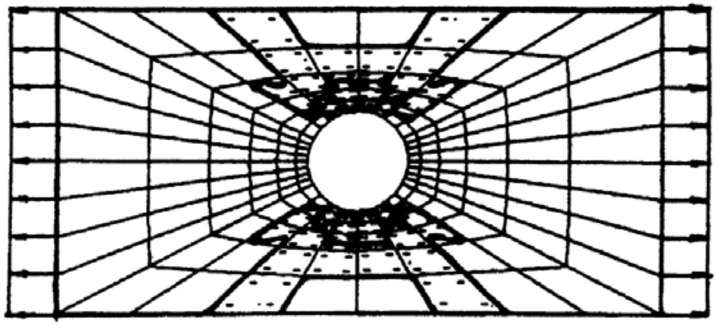
$$\alpha = 1$$



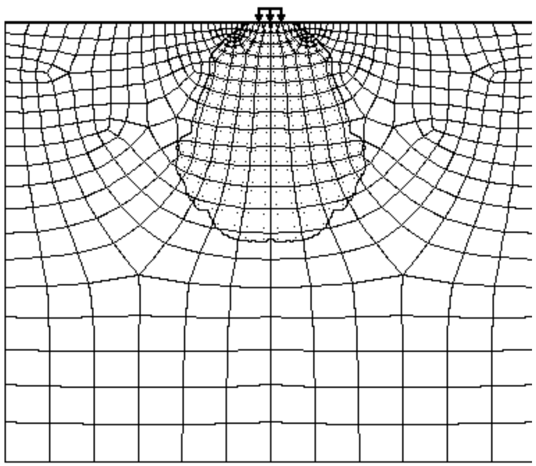
(a) $\alpha=1,\ b=1$

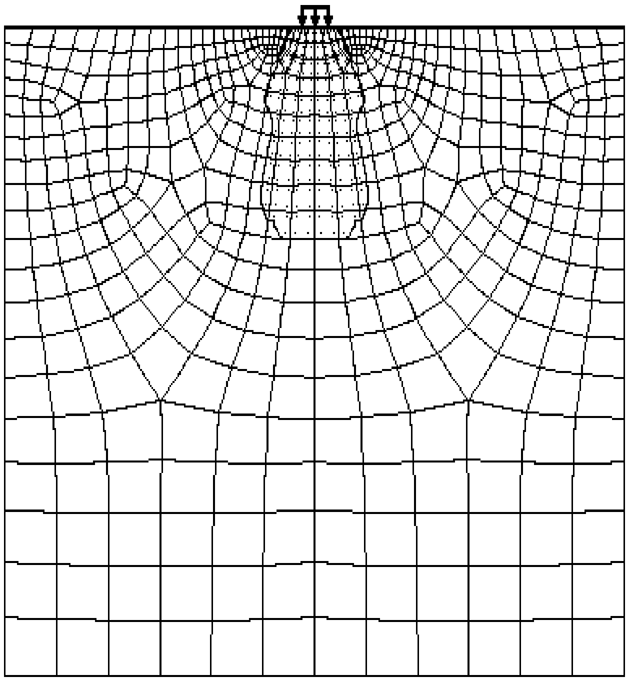


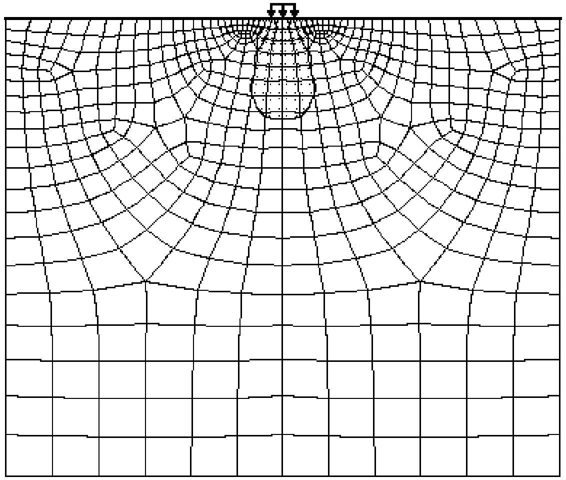
(b) $\alpha=1,\ b=0.5$



(c) $\alpha=1,\ b=0$ (Tresca)







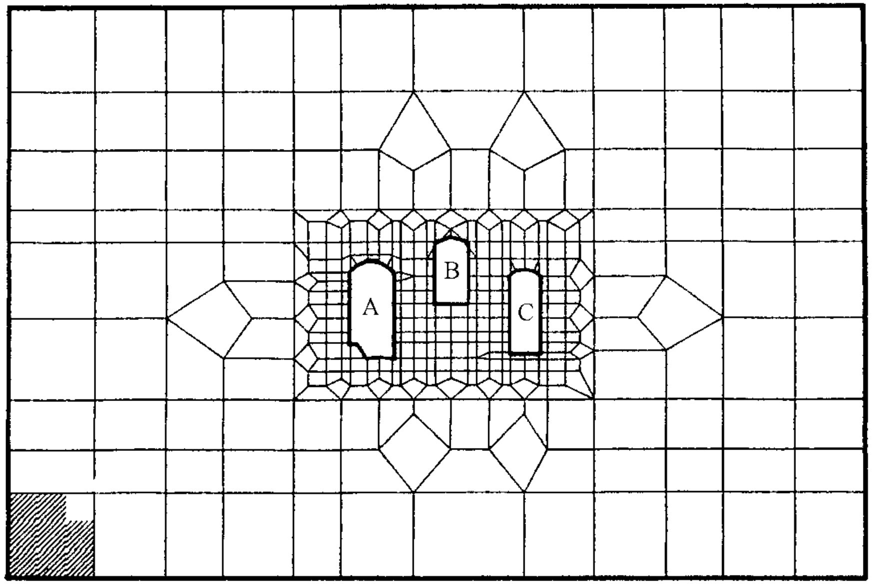
UST: b=0 (Mohr-Coulomb)

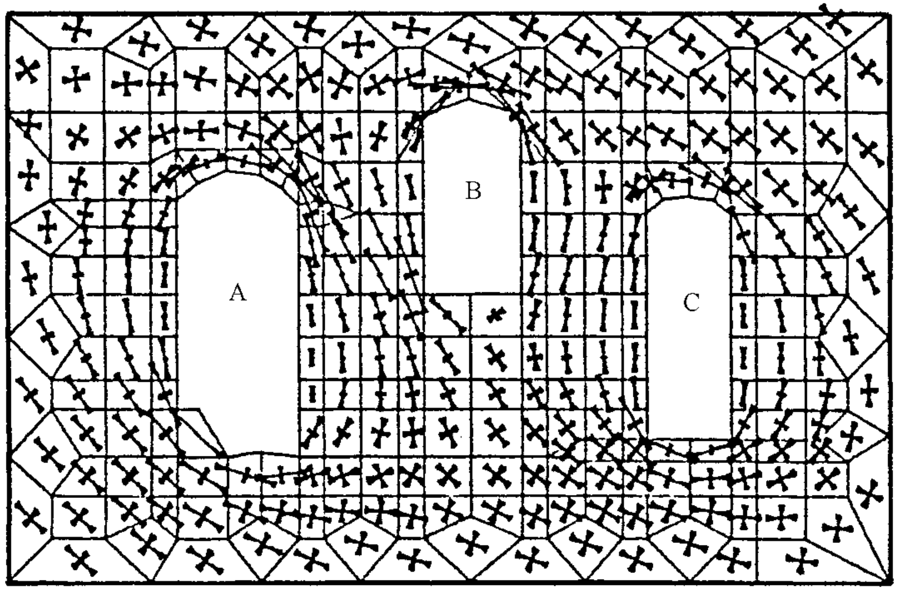
UST: b=0.5

UST: b=1

Plastic zone of weightless soil under the same load by using the unified strength theory

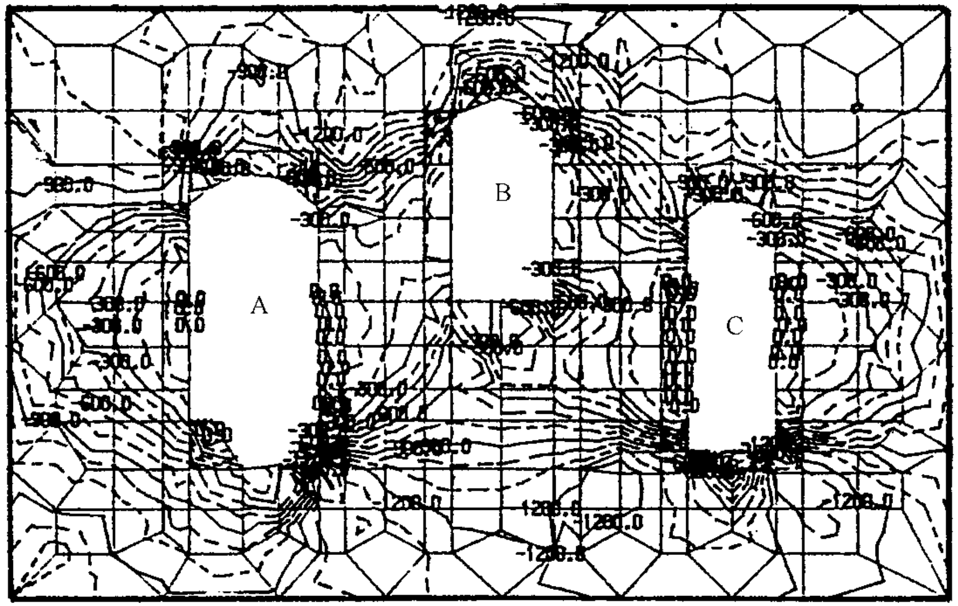
5. Applications> Numerical Solution

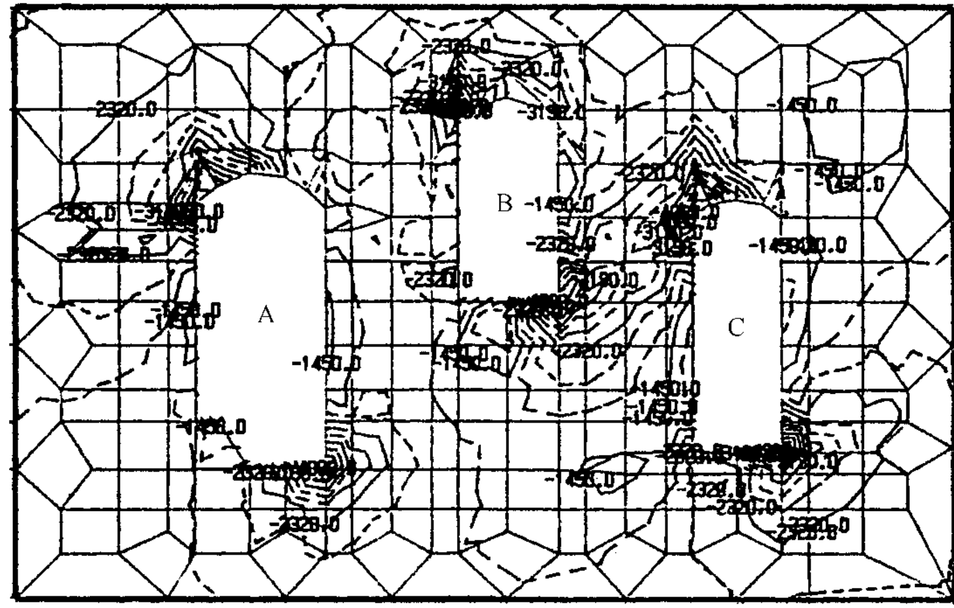




Underground cave and finite element mesh

Principal stress trace around underground cave



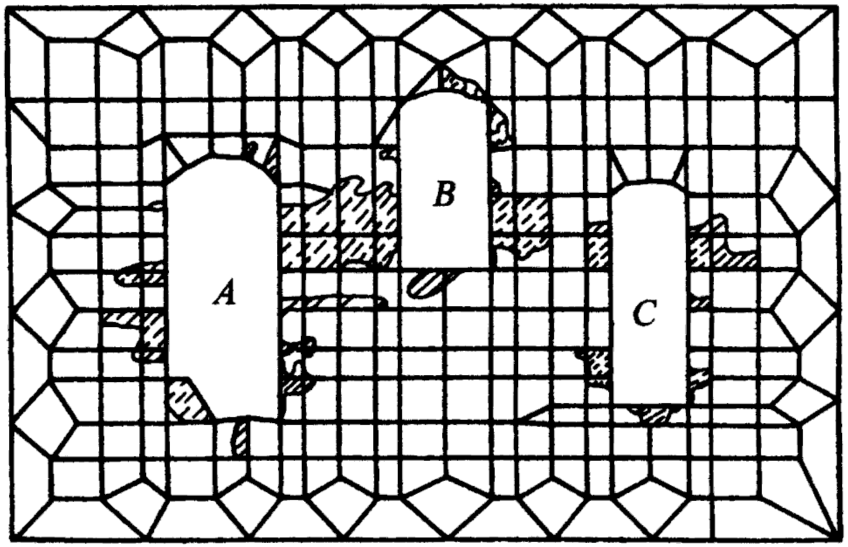


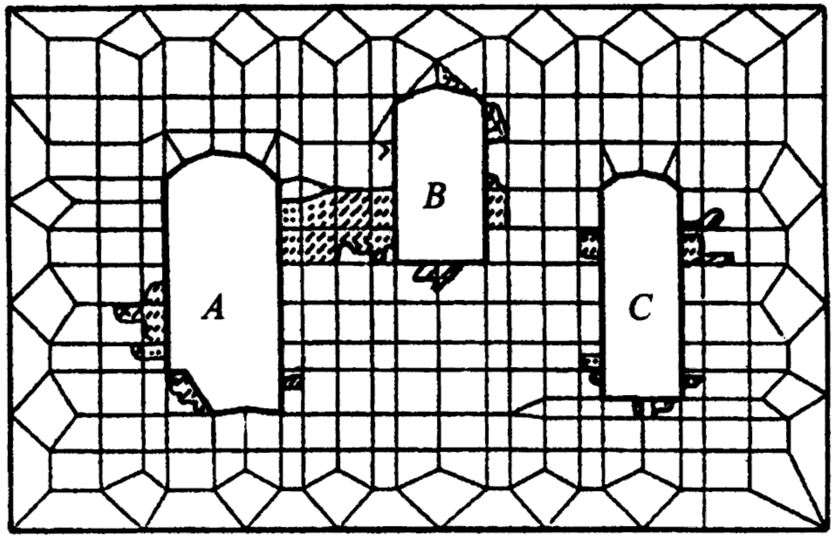
The maximum principal stress σ_1 around the cave

The principal stress σ_2 around the cave

5. Applications> Numerical Solution

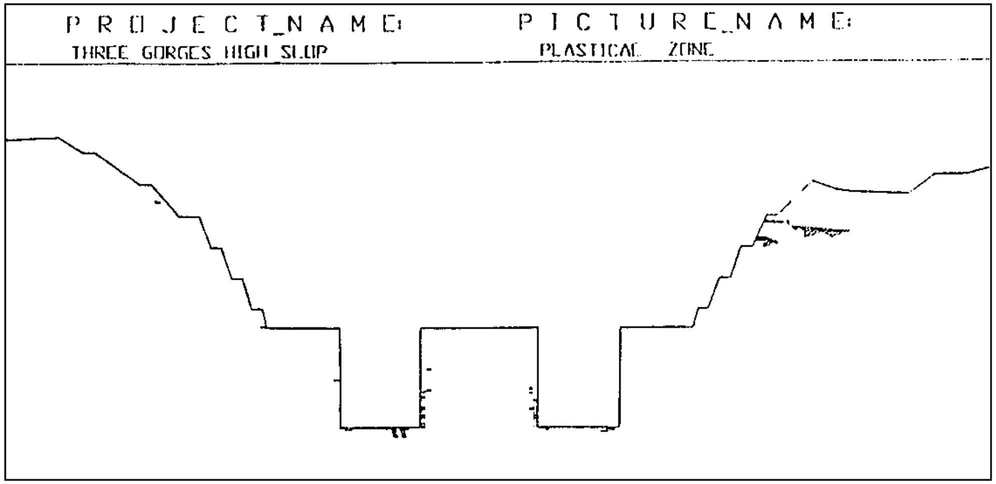
Plastic zone of rock excavation (A large power station at Yellow River)
b=0 (Mohr-Coulomb theory)



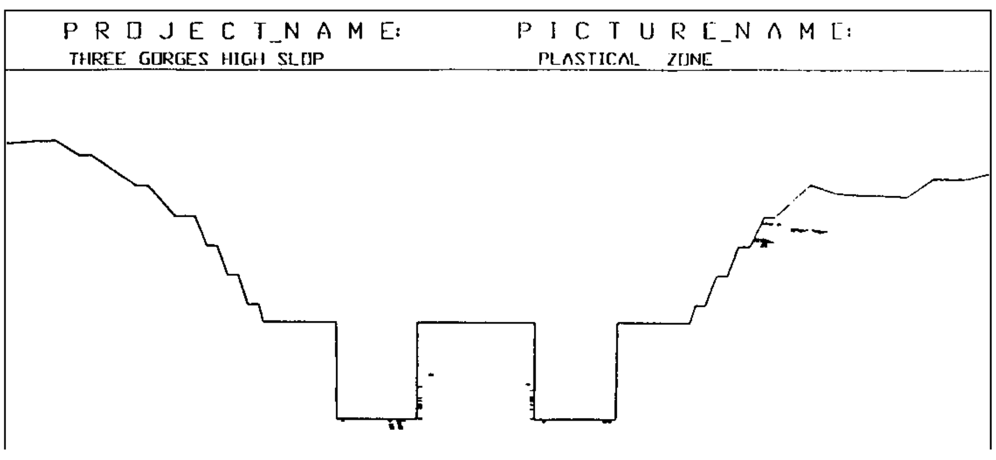


Plastic zone of rock excavation (A large power station at Yellow River)
b=1 (Twin-shear theory)

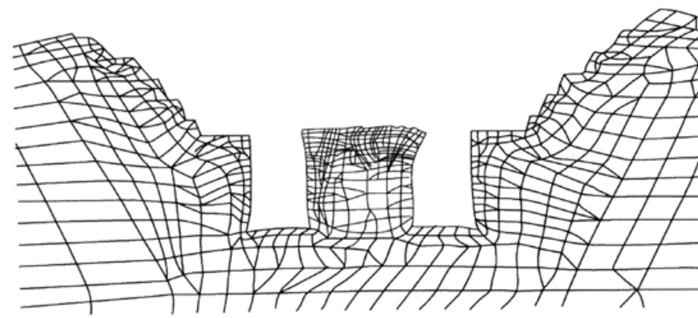
5. Applications (Ship lock of the Yangtze Three Georges)

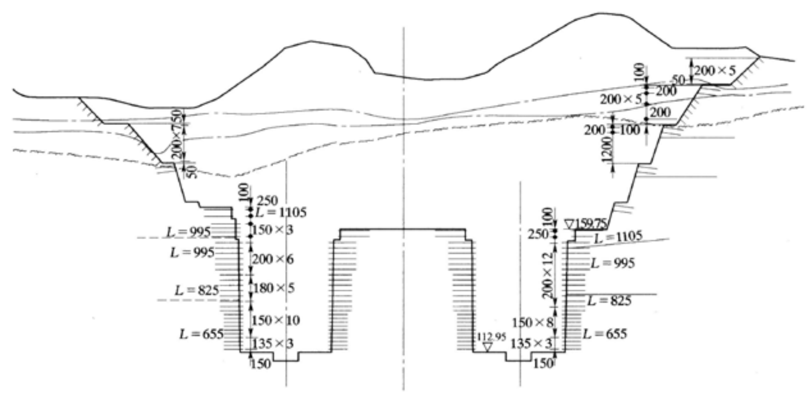


Plastic zone of excavation b=0 (Mohr-Coulomb theory)

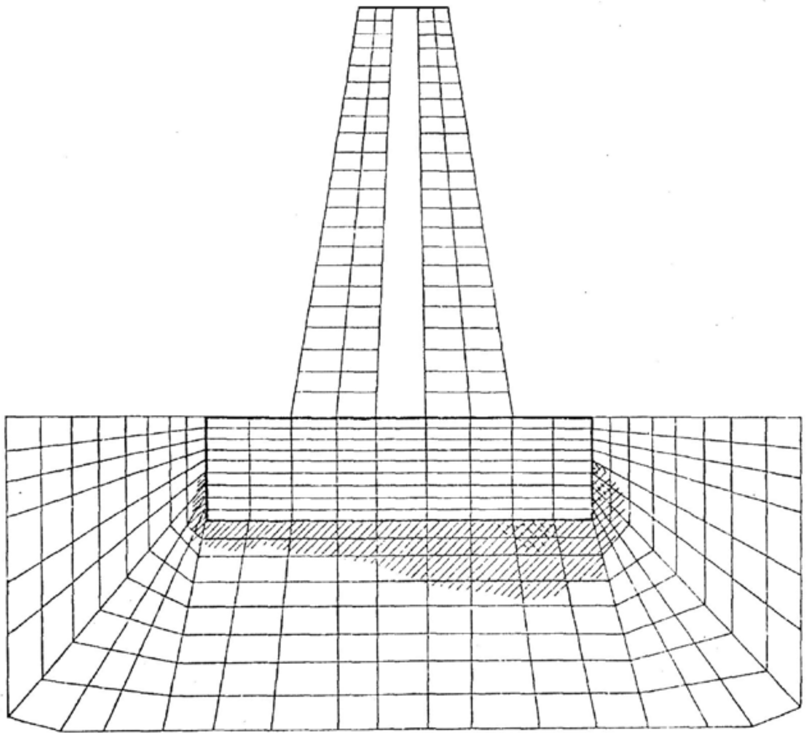


Plastic zone of excavation b=1 (Twin-shear theory

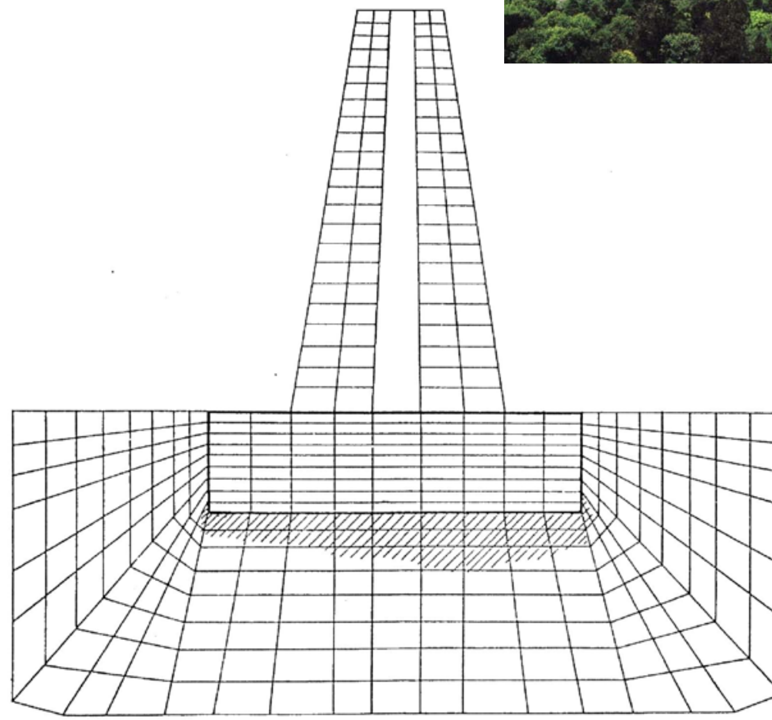




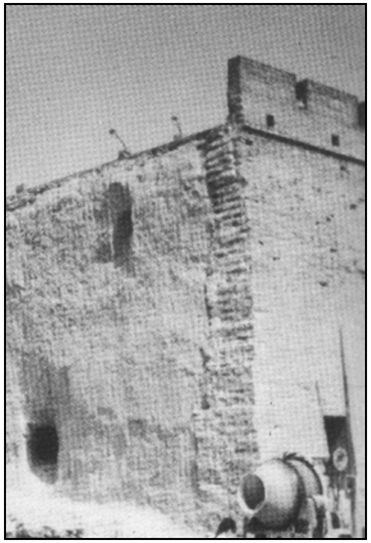
Deformation Analysis of foundation of Big Goose Pagoda

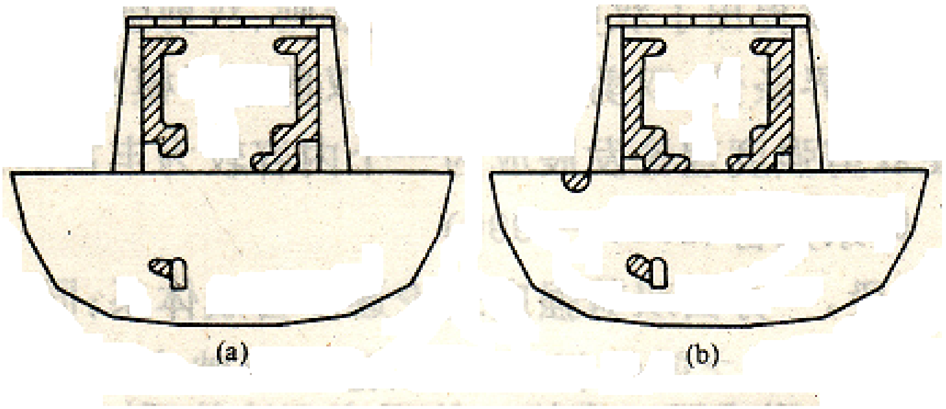


UST b=0 (Mohr-Coulomb)



UST b=1 (Twin-shear)



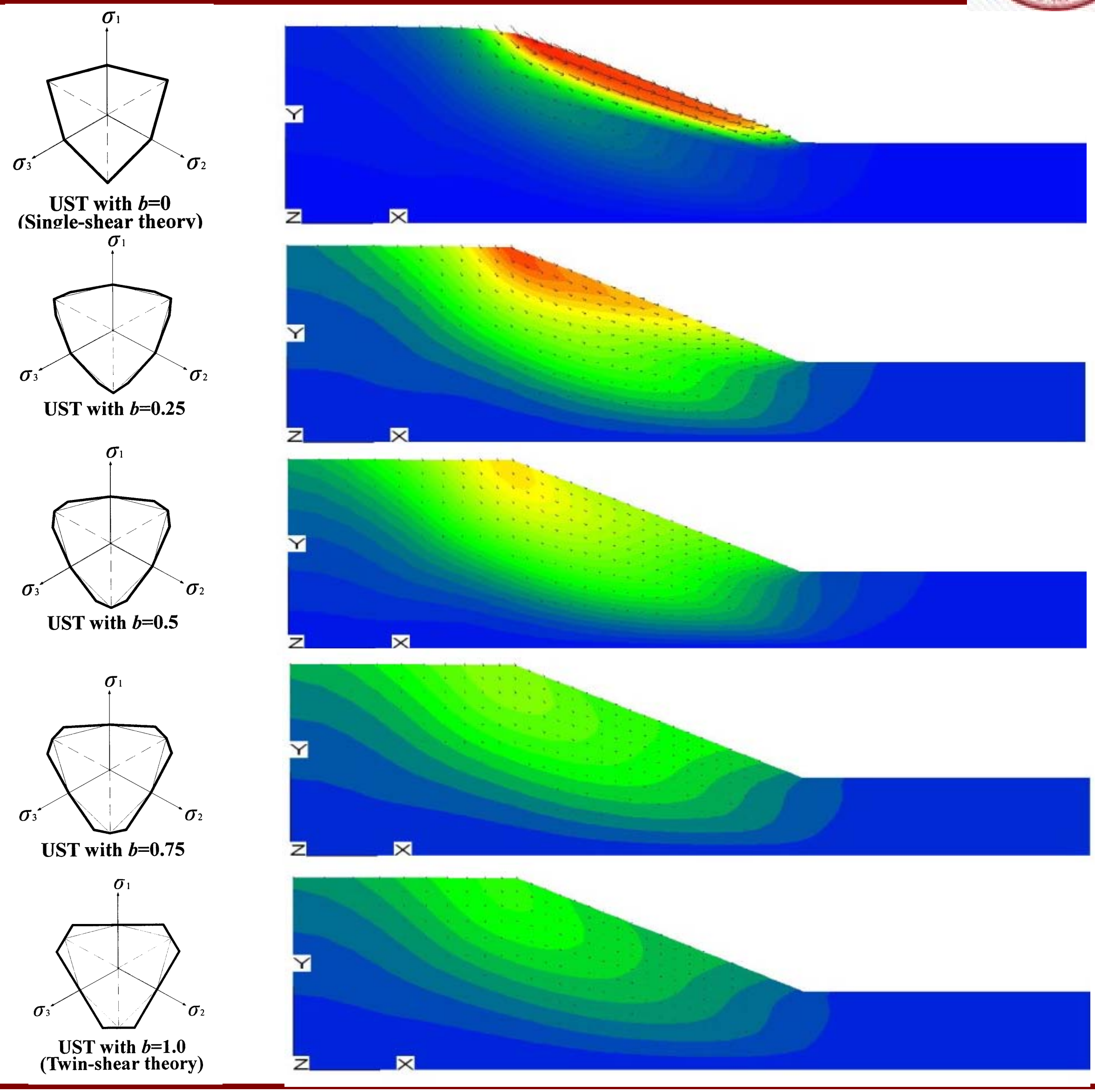


(a) Twin-shear (b) Single shear

Xi'an City Wall

5. Applications> Numerical Solution

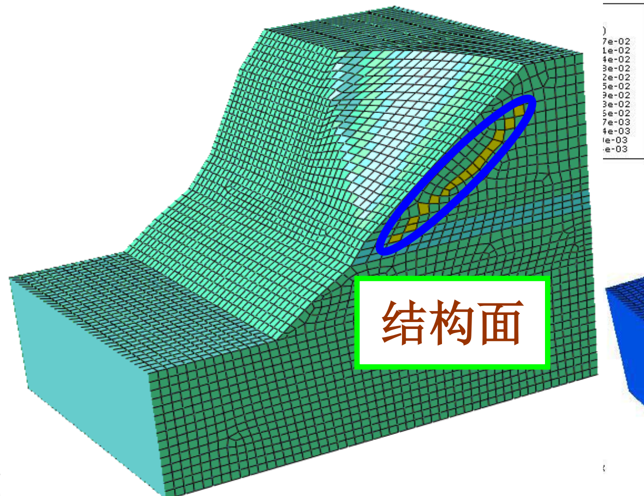
Shear strain spread with several yield criteria for a slope

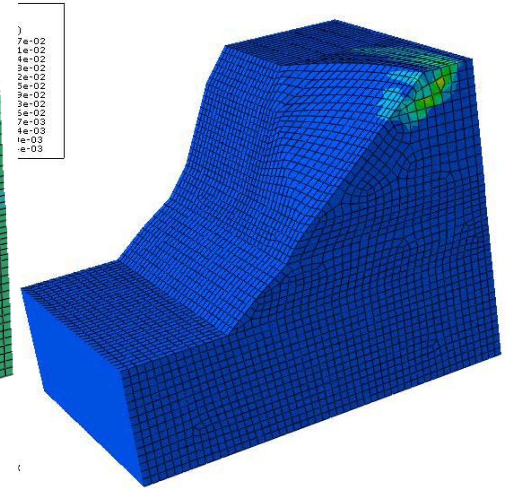


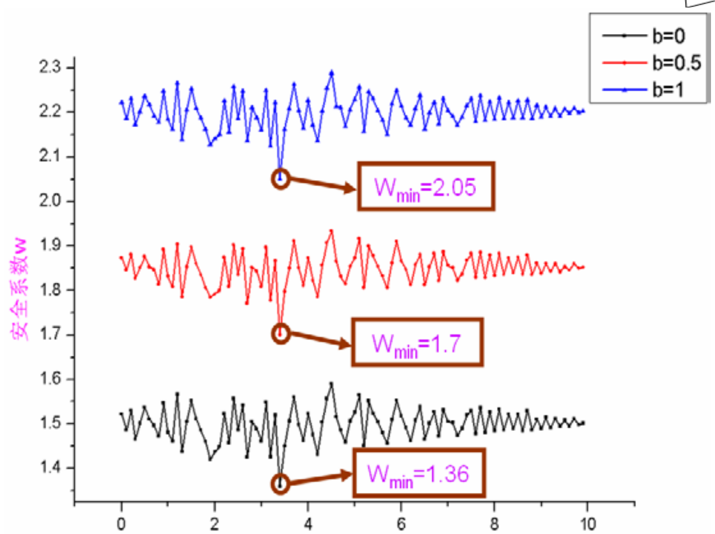
5. Applications> Numerical Solution

Dynamic safety factor

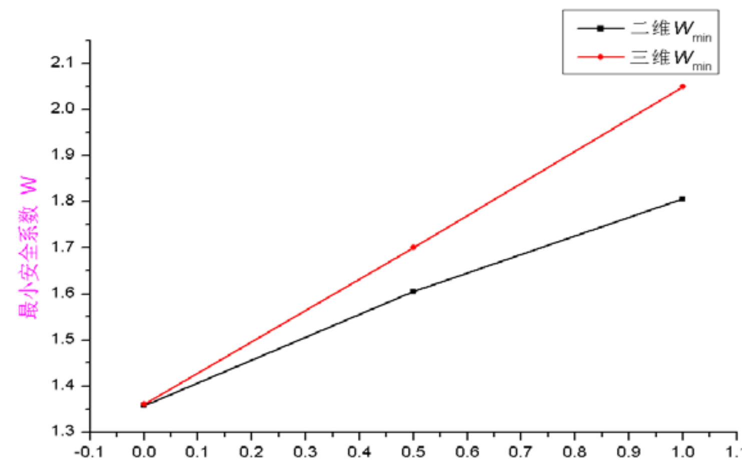
$$w = \frac{\int_0^l (\sqrt{J_2} - 3\alpha p - k) dl}{\int_0^l \tau dl}$$







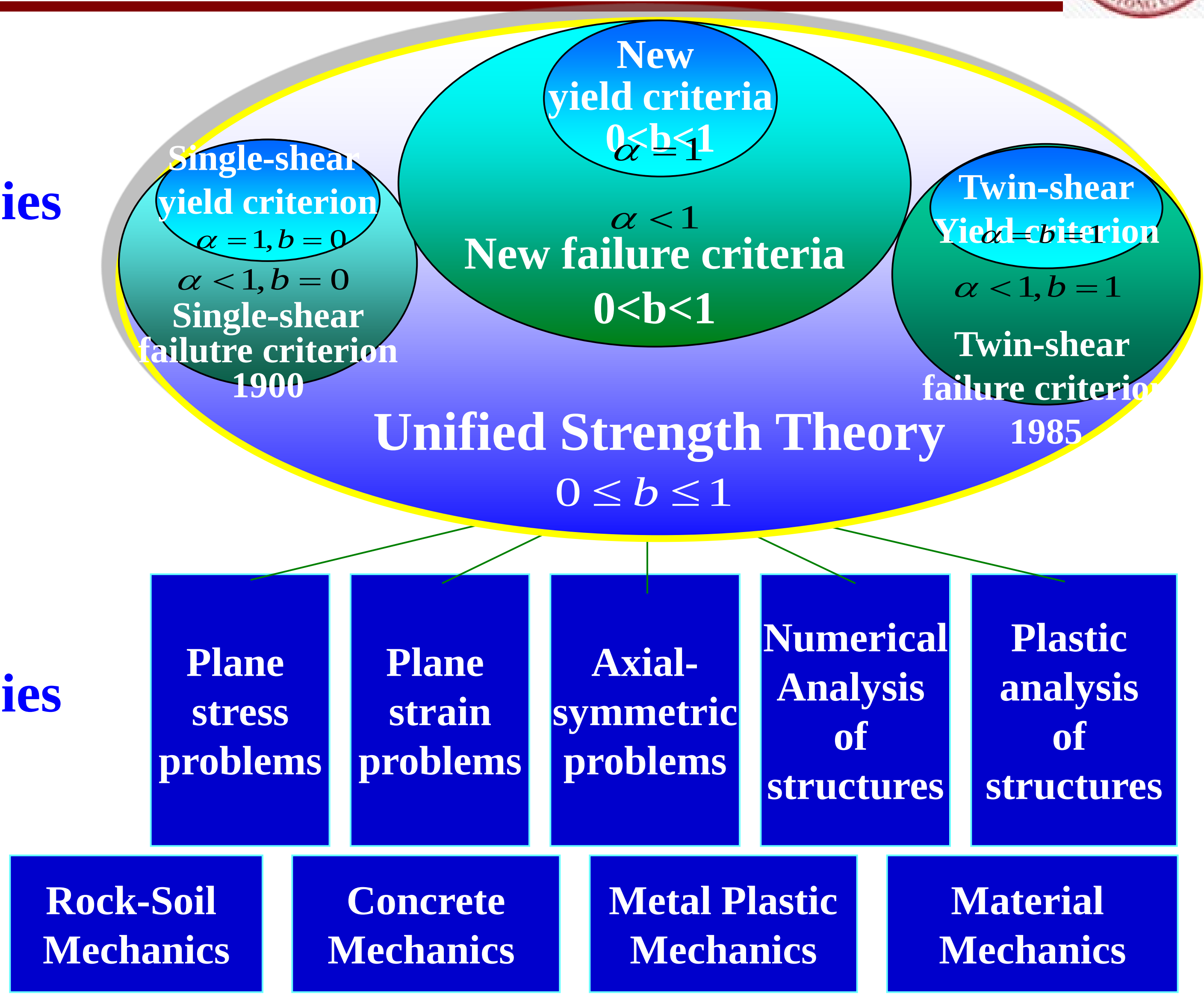
Safety factor cures for different b



Safety factors with b

Summary

Strength Theories for Materials



Strength Theories for Structures

Evaluation

澳大利亚Griffith大学教授

“The UTS (统一强度理论) is advantageous over other failure criteria because it encompasses all other established criteria as special cases. Or, such criteria are merely the linear approximations of the UST. Moreover, the parameters of the UST are easily obtained by experiments.”

Xuesong Zhang, Hong Guan, Yew-Chaye Loo (Dean, School of Engineering, Griffith University Gold Coast Campus, Australia)
“UST failure criterion for punching shear analysis of reinforcement concrete slab-column connections”.
: *Computational Mechanics – New Frontiers for New Millennium*, Valliappan S. and Khalili N. eds. Elsevier Science Ltd, 2001, 299-304.

“统一强度理论超越其他各种准则之处在于它包含了所有其他已经建立的准则并把他们作为特例，或者是统一强度理论的线性逼近。此外，统一强度理论的参数可以由实验较容易得到。”

Evaluation

新加坡南洋理工大学教授

"The beauty of the twin-shear-unified strength theory is her feasibility in difining the convex shape of the surface. Setting the value of the controllable convex parameter *b* to 0 or 1 yields the lower and upper limit of the convex shape function. For any arbitrary value of *b*, the shape function can be written in the following form:

……"

Fan S. C. and Qiang, H.F. ,
"Normal high-velocity impacton concrete slabs-a simulation using the meshless SPH procedures".
Computational Mechanics – New Frontiers for New Millennium,
Valliappan S. and Khalili N. eds. Elsevier Science Ltd, 2001, 1457-1462.

"双剪统一强度理论的美 在于她在确定外凸面形状时的可行性。取可控制的外凸参数*b*为0或1，可得出外凸形状函数的下限和上限，形状函数可写为：

……"

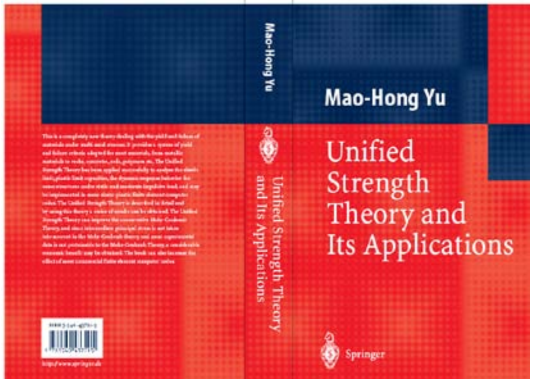
References

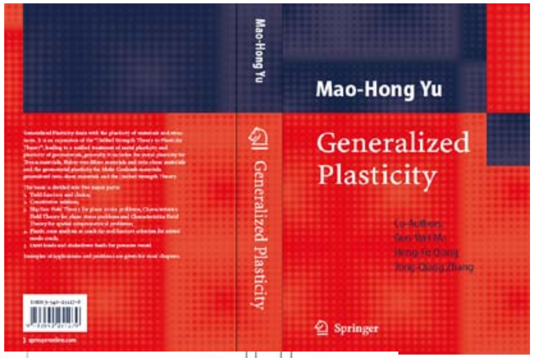
More details can be found in the monograph published by Springer in Berlin

- Mao-Hong Yu, “***Unified Strength Theory and Its Applications***”, Berlin: Springer, 2004

- Mao-Hong Yu, et al. “***Generalized Plasticity***”, Berlin: Springer, 2006

- Mao-Hong Yu, et al. “***Structural Plasticity***”, ZJU-Springer, 2009





Thank you