Summary of lecture 13 - Divergence

Divergence

The *divergence* of a vector field $\mathbf{F} = (F_1, F_2, F_3)$ is the *scalar*,

$$\boxed{\operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}},$$

Divergent field $\iff \operatorname{div} \mathbf{F} > 0$; Convergent $\iff \operatorname{div} \mathbf{F} < 0$,
Incompressible $\iff \operatorname{div} \mathbf{F} = 0$

Laplacian

- *Laplacian* of a scalar field f is

$$\boxed{\nabla^2 f = \nabla \cdot (\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}}.$$

- Laplacian of a vector field $\mathbf{F} = (F_1, F_2, F_3)$ is,

$$\boxed{\nabla^2 \mathbf{F} = (\nabla^2 F_1, \nabla^2 F_2, \nabla^2 F_3)}.$$