

ALGEBRAIC TOPOLOGY IV || MICHAELMAS 2019
PROBLEM SHEET 6

Please solve these problems during week 7. Problems 1 and 2 will form part of the next submission.

Problem 1. Prove the five lemma (you could copy it from somewhere but that seems pointless).

Five Lemma. Consider the commutative diagram of abelian groups and homomorphisms, with exact rows and vertical maps isomorphisms as shown.

$$\begin{array}{ccccccccc} A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & D & \longrightarrow & E \\ \downarrow \cong & & \downarrow \cong & & \downarrow & & \downarrow \cong & & \downarrow \cong \\ A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & D' & \longrightarrow & E' \end{array}$$

Then the remaining vertical homomorphism $C \rightarrow C'$ is a homomorphism.

Problem 2. Let G_* be a homology theory. Let $\Gamma := G_0(\{\text{pt}\})$. Compute the G_* homology of S^1 .

Problem 3. Deduce the Mayer Vietoris theorem from the axioms for homology.