

LMS Undergraduate Summer School
Binary quadratic forms
Exercise sheet 2

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25–26 August 2022

1. For each of the following forms (a, b, c) , find a reduced form equivalent to it:
 - (a) $(12, -14, 9)$;
 - (b) $(30, -25, 6)$;
 - (c) $(21, -11, 3)$.
2. For each of the following d find all reduced positive definite binary quadratic forms of discriminant d :
 - (a) -3 ;
 - (b) -7 ;
 - (c) -11 ;
 - (d) -20 ;
 - (e) -23 ;
 - (f) -24 .
3. For each of the following forms f and g , decide whether f and g are equivalent:
 - (a) $f(x, y) = 4x^2 + 3xy + y^2$, $g(x, y) = 2x^2 + 5xy + 4y^2$;
 - (b) $f(x, y) = 7x^2 - 5xy + 2y^2$, $g(x, y) = 5x^2 + 7xy + 4y^2$.
4. Suppose that $d = -4 \prod_{i=1}^s p_i$ is an even fundamental discriminant, where p_i are distinct prime numbers. Show that there exist at least 2^{s-1} distinct equivalence classes of binary quadratic forms of discriminant d . Hint: count the reduced forms with $b = 0$.
5. If you know how to manipulate Legendre symbols, then classify all prime numbers that are representable by the quadratic form $3x^2 + 3xy + y^2$.