Summary of Lecture 20- 30/11/05 -Linear second order ODEs

Reduction of order method

Consider a linear equation of the form

$$a(x)\frac{d^2y}{dx^2} + b(x)\frac{dy}{dx} + c(x)y = f(x). \tag{1}$$

If RHS$\equiv 0$, we say that the ODE is *homogeneous*

If u is a solution to the homogeneous ODE then we try $y = uv$ as a solution to (1). We then introduce $w = v'$ and solve the w equation and back substitute to find y.

Special cases

Euler equation

$$ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = f(x)$$

The solution to the homogeneous equation has the form $y = x^k$. Once k is found we can use reduction of order to solve the full Euler equation.

Constant coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = f(x)$$

The solution to the homogeneous equation has the form $y = e^{\lambda x}$. If it is possible to guess a solution to the full equation then the general solution is $y = \mathrm{CF} + \mathrm{PI}$. Otherwise use reduction of order.