ALGEBRAIC TOPOLOGY IV ‖ EPIPHANY 2020
PROBLEM SHEET 5

Please hand in Problem 1 from this sheet and the two problems from the previous sheet on Monday 24th February, in the lecture or at my office by 5pm that day.

This problem sheet is on applications of Poincaré duality and the universal coefficient theorem.

Problem 1. Decide whether or not there exists a closed, orientable manifold M whose non-trivial homology groups are as listed; all other homology groups are trivial.

(a) $H_0(M) \cong H_5(M) \cong H_2(M) \cong H_3(M) \cong \mathbb{Z}$.

(b) $H_0(M) \cong H_4(M) \cong \mathbb{Z}$, $H_2(M) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}/2$.

Problem 2. Prove that the Euler characteristic of a closed, orientable manifold of odd dimension $2n + 1$ is zero.

Problem 3. Let X be a closed, oriented, connected 4-manifold with $H_1(X) = 0$ and $\chi(X) = 10$. What are the homology groups of X?