SCALAR VALUED FUNCTIONS

Eg: $f(x,y,z) = x^2 y \sin(z)$ In General, $f: \mathbb{R}^n \to \underbrace{\mathbb{R}}_{\text{Scalar}}$.

$$\boxed{\operatorname{grad} f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)}$$

This gives information on how f changes in each of the $x, y,$ and z directions seperately, but what about in some general direction $\underline{u}$. ($\underline{u}$ = unit vector).

DIRECTION DERIVATIVE
$$\boxed{\frac{\partial f}{\partial \underline{u}} = \underline{u} \cdot \nabla f = u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y} + u_3 \frac{\partial f}{\partial z}.}$$

Weighted in each derection by u_i.

VECTOR VALUED FUNCTIONS

Eg: $\underline{F}(x,y,z) = (x^2, y^2 z, xyz)$. In General, $f: \mathbb{R}^n \to \overbrace{\mathbb{R}^m}^{\text{VECTORS}}$ where $m > 1$.

Simplest form: PARAMETRIC EQUATION:

eg: $\underline{r}(t) = (4-t, 4-t, 4)$

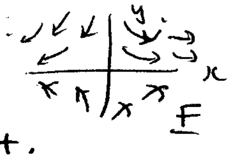
So $\frac{\partial r}{\partial t} = (-t, -t, 0)$ i.e. $\frac{\partial r}{\partial t} = \left(\frac{\partial r_1}{\partial t}, \frac{\partial r_2}{\partial t}, \frac{\partial r_3}{\partial t}\right)$

SOLUTION: Let $\underline{F} = (F_1, F_2, F_3)$.

$$\boxed{1.\ \operatorname{div} \underline{F} = \nabla \cdot \underline{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}}$$

$\nabla \cdot \underline{F} = 0 \Leftrightarrow \underline{F}$ is incompressible

eg:



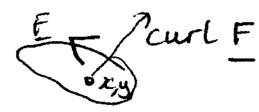
$\nabla \cdot \underline{F} < 0$ convergent

$\nabla \cdot \underline{F} > 0$ divergent.

$$\nabla^2 = \nabla \cdot (\nabla f) = \text{LAPLACIAN} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\boxed{2.\ \operatorname{Curl} \underline{F} = \nabla \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}}$$

$\nabla \times \underline{F} = 0 \Leftrightarrow \underline{F}$ is irrotational.



If $\nabla \times \underline{F} = 0$ ~~then~~ and $\underline{F}$ is defined everywhere then $\underline{F}$ is <u>conservative</u> and there exists a scalar field ϕ s.t. $\nabla \phi = \underline{F}$