ALGEBRAIC TOPOLOGY IV ‖ MICHAELMAS 2019
PROBLEM SHEET 7

Please solve these problems during week 8, in particular Problems 1 and 2 from the last sheet and Problems 1 and 2 from this sheet.

Problem 1. Show that the infinite sphere S^∞ is contractible.

Problem 2. Describe a cell structure on $\mathbb{RP}^n$ with one k cell for every $k = 0, 1, \ldots, k$.

Problem 3. Show that the group $\mathbb{Z}/2$ acts freely on S^2. Show that moreover $\mathbb{Z}/2$ is the only nontrivial group that can act freely on S^2.

A group G acts on a space X if for every $g \in G$ there is a homeomorphism $h_g \colon X \to X$ such that $h_e = \mathrm{Id}$ and $h_{g \cdot g'} = h_g \circ h_{g'}$. The action is free if h_g has no fixed points for every $g \neq e$ in G. Hint: use the degree to define a homomorphism $G \to \mathbb{Z}/2$.