Summary of Lecture 17 -ODEs

- An *ordinary differential equation* (ODE) is a relationship between a function of one variable e.g. $u(t)$ and its derivatives.
- The *general solution* includes all possible solutions—includes arbitrary constants (ODE).
- A solution without arbitrary constants/functions is called a *particular solution*. This may be found by giving extra conditions in the form of initial.

Separable first order ODEs

Have form

$$\frac{dy}{dx} = f(x)g(y)$$

and are solved by rewriting as

$$\int \frac{1}{g(y)}\, dy = \int f(x)\, dx$$

Exact ODEs

$$P(x, y) + Q(x, y)\frac{dy}{dx} = 0$$

is exact if there exists some $\phi(x, y)$ such that

$$P = \frac{\partial \phi}{\partial x} \text{ and } Q = \frac{\partial \phi}{\partial y}.$$

Then $\phi(x, y) = C$ (a constant) is the general solution to the ODE.

$\phi(x, y)$ exists if and only if

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$