ALGEBRAIC TOPOLOGY IV $\|$ MICHAELMAS 2019
PROBLEM SHEET 2

Please solve these problems during week 3. Problems 1,2,and 3 will form part of the next submission.

Problem 1.

(1) If $0 \to \mathbb{R}^a \to \mathbb{R}^b \to \mathbb{R}^c \to 0$ is an exact sequence, prove that $a - b + c = 0$.

(2) If $0 \to \mathbb{R}^a \to \mathbb{R}^b \to \mathbb{R}^c \to \mathbb{R}^d \to 0$ is an exact sequence, prove that $a+b+c+d$ is even.

Problem 2. Show that S^n is homotopy equivalent to $\mathbb{R}^{n+1} \setminus \{\text{pt}\}$.

Problem 3. Prove that a convex subset X of $\mathbb{R}^n$ is homotopy equivalent to the one point space.

Problem 4.

(1) If $0 \to A \to B \to \mathbb{Z} \to 0$ is an exact sequence of abelian groups, prove that $B \cong A \oplus \mathbb{Z}$.

(2) If $0 \to \mathbb{Z} \to G \to \mathbb{Z} \to \mathbb{Z} \to \mathbb{Z} \to 0$ is an exact sequence of abelian groups, identify the isomorphism type of G.

Problem 5. Prove that the long exact sequence in homology coming from a short exact sequence of chain complexes $0 \to C_* \to D_* \to E_* \to 0$ is indeed exact.

Problem 6. Prove that homotopy equivalence is an equivalence relation on spaces.

Problem 7. Group the SANS SERIF CAPITAL letters of the alphabet according to their homeomorphism classes, and according to their homotopy equivalence classes.