**ALGEBRAIC TOPOLOGY IV ‖ EPIPHANY 2020**
**PROBLEM SHEET 1**

Please hand in problems 2 and 4 from MT Problem Sheet 8 and parts (i), (ii), (iv), (v), and (vi) from the problem on this sheet, in lecture on Monday 27th January or at my office by 5pm that day.

**Problem 1.** Let $F = F_{\{a,b\}}$ be the fundamental group of $X := S^1 \vee S^1$, with $a, b$ generators of $\pi_1(S^1)$ for each of the $S^1$ wedge summands.

For each of the following subgroups, sketch the covering space $\widetilde{X}$ of $S^1 \vee S^1$ corresponding to that subgroup, that is such that $p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$ equals that subgroup, where $p\colon \widetilde{X} \to X$ is the covering map:

(i) $\langle 1 \rangle$;

(ii) $\langle a^2 \rangle$;

(iii) $\langle aba^{-1}b^{-1} \rangle$;

(iv) $\langle b^n a b^{-n} \mid n \in \mathbb{Z} \rangle$;

(v) $\ker(\phi\colon F \to \mathbb{Z}/3)$ where $\phi(a) = 1 = \phi(b)$;

(vi) $\ker(\psi\colon F \to \mathbb{Z} \oplus \mathbb{Z})$ where $\psi(a) = (1, 0)$ and $\psi(b) = (0, 1)$;

(vii) $\langle a^2, b^2, aba^{-1}b^{-1}, ba^2b^{-1}, ab^2a^{-1} \rangle$.

If $\widetilde{X}$ is non-compact, just sketch a few iterations of the pattern.