ALGEBRAIC TOPOLOGY IV ‖ MICHAELMAS 2019
PROBLEM SHEET 5

Please hand in Problem 1 from this sheet and Problem 1(i) and Problem 2 from the previous sheet on Monday 18th November.

Problem 1. Consider the "word" ALGTOP written in $\mathbb{R}^2$. Compute the relative homology groups $H_n(\mathbb{R}^2, \mathsf{TOP})$, and $H_n(\mathbb{R}^2, \mathsf{ALGTOP})$ for each n. Describe generators, and compute the homomorphism

$$g_* \colon H_n(\mathbb{R}^2, \mathsf{TOP}) \to H_n(\mathbb{R}^2, \mathsf{ALGTOP})$$

induced by the map of pairs $g \colon (\mathbb{R}^2, \mathsf{TOP}) \to (\mathbb{R}^2, \mathsf{ALGTOP})$.

Problem 2. Compute the homology of the closed, oriented surface Σ_g of genus g. I suggest an approach by induction as follows. Write $T := S^1 \times S^1$. We can obtain the surface of genus $g+1$ from a surface of genus g by the *connected sum* operation

$$\Sigma_g \cong \Sigma_{g-1} \# T.$$

The operation $\#$ is defined as follows, for closed surfaces S_1 and S_2. Remove discs from each, and take the closure, to obtain $\overline{S_1 \setminus D^2}$ and $\overline{S_2 \setminus D^2}$. Glue them together (form the quotient space) along their common boundary to obtain the connected sum

$$S_1 \# S_2 = \overline{S_1 \setminus D^2} \cup_{S^1} \overline{S_2 \setminus D^2}.$$