# Summary of Lecture 19- 28/11/05 -First order ODEs

### Separable equations

These can be written in the form:

$$\frac{dy}{dx} = f(x)g(y).$$

The solution is found by solving $\int 1/g(y)dy = \int f(x)dx$.

### Exact equations

These have the form

$$P(x, y) + Q(x, y)\frac{dy}{dx} = 0 \qquad (*)$$

such that $\partial P/\partial y = \partial Q/\partial x$. The solution of this problem is $\phi(x, y) = C$, where $\phi$ satisfies $\partial\phi/\partial x = P$ and $\partial\phi/\partial y = Q$.

### Integrating factors

If the equation $(*)$ is *not* exact then it can be converted to an exact equation by multiplying through by $\mu(x, y)$, *the integrating factor*, to give

$$(\mu P) + (\mu Q)\frac{dy}{dx} = 0. \qquad (**)$$

Both equations $(*)$ and $(**)$ have the same solution. To solve $(*)$ you now use the method for exact equations on the new ODE you have obtained.

To find $\mu$, solve

$$\frac{\partial}{\partial y}(\mu P) = \frac{\partial}{\partial x}(\mu Q).$$