ERRATUM TO THE ARTICLE: EMBEDDED SURFACES WITH INFINITE CYCLIC KNOT GROUP.

ANTHONY CONWAY AND MARK POWELL

Given a $\mathbb{Z}$-surface $\Sigma \subset X$ with (possibly empty) connected boundary [CP23, Theorems 1.7 and 1.8] claim that any surface $\Sigma_n(\alpha, J)$ obtained by n-roll 1-twist rim surgery on Σ is topologically ambiently isotopic to Σ. These results and their proofs are correct for $n = 0$ (i.e. for surfaces obtained by 1-twist rim surgery) but are false in general for $n \neq 0$. We are grateful to Yujie Lin for bringing this mistake to our attention and for pointing us to the work of Teragaito that provides a counterexample [Ter94]. Namely, the proofs of [Ter94, Theorems 1, 2, and 3] show that there are examples of n-roll 1-twist knots in S^4 whose knot group is not $\mathbb{Z}$. The error in our proof originates in [CP23, Lemma 8.2] where we erroneously claim that an application of [Kim06, Proposition 3.3] shows that $\pi_1(\Sigma_n(\alpha, J)) \cong \mathbb{Z}$ for any $n \in \mathbb{Z}$. In fact [Kim06, Proposition 3.3] is only stated for $n = 0$ and the other reference we had in mind, namely the proof [KR08, Proposition 3.1], only works for $\mathbb{Z}_d$-surfaces with $\mathbb{Z}_d$ a finite group. No other theorem of the article is affected.

References

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The University of Texas at Austin, Austin TX 78712
E-mail address: anthony.conway@austin.utexas.edu

School of Mathematics and Statistics, University of Glasgow, United Kingdom
E-mail address: mark.powell@glasgow.ac.uk