Summary of lecture 5 - Critical points & classification

- (a, b) is a *critical point (stationary point)* of f provided $f_x(a, b) = 0$ **and** $f_y(a, b) = 0$.
- The sign of

$$\Delta(h, k) = f(a + h, b + k) - f(a, b)$$

  reveals whether (a, b) is a local maximum or minimum.
- If $\Delta(h, k) > 0$ for all $(h, k) \neq (0, 0)$ sufficiently close to $(0, 0)$, (a, b) is a local minimum. If $\Delta(h, k) > 0$ then (a, b) is a local maximum,
- otherwise, (a, b) is a saddle point.
- To classify a critical point we first use the second derivative test and if $D = 0$ then we use *first principals* and look at $\Delta(h, k)$.

2nd derivative test

Let (a, b) be a critical point of f and let $A = f_{xx}$ and $D = f_{xx}f_{yy} - f_{xy}^2$, where all derivatives are evaluated at (a, b). Then

1. If $A > 0$ and $D > 0$ then (a, b) is a minimum point,
2. If $A < 0$ and $D > 0$ then (a, b) is a maximum point,
3. If $D < 0$ then (a, b) is a saddle point,
4. If $D = 0$ then no conclusion about the nature of (a, b) is made.