Summary of Lecture 18- 23/11/05 -Integrating factors

• If an ODE of the form

$$P(x,y) + Q(x,y)\frac{dy}{dx} = 0$$

is *not* exact then it can be converted to an exact equation by multiplying through by $\mu(x, y)$

$$(\mu P) + (\mu Q)\frac{dy}{dx} = 0. \tag{**}$$

Both equations have the same solution.

• The trick is finding the integrating factor. You need to find μ such that

$$\frac{\partial}{\partial y}(\mu P) = \frac{\partial}{\partial x}(\mu Q).$$

• The important special case of linear ODEs:

$$\frac{dy}{dx} + a(x)y = b(x),$$

• is seen in first year and has an integrating factor

$$\mu = \exp\left(\int a\,dx\right).$$

• See the web (teaching $\rightarrow 2x \rightarrow$ Lecture Notes/Summaries) for a derivation of this.