Summary of lecture 4 - Chain rule for two variables

• The chain rule for functions of two variables: Let F(x, y) = f(u(x, y), v(x, y)). Then

$$\frac{\partial F}{\partial x} = \frac{\boxed{\partial u}}{\partial x} \frac{\partial f}{\partial u} + \frac{\boxed{\partial v}}{\partial x} \frac{\partial f}{\partial v}, \quad and$$
$$\frac{\partial F}{\partial y} = \frac{\boxed{\partial u}}{\partial y} \frac{\partial f}{\partial u} + \frac{\boxed{\partial v}}{\partial y} \frac{\partial f}{\partial v}.$$

• Special cases: If F(x, y) = f(u(x, y)) then

$$\frac{\partial F}{\partial x} = \frac{\partial u}{\partial x} \frac{df}{du};$$

• if F(x) = f(u(x), v(x)) then

$$\frac{dF}{dx} = \frac{du}{dx}\frac{\partial f}{\partial u} + \frac{dv}{dx}\frac{\partial f}{\partial v}.$$

• Partial derivatives are written as ordinary derivatives when used on functions of one variable.