ALGEBRAIC TOPOLOGY IV ∥ MICHAELMAS 2019
PROBLEM SHEET 4

Problem 1(i) and Problem 2 will form part of the next hand in.

Problem 1. *Homology versus knot theory.* Let $0 < n < m$. Consider a knot $S^n \subset S^m$ that is *flat*, meaning that there is an embedding

$$S^n \times D^{m-n} \subset S^m$$

with $S^n \times \{0\}$ the original knot. For the following values of n, m, compute the homology groups $H_k(E)$ of the exterior

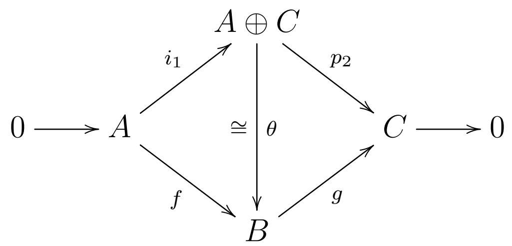
$$E := \overline{S^m \setminus (S^n \times D^{m-n})},$$

for every k. Hence compute the homology groups of the complement $S^m \setminus S^n$.

(i) $n = 1$, $m = 3$, that is a usual knot $S^1 \subset S^3$.
(ii) $n = 2$, $m = 3$, embeddings $S^2 \subset S^3$.
(iii) $n = 1$, $m = 4$, a knot in 4-dimensional space.

Interpret the answer to (i) from the point of view of knot theory.

Problem 2. *Splitting of short exact sequences.* Let $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ be a short exact sequence of abelian groups. Show that there exists a homomorphism $s\colon C \to B$ with $\mathrm{Id} = g \circ s\colon C \to C$, if and only if there is an isomorphism $\theta\colon A \oplus C \to B$ such that



commutes, where $i_1\colon a \mapsto (a, 0)$ and $p_2\colon (a, c) \mapsto c$.