Problem 1. We showed that $F \times B \to B$ is a fibration. Show that the lifted homotopy h need not be unique.

Problem 2. Show that pullbacks of fibrations are fibrations.

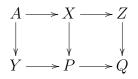
Problem 3. Show that the composition of two fibrations is a fibration.

Problem 4. Let \mathbb{F} be either \mathbb{R} , \mathbb{C} or \mathbb{H} , in which cases let d = 1, 2 or 4 respectively i.e. d is the real dimension of \mathbb{F} . For any $n \ge 1$, there is a map

$$\begin{array}{rccc} S^{d(n+1)-1} & \to & \mathbb{F}P^n \\ (x_0, \dots, x_n) & \mapsto & [x_0, \dots, x_n]. \end{array}$$

Here points on the sphere are coordinates in \mathbb{F}^{n+1} with $\sum_{i=0}^{n} |x_i|^2 = 1$, and the points of projective space are given in homogeneous coordinates. Show that these maps, called *Hopf fibrations* are fibrations. What are the fibres? Show that $\mathbb{F}P^1 \cong S^d$.

Problem 5. Consider the diagram



- (i) Suppose that the two small squares are pullback squares. Show that the large rectangle with corners A, Z, Y and Q is also a pullback square.
- (ii) Suppose that the large rectangle and the right hand square is a pullback square. Show that the left hand square is also a pullback square.