Summary of lecture 10-Parametric curves

Vector and scalar valued functions:

Consider $\mathbf{f} : D \to \mathbb{R}^m$ where $D \subset \mathbb{R}^n$.

- f is a *scalar-valued* function if $m = 1$, i.e. $\mathbf{f} : D \to \mathbb{R}$.
- f is a *vector-valued* function if $m = 2$ or 3 or more, e.g. $\mathbf{f} : D \to \mathbb{R}^2$.
- The simplest vector-valued functions have the form $\mathbf{f} : I \to \mathbb{R}^2$, where $I \subset \mathbb{R}$. This is a parametric curve.
- Example: The curve $y = x^2 + 2x$ is parameterized by $x = t$ giving $y = t^2 + 2t$. The parametric curve is then $\mathbf{f}(t) = (t, t^2 + 2t)$, where $t \in \mathbb{R}$ is the *parameter*.

Differentiation of vector-valued functions:

- Is component wise. E.g. If $\mathbf{f}(t) = (x(t), y(t), z(t))$ then

$$\frac{d\mathbf{f}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

- The vector $\frac{d\mathbf{f}}{dt}$ lies along the tangent to the curve at $\mathbf{f}$.