Topologie Algébrique II: Problem sheet 2

Problem 1. Suppose that $f \sim g\colon X \to Y$. Show that the mapping cones are homotopy equivalent $C_f \simeq C_g$.

Problem 2. For every space X, show directly that the inclusion of X into its cone CX is a cofibration.

Problem 3. Show that the composition of two cofibrations is a cofibration.

Problem 4. Let $f\colon A \to X$ be any map, and let $M_f = X \cup_{A\times\{1\}} A \times I$ be the mapping cylinder. Show that $X \simeq M_f$, then show that the inclusion $i\colon A \to A \times \{0\} \subset M_f$ is a cofibration.

Problem 5. Let A be a contractible subspace of X and suppose that the inclusion is a cofibration. Show that the quotient map $X \to X/A$ is a homotopy equivalence.