ALGEBRAIC TOPOLOGY IV ‖ MICHAELMAS 2019
PROBLEM SHEET 3

Please hand in Problems 1 – 3 from sheet 2 and Problem 1 – 2 from this sheet, either at the lecture on Monday 4th November or by 5pm that day at CM233.

Problem 1. Use the Mayer-Vietoris sequence to compute the homology groups of the torus $H_n(S^1 \times S^1)$, for every $n \in \mathbb{N}_0$. Do the same for the Klein Bottle.

Problem 2. Compute the homology $H_n(\mathbb{M})$ of the Möbius band $\mathbb{M}$, for every $n \in \mathbb{N}_0$.

Problem 3. Compute the homology of the products $S^p \times S^q$, also using the Mayer-Vietoris sequence and induction, since $S^p \times S^q = S^p \times D^q \cup_{S^p \times S^{q-1}} S^p \times D^q$.

Problem 4. Let C be the *comb space* consisting of points in the plane

$$\{(x, 0) \mid 0 \le x \le 1\} \cup \{(\frac{1}{n}, y) \mid n \in \mathbb{N}, 0 \le y \le 1\} \cup \{(0, y) \mid 0 \le y \le 1\}.$$

(i) Let $x_0 = (0, 1)$ and let $x_1 = (0, 0)$. Show that C is contractible relative to the base point x_1 but not relative to x_0. (Being contractible relative to a base point z means that the homotopies have to keep the point z fixed.)

(ii) Form a space by taking two copies of C and identifying the two corresponding points x_0. Is this space contractible?