## Summary of Lecture 22- 7/12/05 -2nd order PDEs

### Solving second order PDEs using change of variables

- The techniques are the same as for first order PDEs
- but application of the chain rule is a 'bit' more complicated
- e.g.

$$z_{xx} = \frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial z}{\partial u} + \frac{\partial v}{\partial x}\frac{\partial z}{\partial v}\right)$$
$$= \frac{\partial^2 u}{\partial x^2}\frac{\partial z}{\partial u} + \frac{\partial u}{\partial x}\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial u}\right) + \frac{\partial^2 v}{\partial x^2}\frac{\partial z}{\partial v} + \frac{\partial v}{\partial x}\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial v}\right)$$

- use the chain rule in two variable to find the first derivative $\partial z/\partial x$. To obtain the second derivative differentiate this result again, but use the product rule. Once this is done use the chain rule again to expand any remaining derivatives of $z$.